

Validation of MC generators using new extractions of ^{12}C electromagnetic longitudinal and transverse response functions

We extract ^{12}C Longitudinal (\mathbf{R}_L) and (\mathbf{R}_T) Nuclear Electromagnetic Response Functions from *all* Electron Scattering Measurements on Carbon for:

1. Testing first principle nuclear theory predictions
2. Provide a platform for verification of electron and neutrino MC generators over the entire kinematic range of interest

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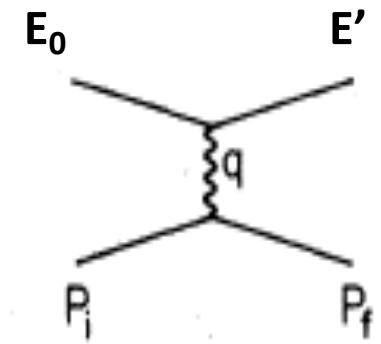
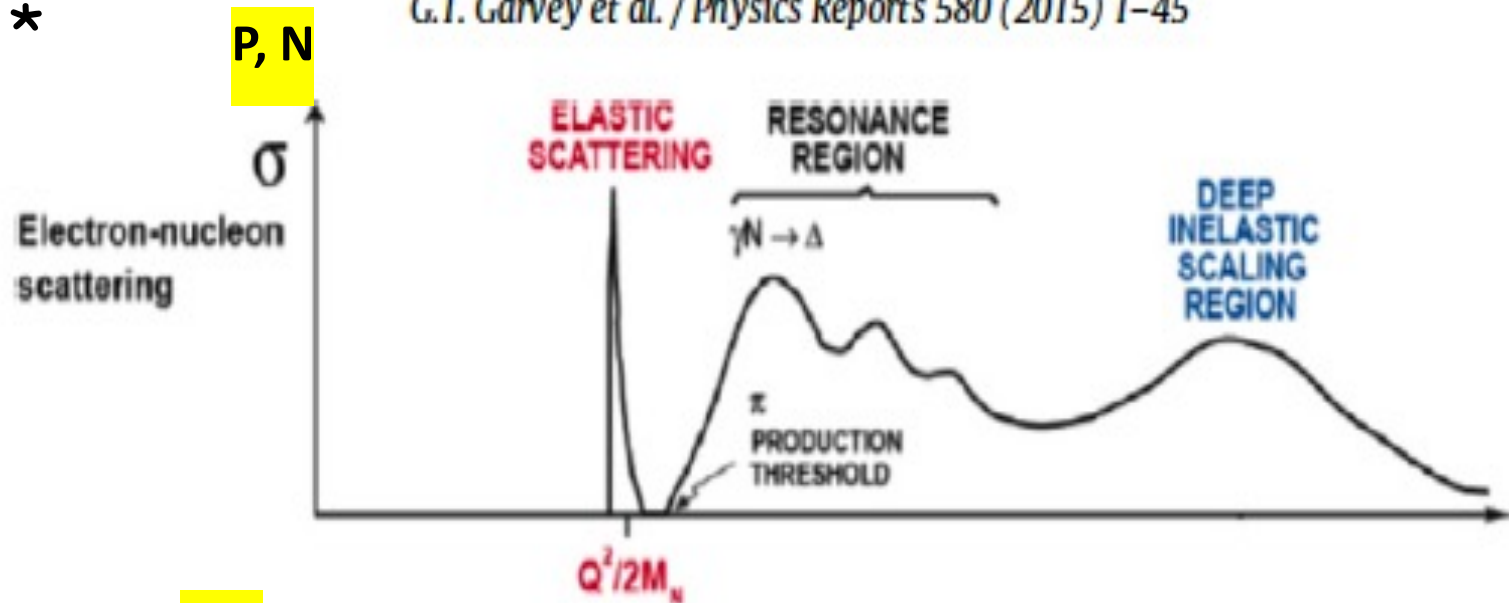
Presented by A. Bodek

2nd Short-Baseline Experiment-Theory Workshop 2024, April 2-5, Santa Fe, NM

25 min + 5 min questions

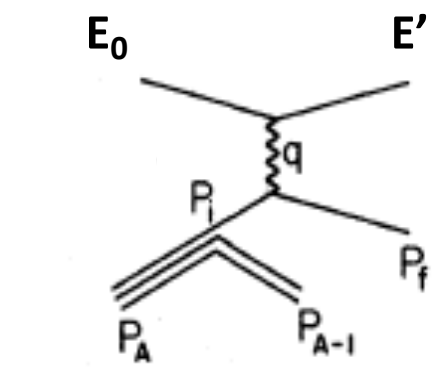
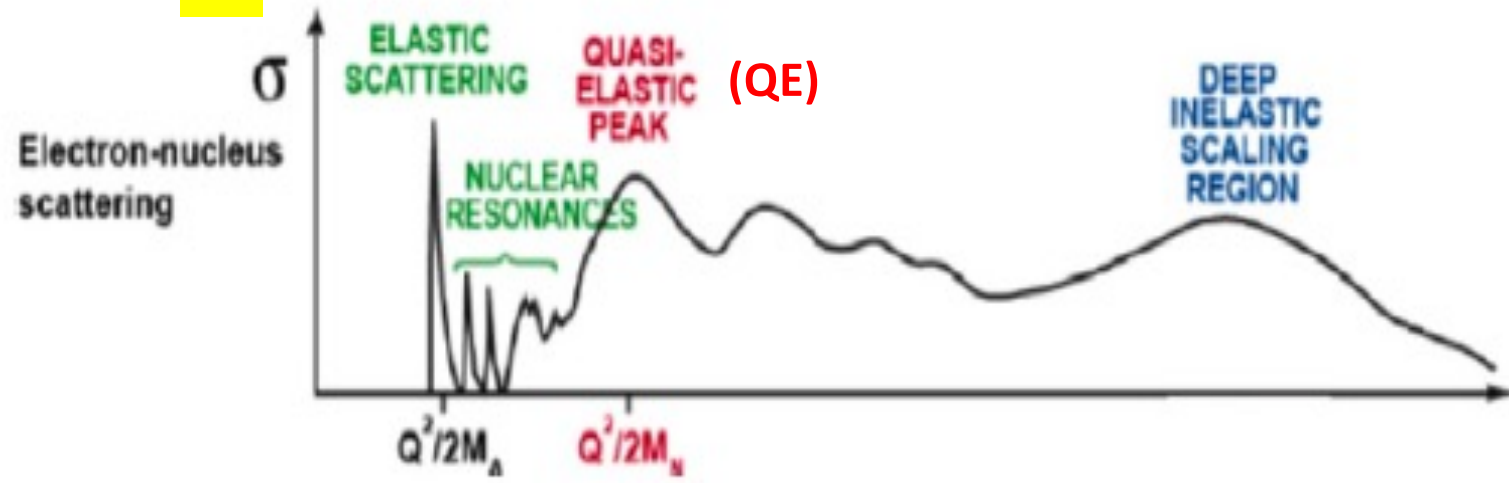
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(i) (or ν)

^{12}C



(i) (or ν)

Nuclear corrections affect enhance the Transverse Response (R_T) and quench the Longitudinal Response (R_L) in QE scattering. There are three different formalisms to parametrize these two responses. (1) R_L and R_T (2) F_1 and F_L (3) σ_L and σ_T .

In this analysis we use R_L and R_T .

★

Nuclear Physics

This description is primarily used in the nuclear excitation and QE regions. The electron scattering differential cross section is written in terms of longitudinal ($\mathcal{R}_L(Q^2, \nu)$) and transverse ($\mathcal{R}_T(Q^2, \nu)$) nuclear response functions [24] energy transfer = ν (or ω)

$$\frac{d\sigma}{d\nu d\Omega} = \sigma_M [A \mathcal{R}_L(Q^2, \nu) + B \mathcal{R}_T(Q^2, \nu)] \quad (20)$$

where σ_M is the Mott cross section, $A = (Q^2/\mathbf{q}^2)^2$ and $B = \tan^2(\theta/2) + Q^2/2\mathbf{q}^2$.

Particle Physics

This description is primarily used in the inelastic continuum region. In the one-photon-exchange approximation, the spin-averaged cross section for inclusive electron-proton scattering can be expressed in terms of two structure functions as follows

$$\frac{d\sigma}{d\Omega dE'} = \sigma_M [\mathcal{W}_2(W^2, Q^2) + 2 \tan^2(\theta/2) \mathcal{W}_1(W^2, Q^2)]$$

$$\sigma_M = \frac{\alpha^2 \cos^2(\theta/2)}{[2E_0 \sin^2(\theta/2)]^2} = \frac{4\alpha^2 E'^2}{Q^4} \cos^2(\theta/2) \quad (10)$$

$Q^2 = 4$ -momentum transfer squared

$$Q^2 = (-q)^2 = 4E_0 E' \sin^2 \frac{\theta}{2},$$

$$\nu = E_0 - E'$$

$W^2 =$ final state invariant mass squared

$$W^2 = M^2 + 2M\nu - Q^2$$

$\mathbf{q}^2 = 3$ -momentum transfer squared

$$\mathbf{q}^2 = Q^2 + \nu^2$$

$E_x =$ Excitation energy

$$E_x = \nu - \nu_{\text{elastic}}$$

$$x = Q^2 / (2M\nu).$$

$$\mathcal{F}_1 = M\mathcal{W}_1 \text{ and } \mathcal{F}_2 = \nu\mathcal{W}_2.$$

$$\mathcal{F}_L(x, Q^2) = \mathcal{F}_2 \left(1 + \frac{4M^2 x^2}{Q^2} \right) - 2x\mathcal{F}_1,$$

$$\mathcal{R}_T(\mathbf{q}, \nu) = \frac{2\mathcal{F}_1(\mathbf{q}, \nu)}{M}$$

$$\mathcal{R}_L(\mathbf{q}, \nu) = \frac{\mathbf{q}^2}{Q^2} \frac{\mathcal{F}_L(\mathbf{q}, \nu)}{2Mx}$$

σ_L and σ_T .

A. Description in terms of longitudinal and transverse virtual photon cross sections

This description is often used in the resonance region. In the one-photon-exchange approximation, the spin-averaged cross section for inclusive electron-proton scattering can be expressed in terms of the photon helicity coupling as

$$\frac{d\sigma}{d\Omega dE'} = \Gamma [\sigma_T(W^2, Q^2) + \epsilon\sigma_L(W^2, Q^2)], \quad (6)$$

where σ_T (σ_L) is the cross section for photo-absorption of purely transverse (longitudinal) polarized photons,

$$\Gamma = \frac{\alpha E' (W^2 - M_N^2)}{(2\pi)^2 Q^2 M E_0 (1 - \epsilon)} \quad (7)$$

is the flux of virtual photons, $\alpha = 1/137$ is the fine structure constant, and

$$\epsilon = \left[1 + 2 \left(1 + \frac{\nu^2}{Q^2} \right) \tan^2 \frac{\theta}{2} \right]^{-1} \quad (8)$$

is the relative flux of longitudinal virtual photons (sometimes referred to as the virtual photon polarization). Since Γ and ϵ are purely kinematic factors, it is convenient to define the reduced cross section

$$\sigma_r = \frac{1}{\Gamma} \frac{d\sigma}{d\Omega dE'} = \sigma_T(W^2, Q^2) + \epsilon\sigma_L(W^2, Q^2). \quad (9)$$

All the hadronic structure information is therefore, contained in σ_T and σ_L , which are only dependent on W^2

$$K = \frac{Q^2(1-x)}{2Mx} = \frac{2M\nu - Q^2}{2M} \quad (13)$$

$$\mathcal{F}_1 = \frac{MK}{4\pi^2\alpha} \sigma_T \quad (14)$$

$$\mathcal{F}_2 = \frac{\nu K(\sigma_L + \sigma_T)}{4\pi^2\alpha(1 + \frac{Q^2}{4M^2x^2})} \quad (15)$$

$$\mathcal{F}_L(x, Q^2) = \mathcal{F}_2 \left(1 + \frac{4M^2x^2}{Q^2} \right) - 2x\mathcal{F}_1, \quad (16)$$

$$\mathcal{R}_T(\mathbf{q}, \nu) = \frac{2\mathcal{F}_1(\mathbf{q}, \nu)}{M} = \frac{K}{2\pi^2\alpha} \sigma_T \quad (21)$$

$$\mathcal{R}_L(\mathbf{q}, \nu) = \frac{\mathbf{q}^2 \mathcal{F}_L(\mathbf{q}, \nu)}{Q^2} = \frac{\mathbf{q}^2}{Q^2} \frac{K}{4\pi^2\alpha} \sigma_L \quad (22)$$

★

- **We have initiated a program to extract RL and RT values on various nuclei using all available data.**
Here we report on our extraction for Carbon.
- We extract RL and RT in all regions, nuclear elastic, nuclear excitations, quasielastic, resonance region and inelastic scattering.
- We extract at 18 values of fixed Q^2 values $0 < Q^2 < 3.45 \text{ GeV}^2$ as a function of energy transfer ν .
- And at 18 values of fixed values of 3-momentum transfer $0.1 < \mathbf{q} < 2.78 \text{ GeV}$ as a function of ν
- We energy transfer ν , from $\nu=0$ to the end of the resonance region $W=2.0 \text{ GeV}$

For Carbon we use 16,000 electron scattering and photoproduction cross sections

Goals:

- Test first-principle nuclear theories at fixed values of Q^2 and \mathbf{q} .
- Use extracted RL and RT values to validate MC generators
Since we covers all kinematic regions this is a much preferable way than comparison with a few cross section measurements in a limited sets of kinematic regions.
- **Where there is no data, we provide the values from our universal fit to all electron scattering data.**
The fit will be also be made available for validation of electron and neutrino MC generators

★ **First Step: Christy- Bodek Universal Fit (needed for this analysis))**

- We update the Bosted-Christy fit to **all of the world's electron scattering data** on H, D and nuclear targets to include the **lowest values of energy transfer ν and q^2** (for ^{12}C we fit **~16,000 electron scattering and photoabsorption cross sections**).
- We fit for: **QE cross section (including Transverse Enhancement/MEC and longitudinal low q Quenching)**. **Resonance and pion production, DIS, nuclear excitations**, elastic scattering data. Since the cross sections span a large range of energies and scattering angles, we **extract both the longitudinal RL and transverse RT** contributions, and also get the Coulomb Sum Rule.
- We parameterize both the **Enhancement of the Transverse QE cross section** and the **Quenching of the Longitudinal QE cross section**. **We extract the most precise Coulomb Sum rule as a function of q and compare to theoretical calculations.**
- **The fit can be used in lieu of data to benchmark Monte Carlo predictions** (e.g. for e-H, e-D and e- ^{12}C and e- ^{16}O cross sections, and to is being used **compute radiative corrections for electron scattering experiments.**

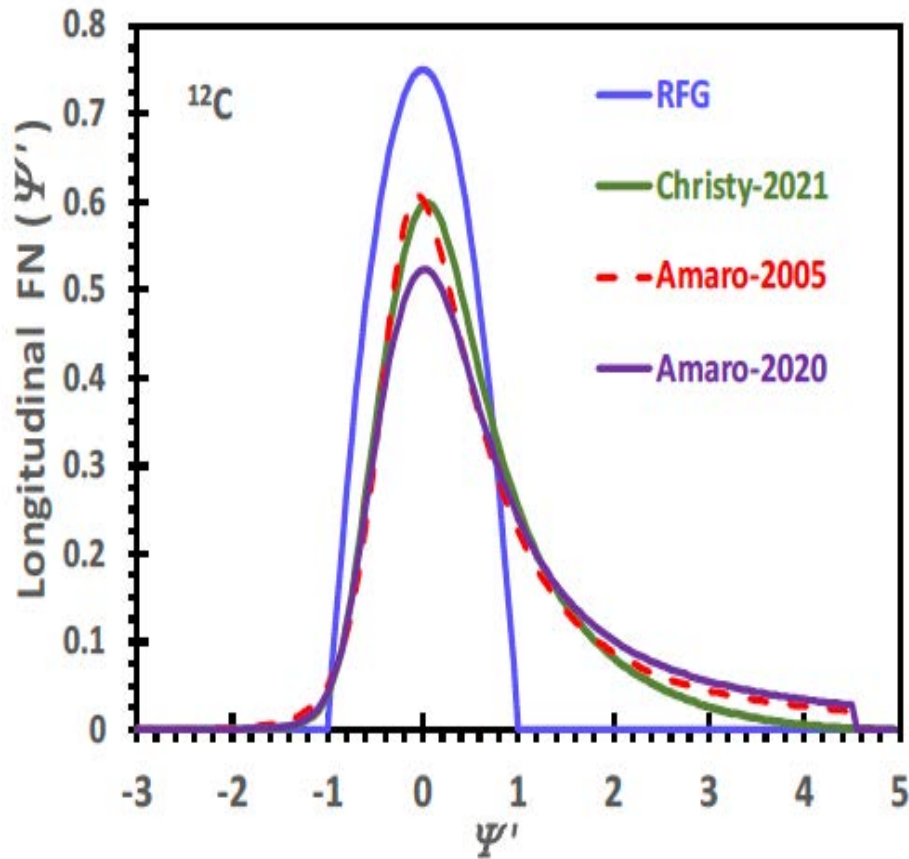
Modeling QE:

Use superscaling- Fit for the longitudinal scaling function parameters in the overall fit

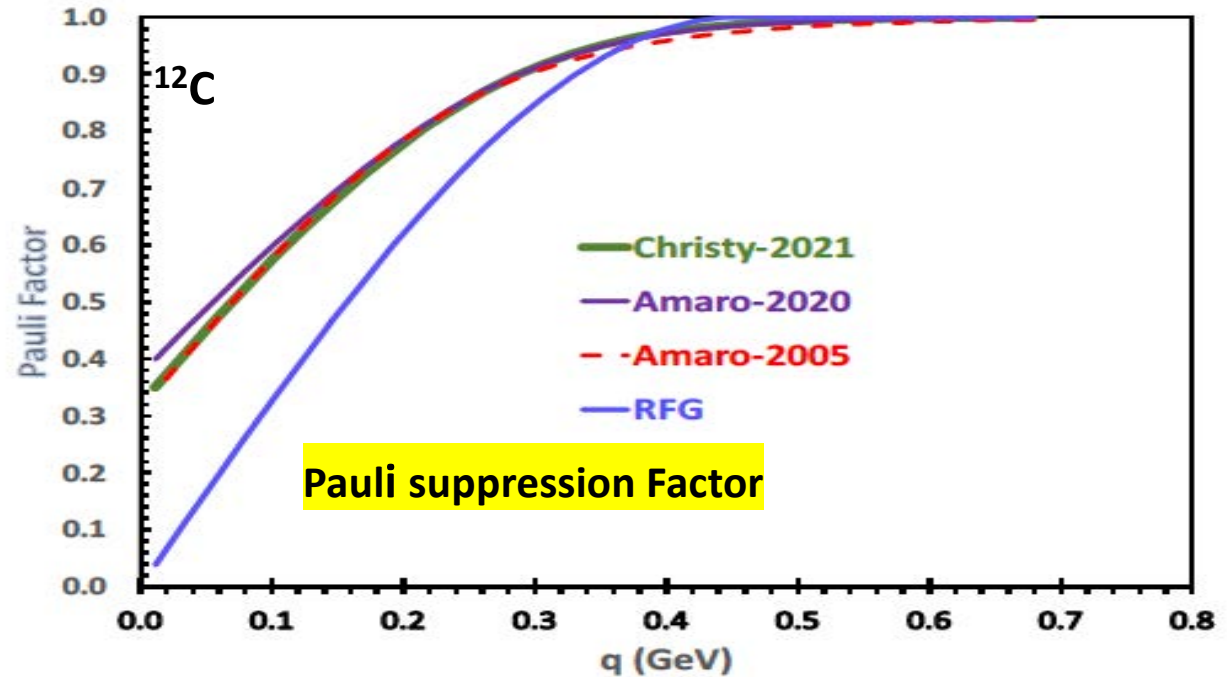
The ψ' superscaling variable is given by the following expression:

$$\psi' \equiv \frac{1}{\sqrt{\xi_F}} \frac{\lambda' - \tau'}{\sqrt{(1 + \lambda')\tau' + \kappa\sqrt{\tau'(1 + \tau')}}}, \quad (16)$$

where $\xi_F \equiv [\sqrt{1 + \eta_F^2} - 1]$, $\eta_F \equiv K_F/M_n$, $\lambda \equiv \nu/2M_n$, $\kappa \equiv |q|/2M_n$ and $\tau \equiv |Q^2|/4M_n^2 = \kappa^2 - \lambda^2$.

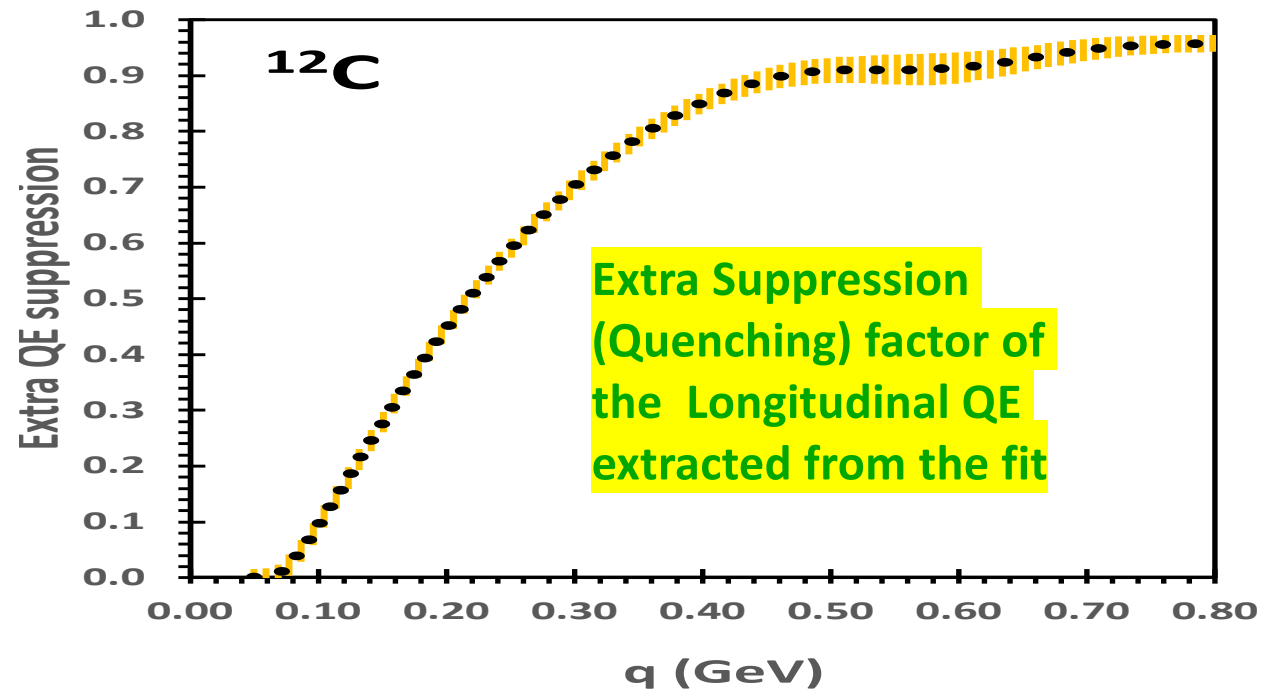


Pauli Suppression Factor. We use the Rosenfelder method.



We find that there is Quenching of the Longitudinal QE cross section (in addition to Pauli) extracted from our fit

Fit is used to remove data sets which do not agree with world data (2 for C12)



★ Christy- Bodek Universal Fit: Carbon
In the Nuclear excitation region

Cross sections for
excitations less than 10
MeV multiplied by (1/6)

Nuclear excitation region

Ex < 50 MeV

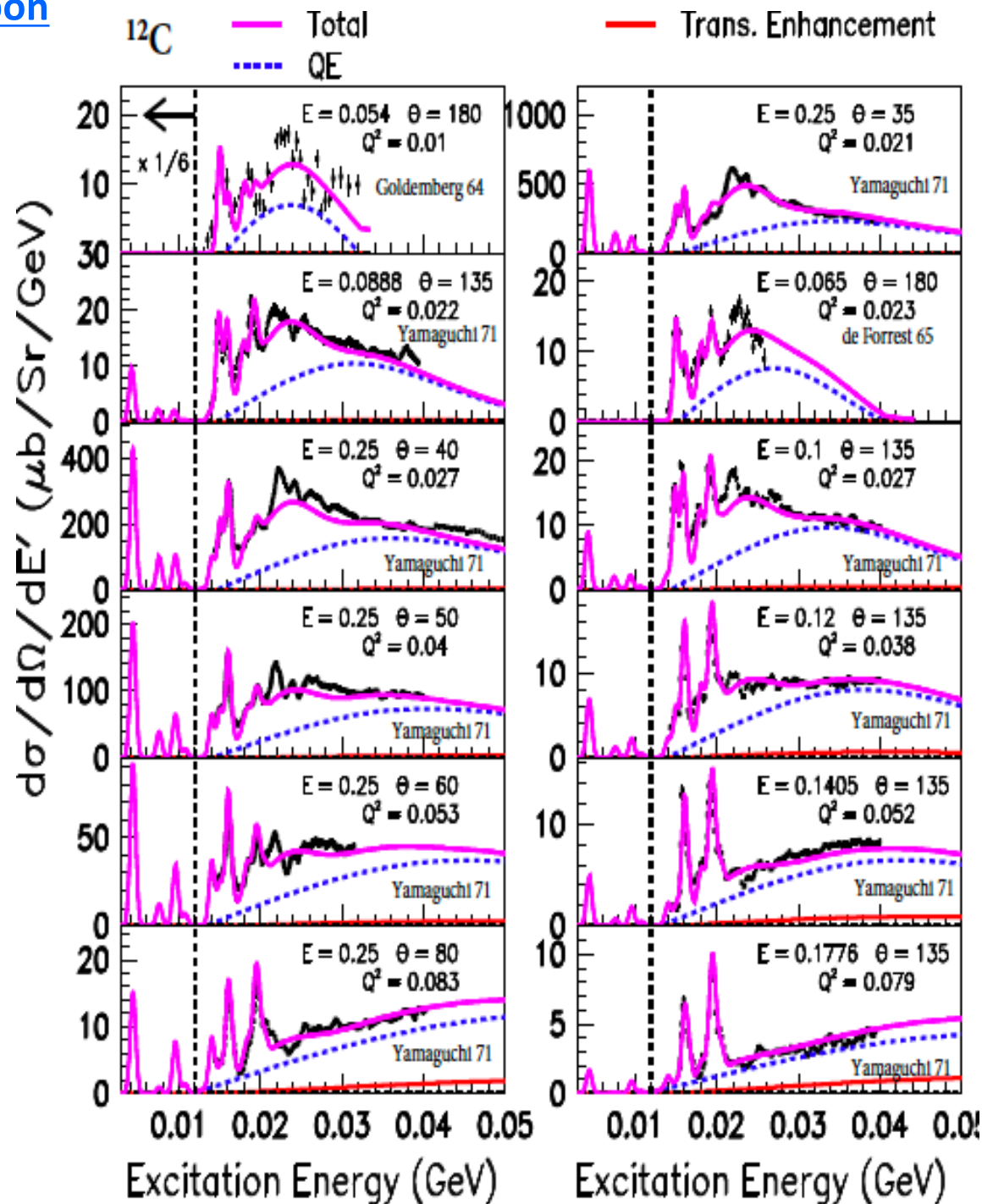
Comparison of our fit to
representative e-C12 data for
 $0.01 < q^2 < 0.08 \text{ GeV}^2$.

Shown: Total including excitations
: solid -----

Quasielastic (QE) contribution:
dashed-----

Transverse Enhancement at large
angles accounts for Meson
Exchange Currents and
Enhancement of Transverse QE
response dashed-----

Electron scattering cross sections



Christy- Bodek Universal Fit

Quasielastic (QE) Region-I

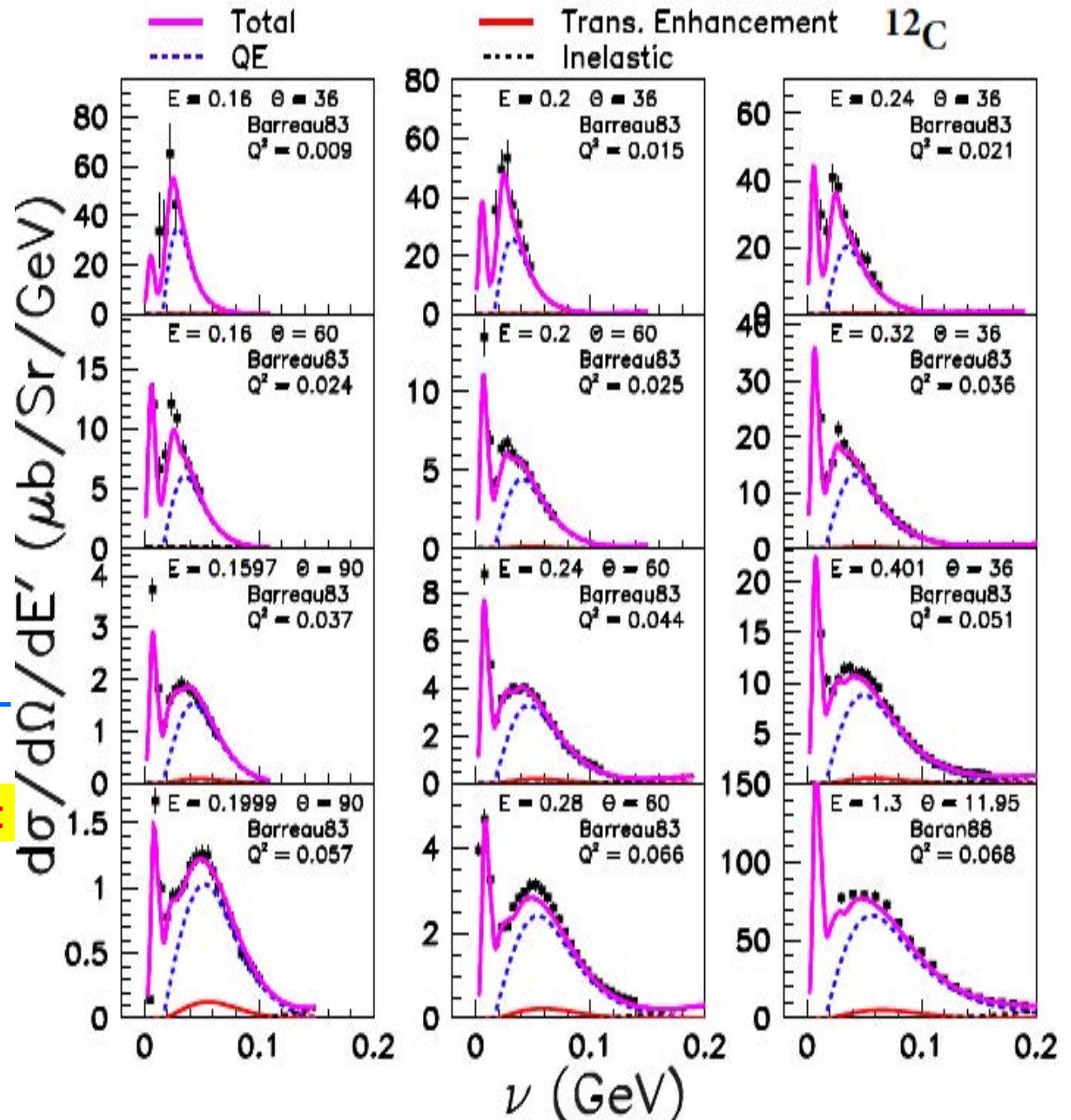
Comparison of our fit to representative e-C12 data

For $\nu < 0.2$ GeV and $0.01 < q^2 < 0.068$ GeV².

Shown: **Total including excitations** solid -----

Quasielastic (QE) contribution dashed -----

Transverse Enhancement at large angles accounts for Meson Exchange Currents and Enhancement of Transverse QE response dashed-----



Christy- Bodek Universal Fit

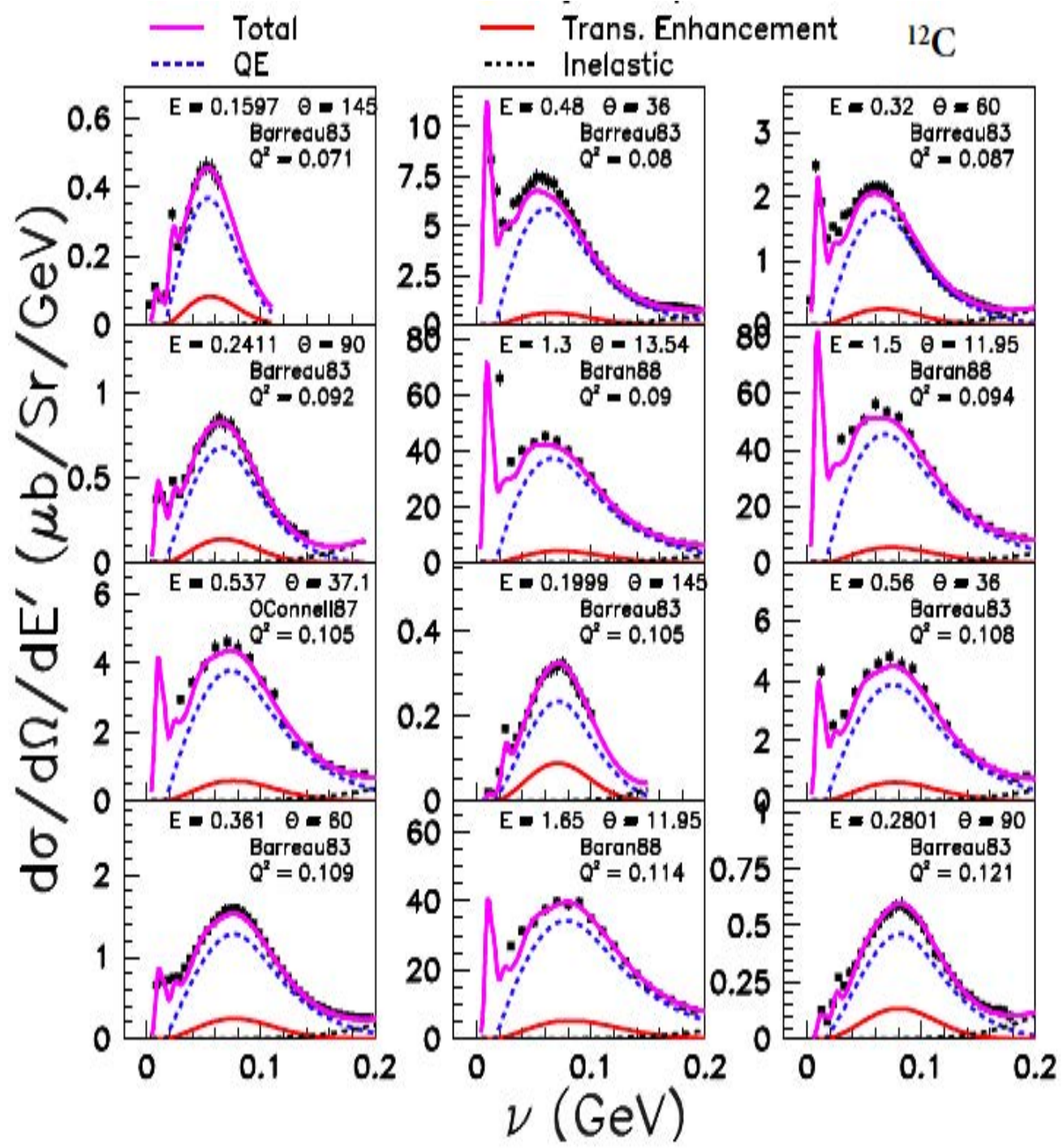
Quasielastic (QE) Region II

Comparison of our fit to representative e-C12 data for $< 0.2 \text{ GeV}$ and $0.071 < q^2 < 0.121 \text{ GeV}^2$.

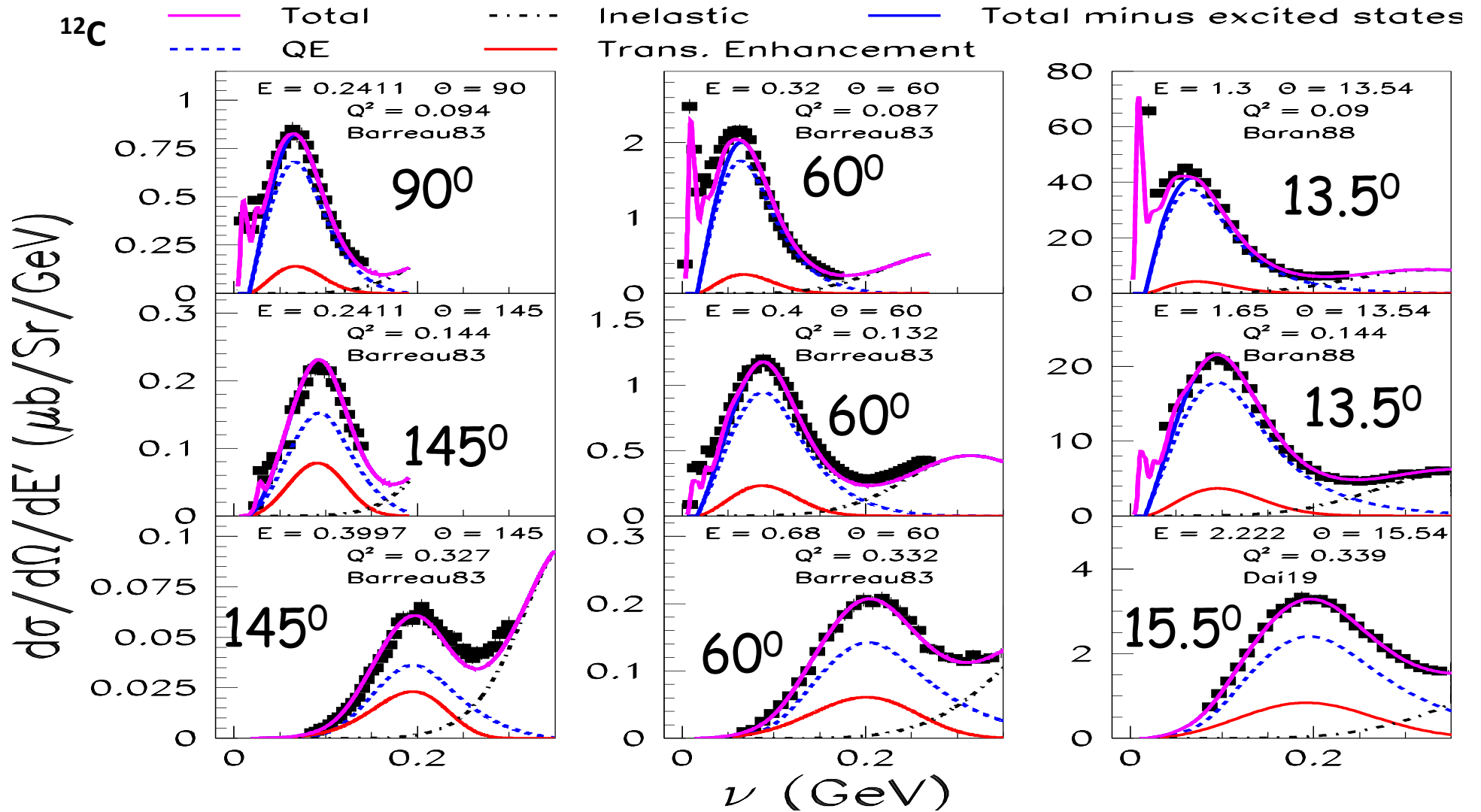
Shown: Total including excitations solid -----

Quasielastic (QE) contribution dashed -----

Transverse Enhancement at large angles accounts for Meson Exchange Currents and Enhancement of Transverse QE response dashed-----



Christy- Bodek Universal Fit - Fit functional forms for RL RT



The overall fit provides R_L and R_T at all values of q

Shown are large and small angle cross sections at the same q that provide the major contribution to the extraction of R_L and R_T at

$q^2=0.09, 0.15$ and 0.35 GeV² ($q=0.3, 0.38$ and 0.57 GeV)

* NEW: Individual R_L R_T extractions (Rosenbluth plots)

We apply Coulomb corrections in the analysis. We bin all cross sections in bins of q (we also do it for bins in Q^2) and apply bin centering corrections using the universal fit. Bin centers are at:

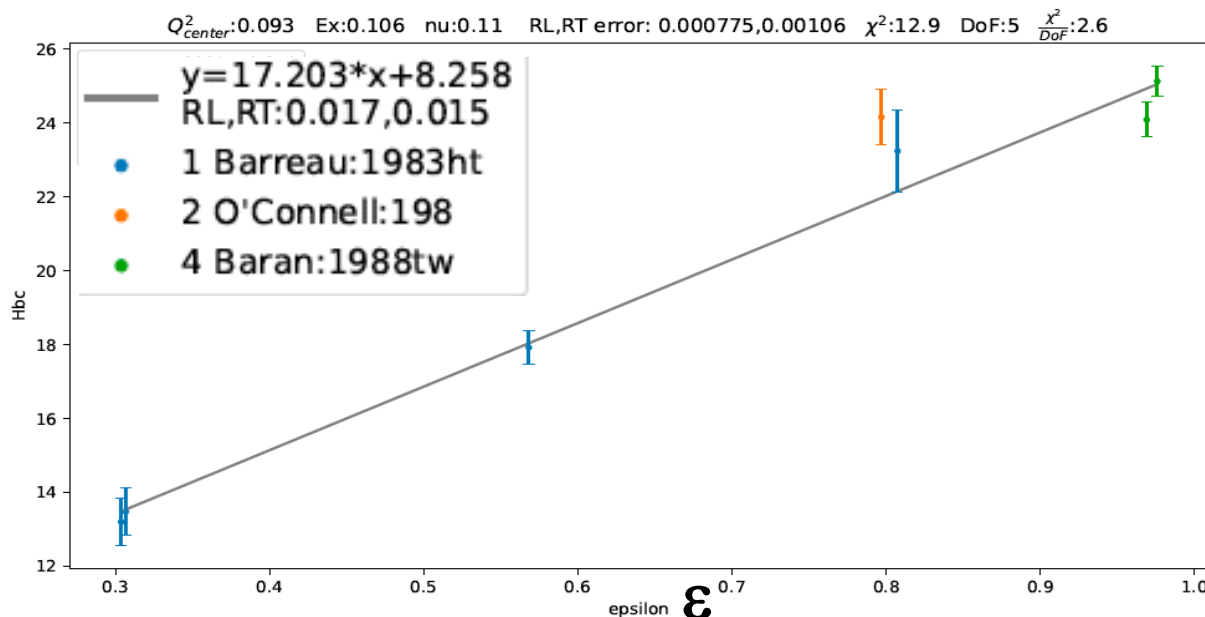
18 q values: 0.100, 0.148, 0.167, 0.205, 0.240, 0.300, 0.380, 0.475, 0.570,

0.649, 0.756, 0.991, 1.659, 1.921, 2.213, 2.500, 2.783, 3.500 GeV

18 Q^2 values: 0.00 (photoproduction), 0.010, 0.020, 0.026, 0.040, 0.056, 0.093, 0.120, 0.160, 0.265, 0.38, 0.50, 0.80, 1.25, 1.75, 2.25, 2.75, 3.25, 3.75 GeV²

2. Bin centering correction in q (or Q^2). For $\nu < 50$ MeV we bin-center the data in bins of Excitation energy E_x , and for $\nu > 50$ MeV we bin-center the data in bins of W^2 . We then perform Rosenbluth fits versus virtual photon polarization ϵ extract R_L and R_T . Later convert E_x and W^2 to ν .

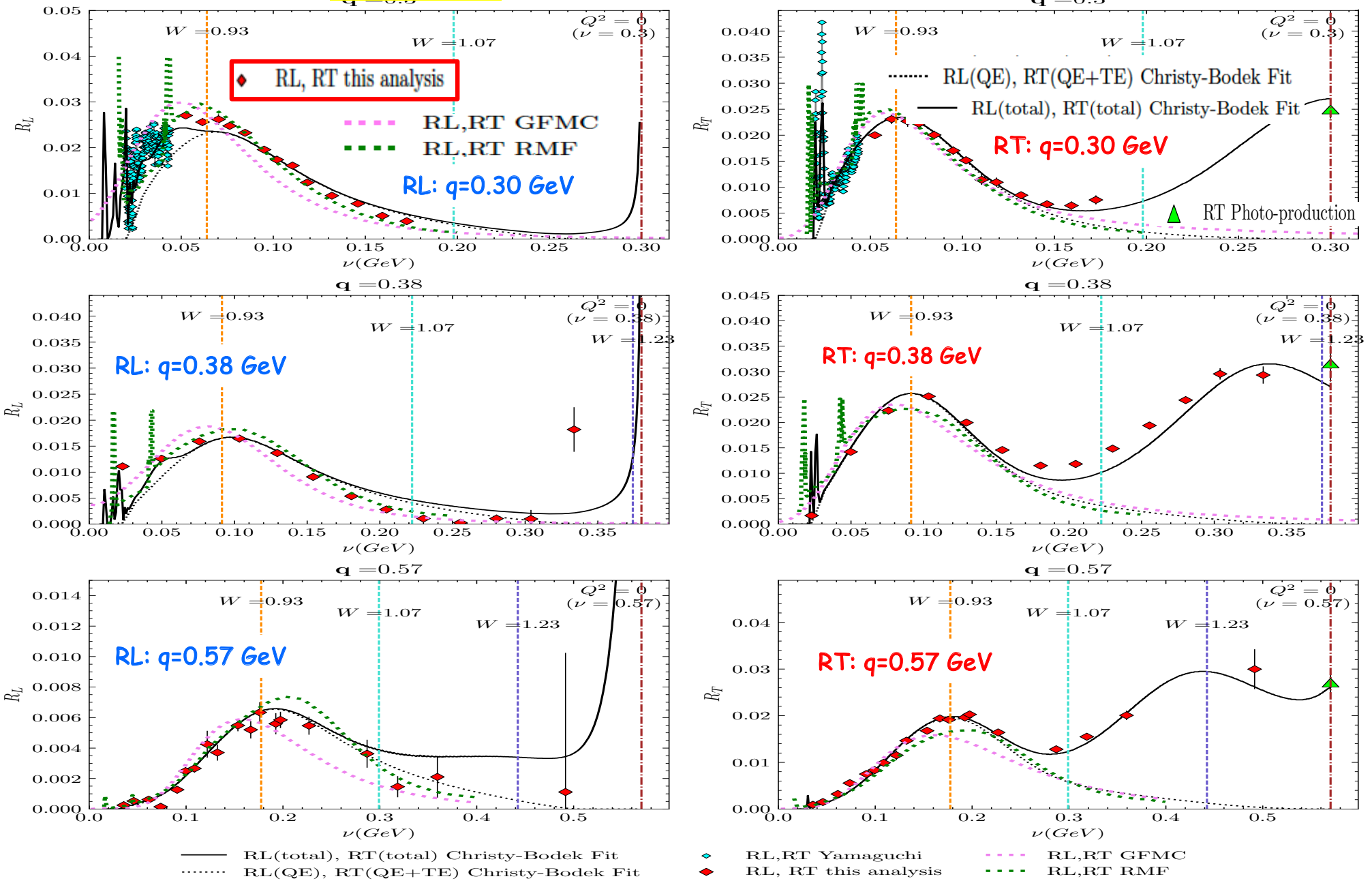
2. The universal fit includes all **16,000 cross sections**, The individual R_L R_T only use a **subset of the data** in which for the same values ν and Q^2 there is data spanning a significant range of angles. Therefore the Universal Fit provide R_L and R_T for a more extended kinematic range



ϵ = virtual photon polarization

$$\epsilon = \left[1 + 2 \left(1 + \frac{\nu^2}{Q^2} \right) \tan^2 \frac{\theta}{2} \right]^{-1}$$

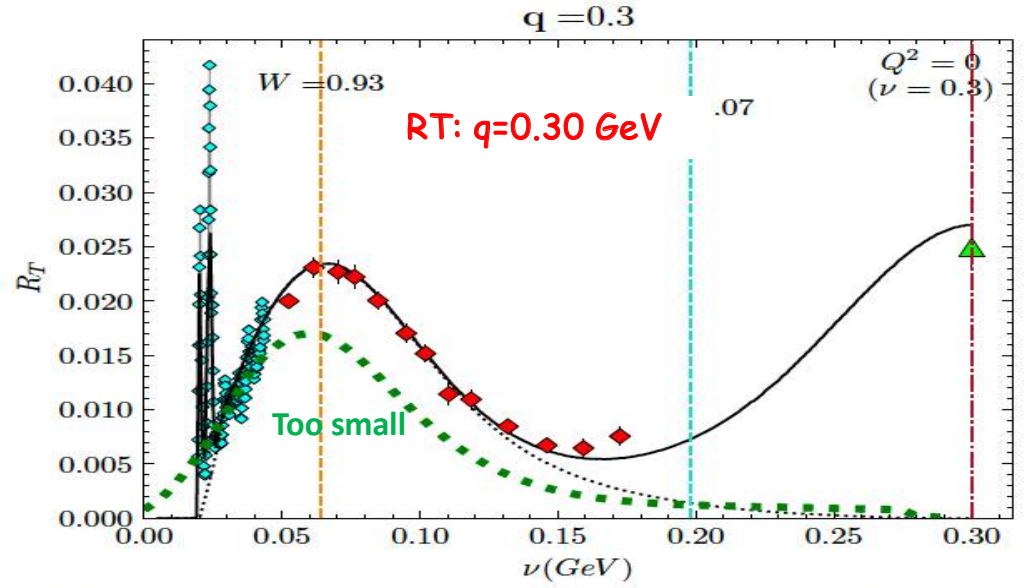
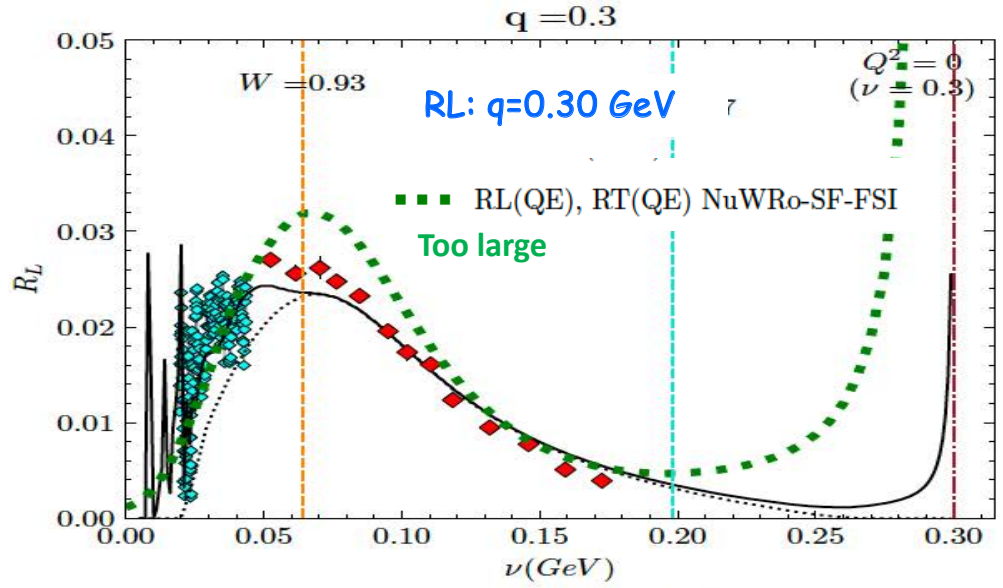
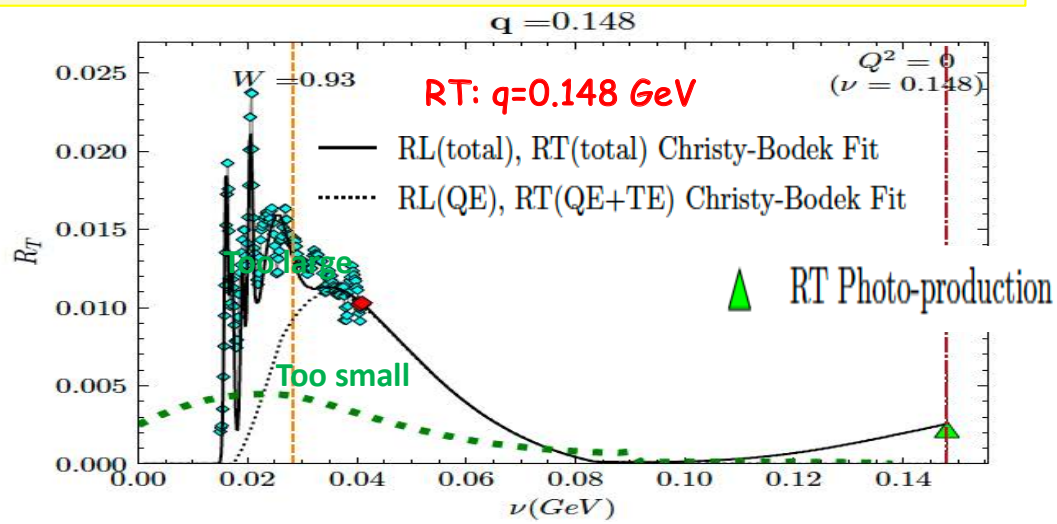
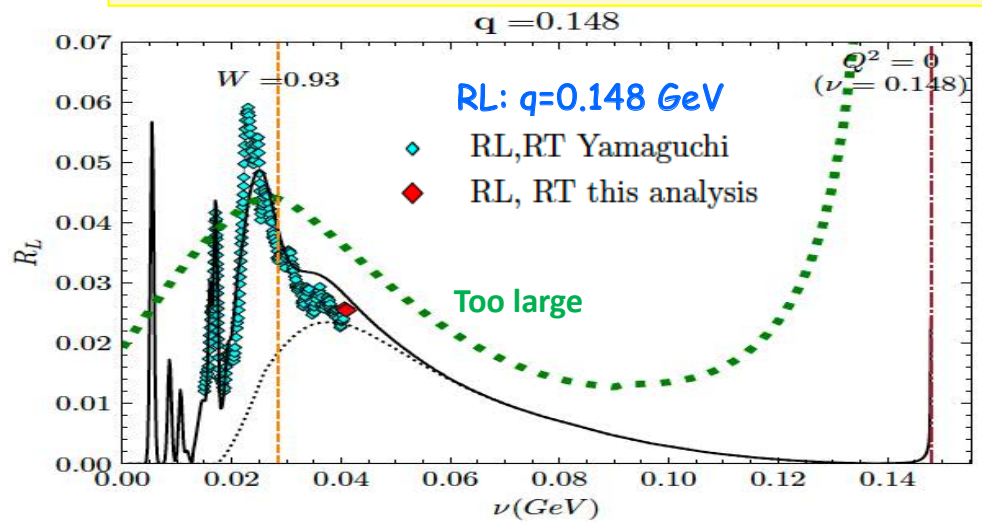
*Comparison to nuclear theory for QE 1p1h process (available for $q = 0.3, 0.38, 0.57$ GeV) only



Shown are theory predictions for QE 1p1h process (including contribution from 1b and 2b currents).
 GFMC (Green's Function Monte Carlo – first principle)..... No nuclear states
 ED-RMF (Energy Depending Relativistic mean field Nuclear states grouped at 2 values of fixed
 QE predictions reasonable but not perfect

* Comparison to **NuWro Neutrino MC generator** for QE (in electron scattering mode)

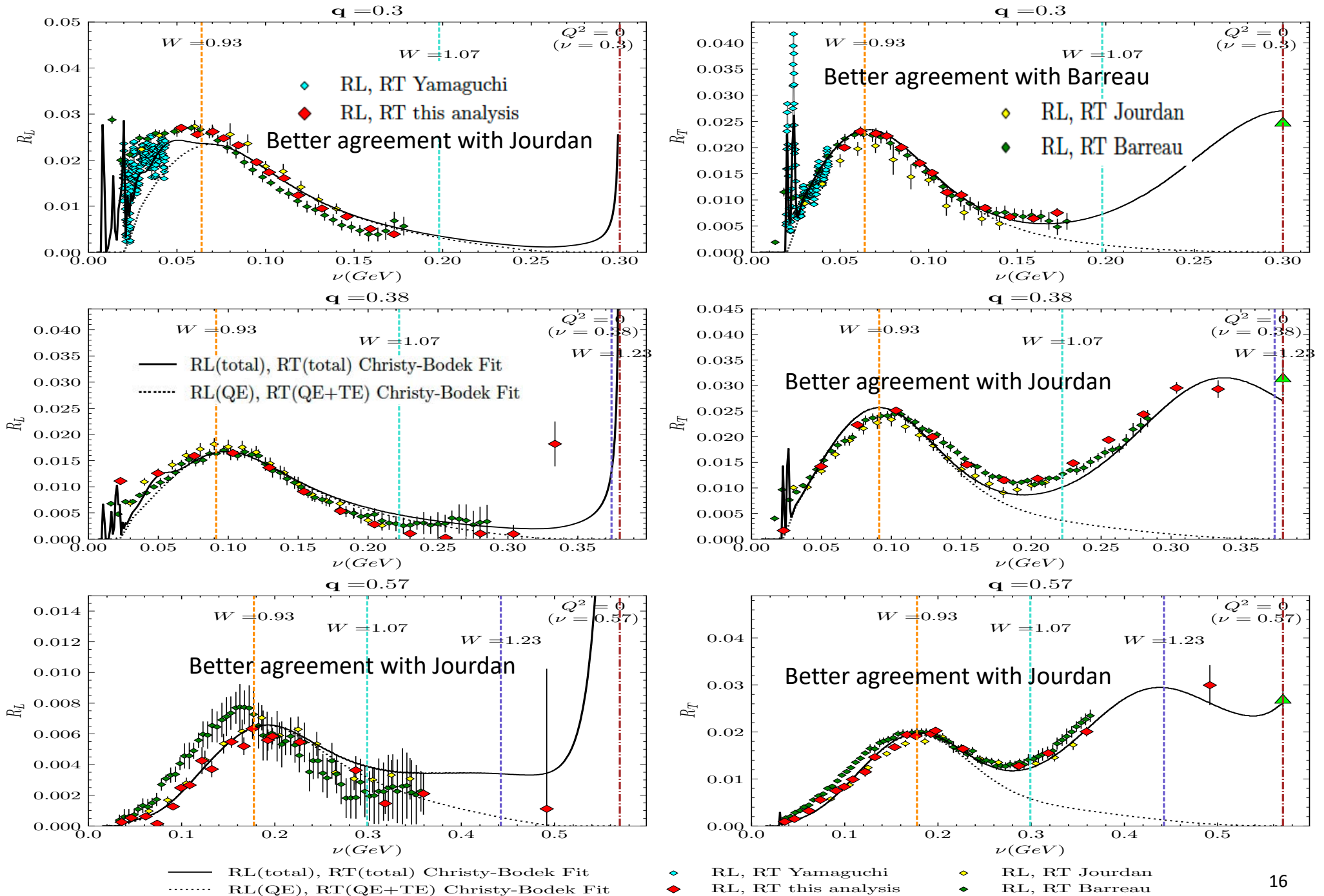
NuWro; R_L is too high (needs longitudinal quenching), R_T is too low (needs Transverse Enhancement/MEC). No nuclear excitations are modeled in any NuWro. Note: **NuWro has Transverse Enhancement/MEC in neutrino but not in electron mode.** ---If MEC is implemented in electron mode, these data can be used to validate the model.



— RL(total), RT(total) Christy-Bodek Fit \diamond RL,RT Yamaguchi \blacksquare RL(QE), RT(QE) NuWro-SF-FSI
 ⋯ RL(QE), RT(QE+TE) Christy-Bodek Fit \blacklozenge RL, RT this analysis \blacktriangle RT Photo-production

Comparisons with GENIE (in electron mode) will be available shortly

Comparison to previous $R_L R_T$ extractions $q = 0.3, 0.38, 0.57$ GeV



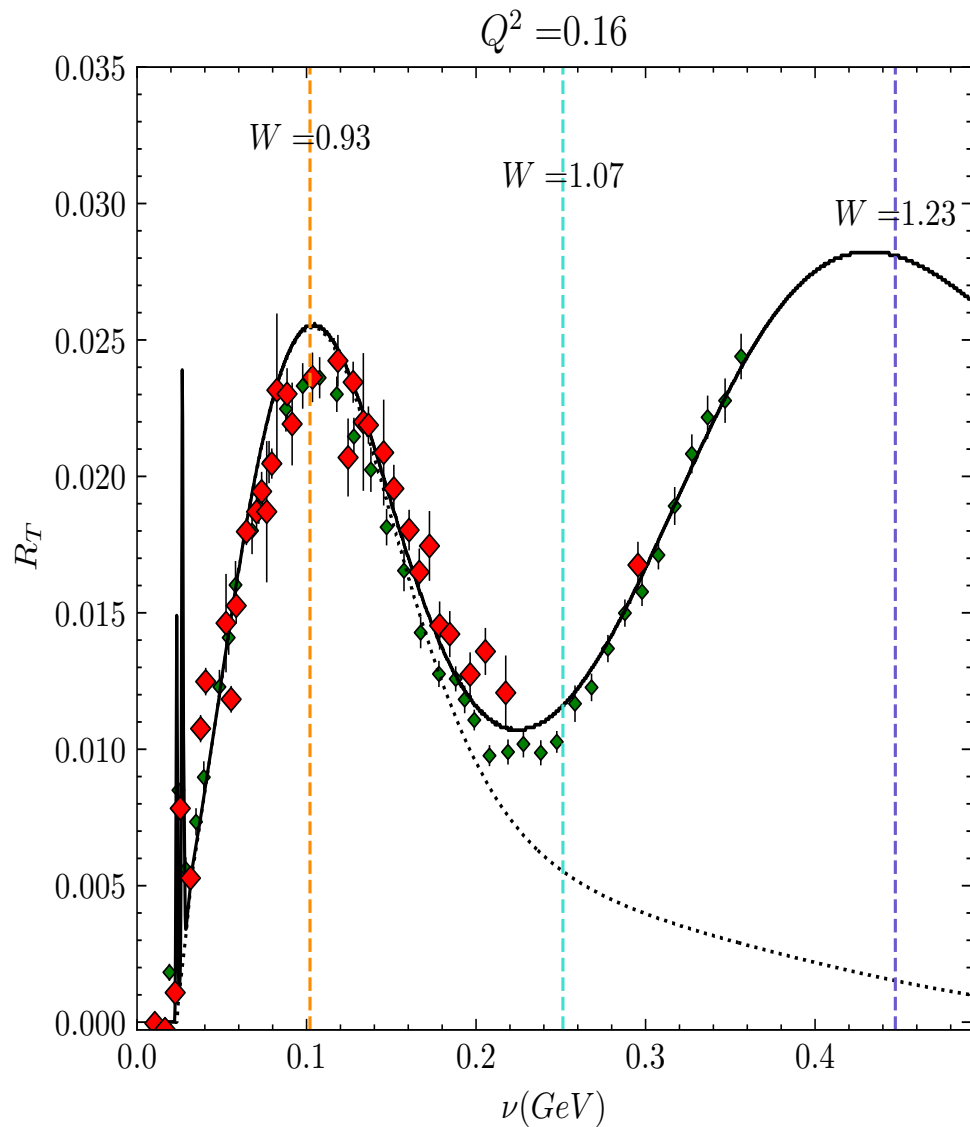
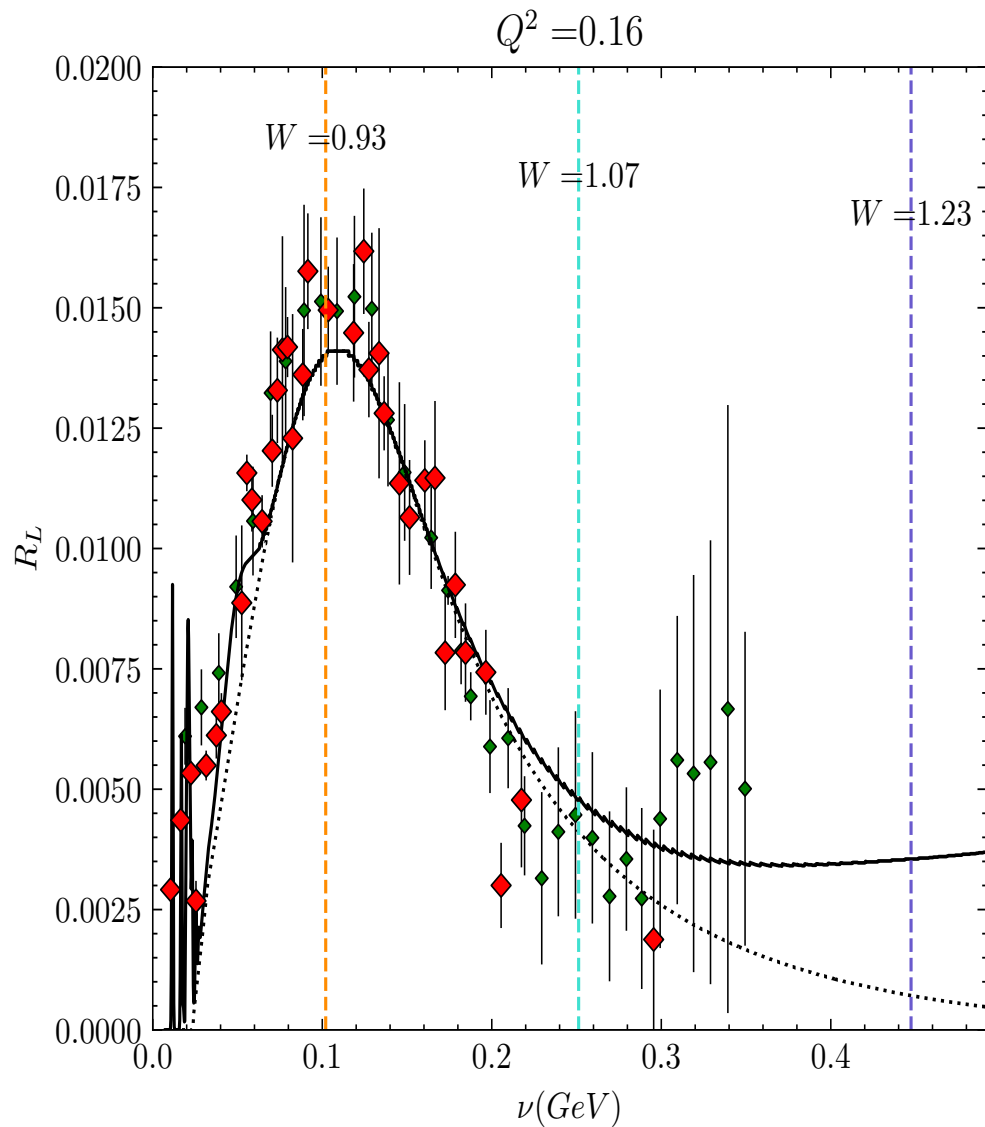
Comparison to previous R_L R_T extractions $Q^2=0.16 \text{ GeV}^2$

◆ R_L, R_T this analysis

◆ R_L, R_T Barreau

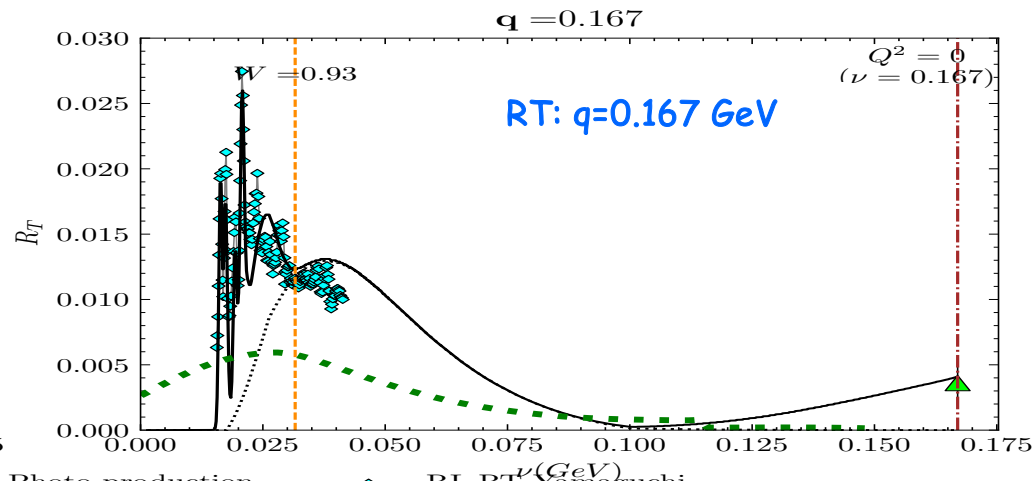
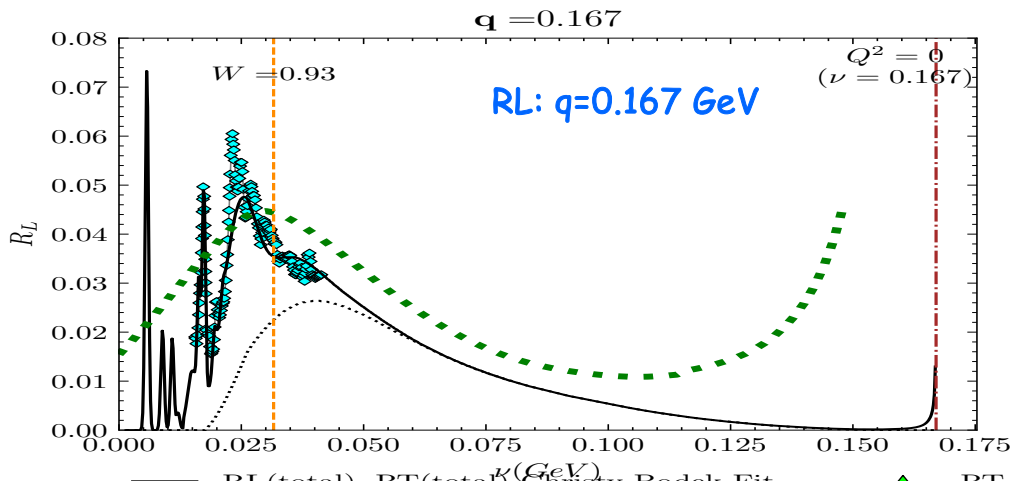
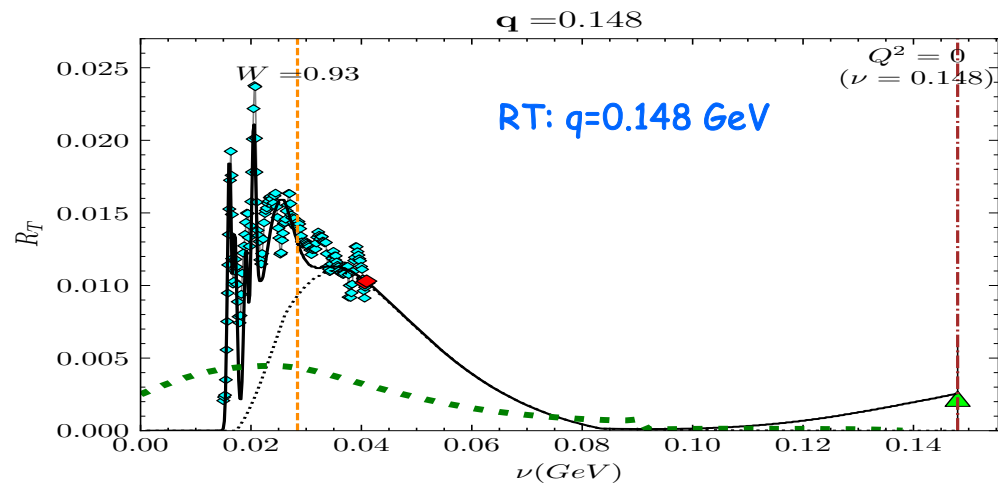
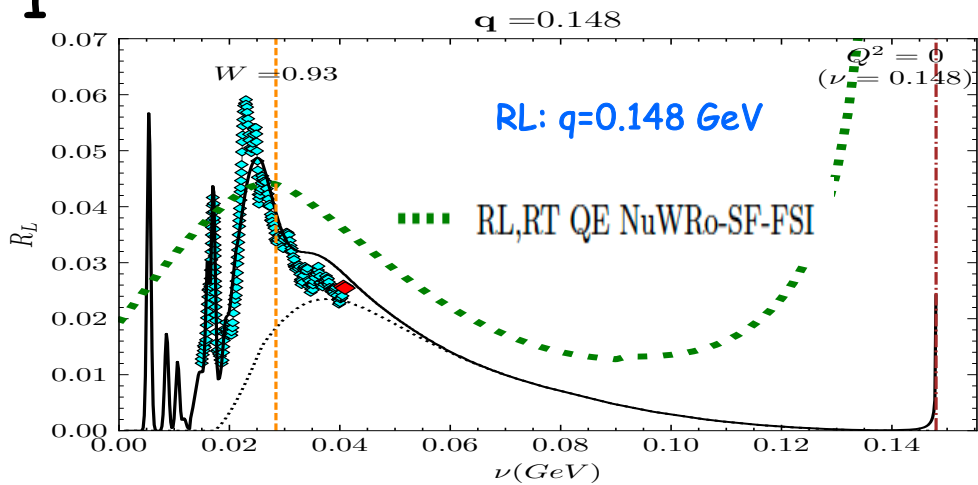
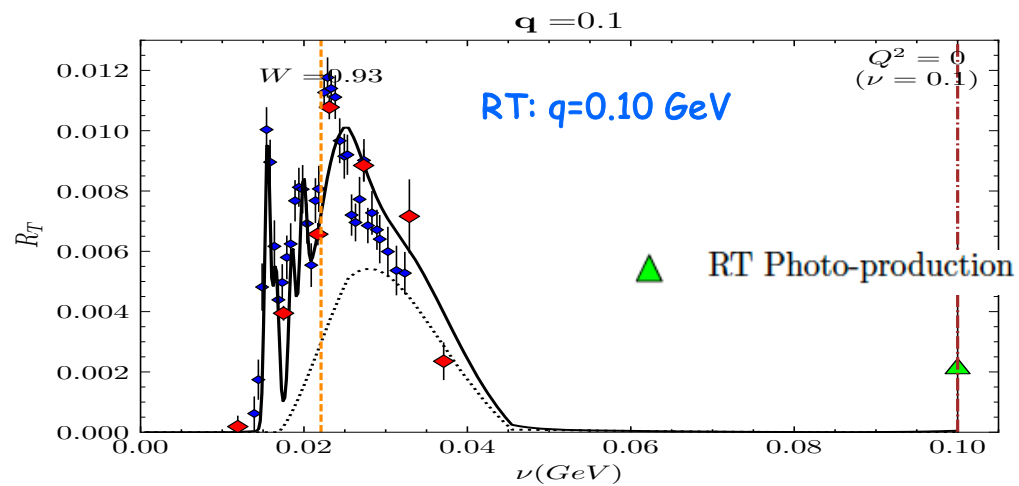
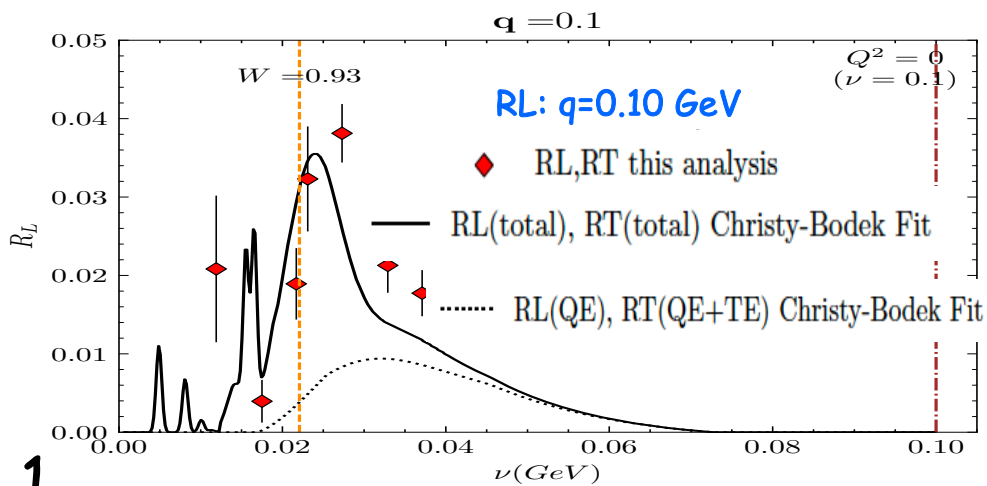
— $R_L(\text{total}), R_T(\text{total})$ Christy-Bodek Fit

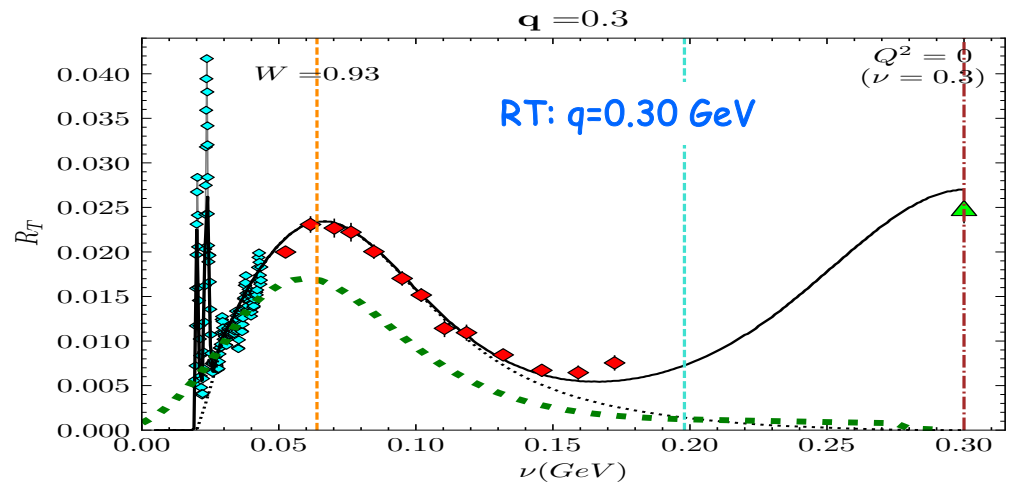
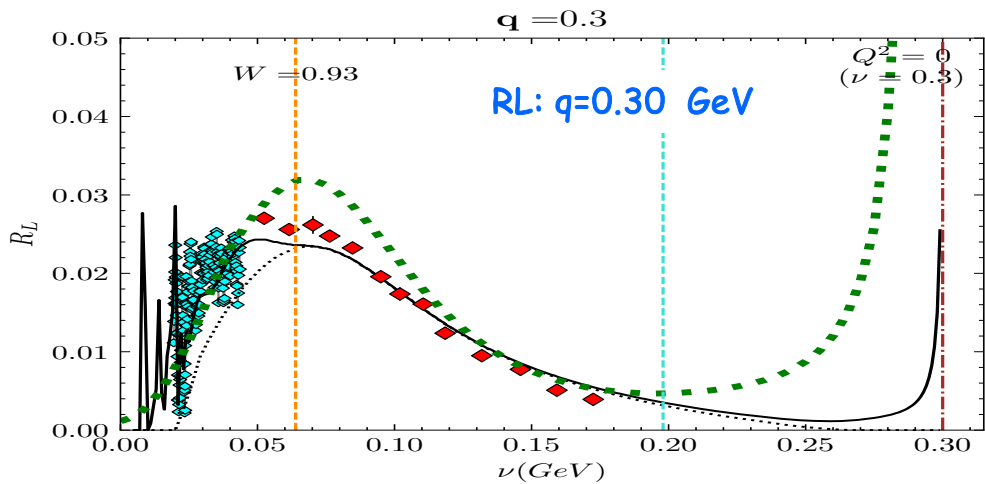
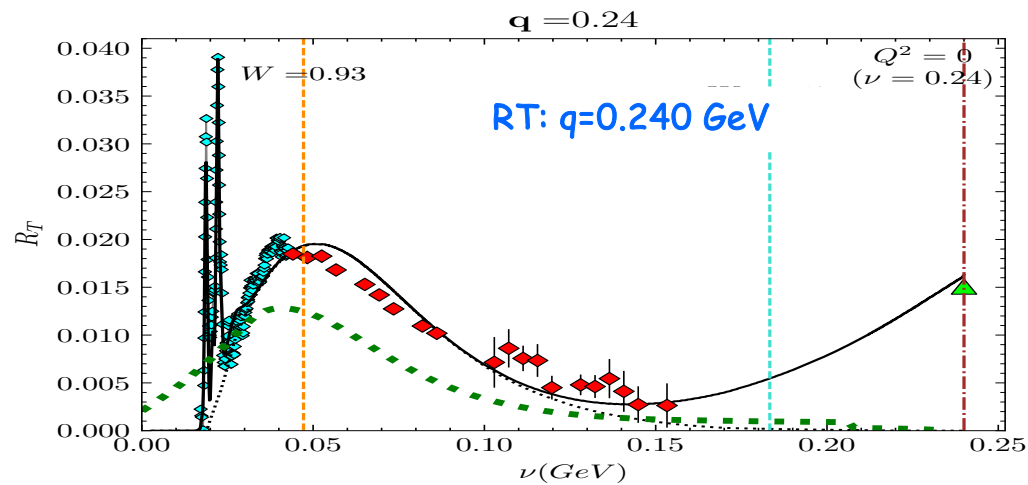
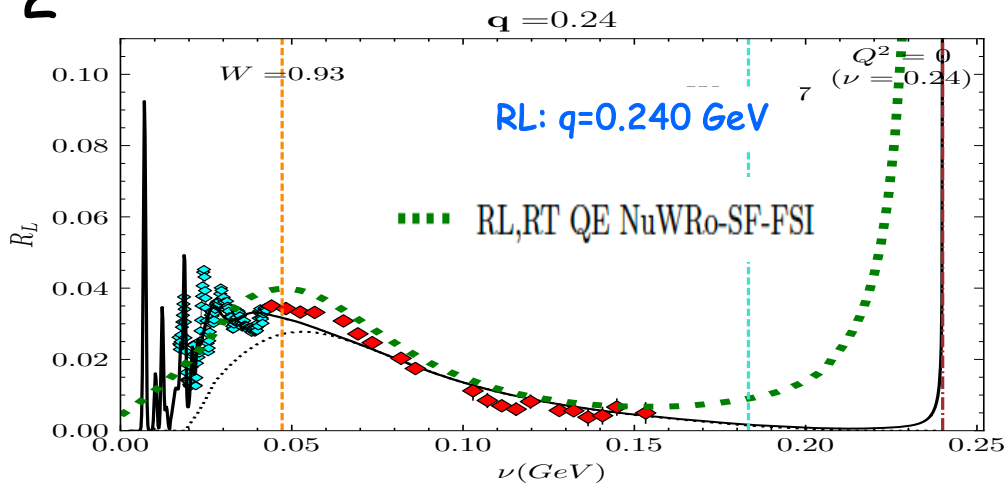
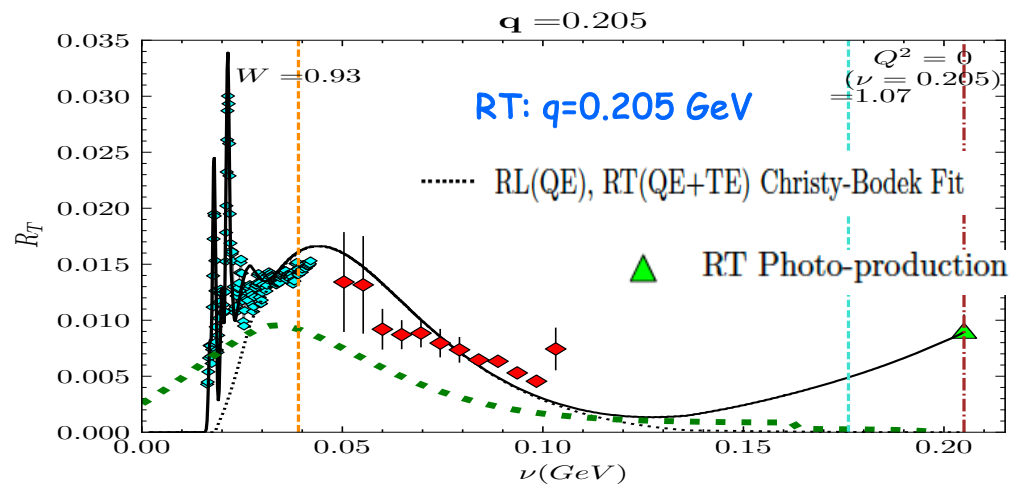
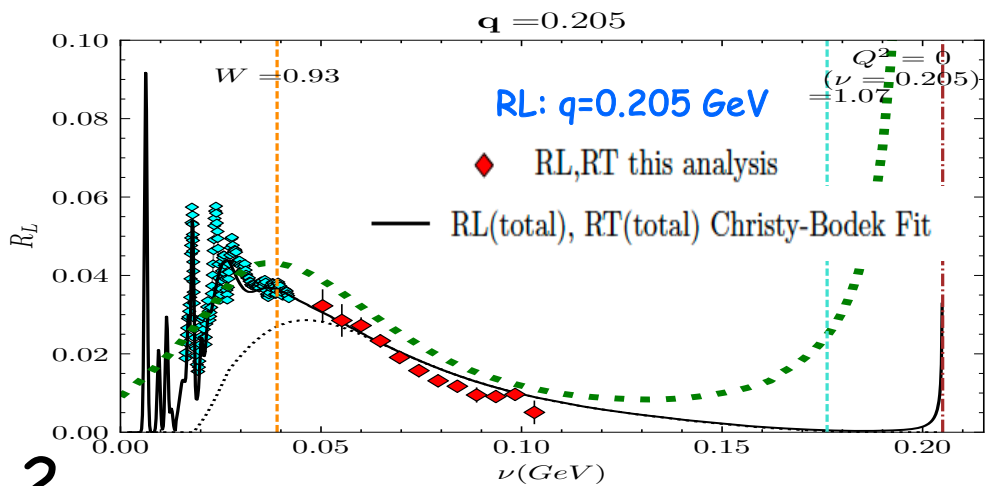
⋯ $R_L(\text{QE}), R_T(\text{QE+TE})$ Christy-Bodek Fit



Extraction of Individual Measurements of R_L and R_T

- The most effective way to validate **theoretical models** and neutrino and electron scattering MC generators is to compare to R_L and R_T measurements at fixed values of \mathbf{q} (or Q^2) versus ν . This way, the entire kinematic range at all scattering angles can be validated.
- The Christy-Bodek universal fit provides fits to R_L and R_T everywhere. However, it is also important to validate the universal fit everywhere.
- The Christy-Bodek universal fit includes all 16,000 cross sections measurements. In order to extract we need cross section measurements at the same \mathbf{q} (or Q^2) and ν , but at both at small and large.
- Therefore, only a subset of the cross sections can be used in individual R_L and R_T extractions since some cross sections are **only measured at small angles** and some are **only measured at large angles** (but at different \mathbf{q} (or Q^2) and ν). However, all cross sections are included in the Christy-Bodek universal fit .





— RL(total), RT(total) Christy-Bodek Fit

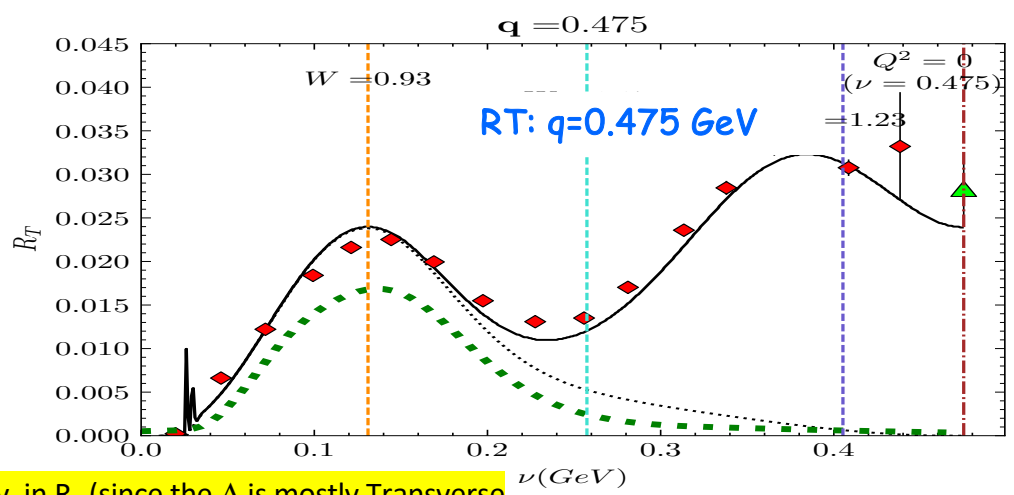
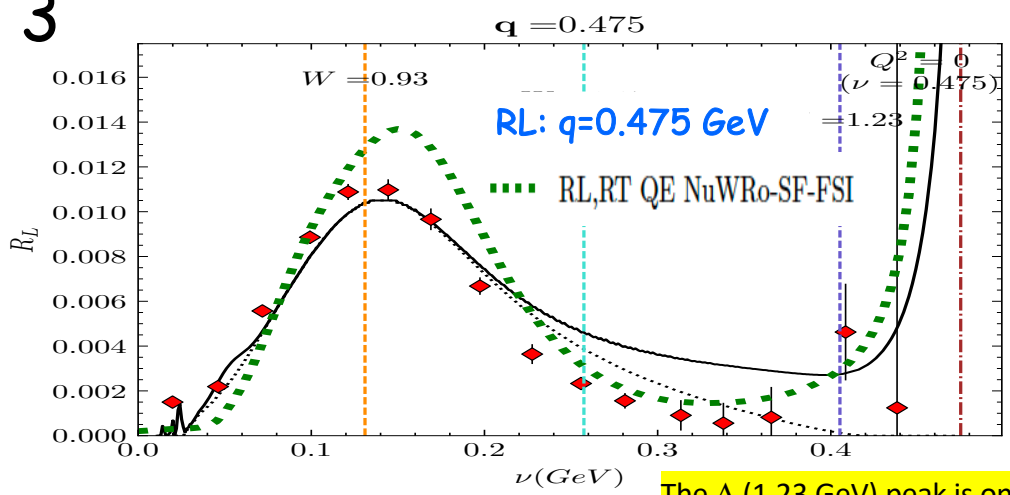
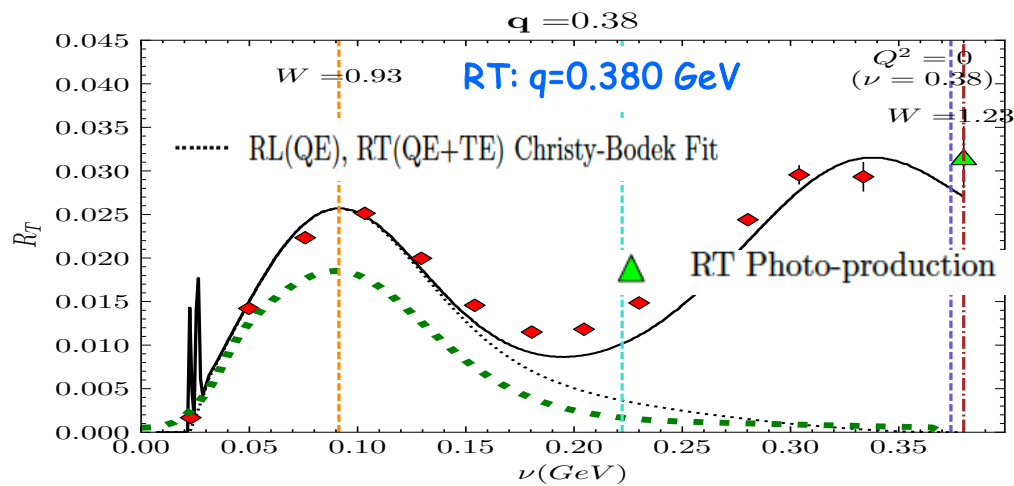
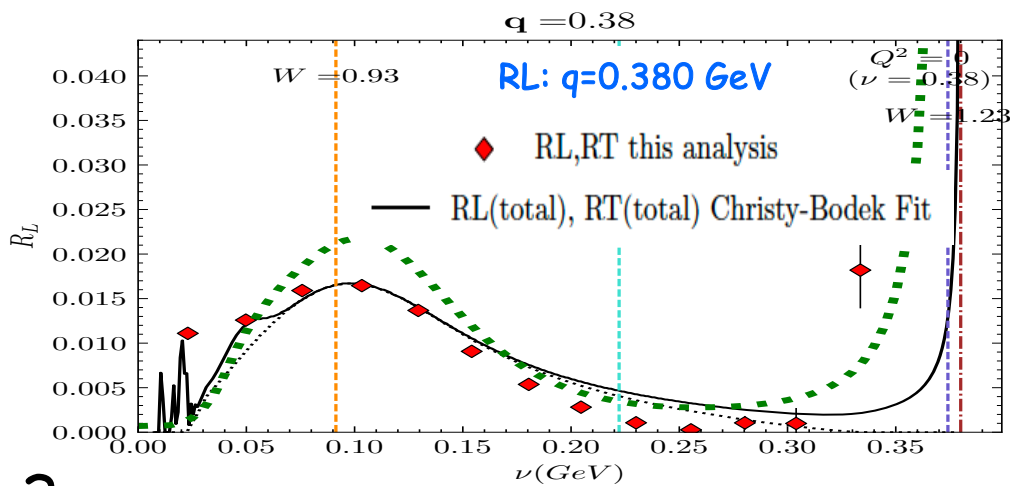
..... RL(QE), RT(QE+TE) Christy-Bodek Fit

◆ RL,RT Yamaguchi

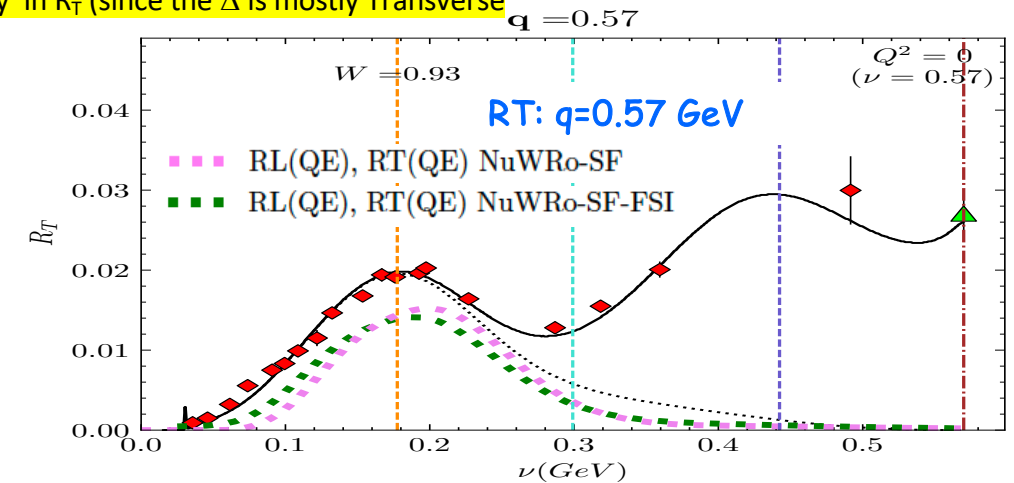
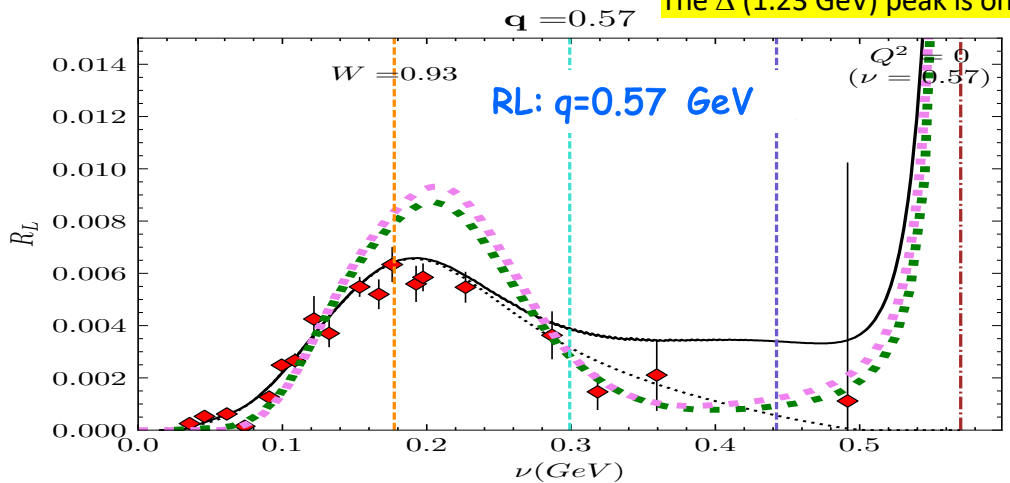
◆ RL, RT this analysis

..... RL(QE), RT(QE) NuWRo-SF-FSI

▲ RT Photo-production



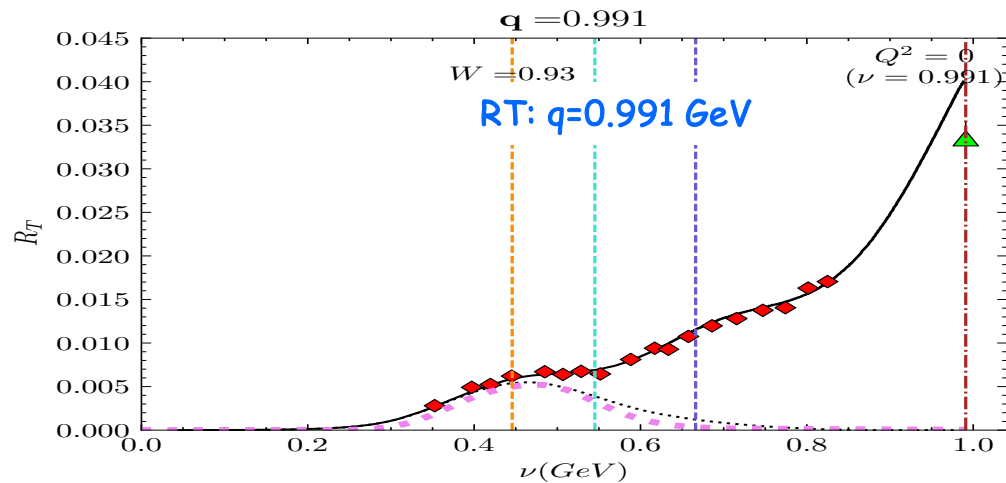
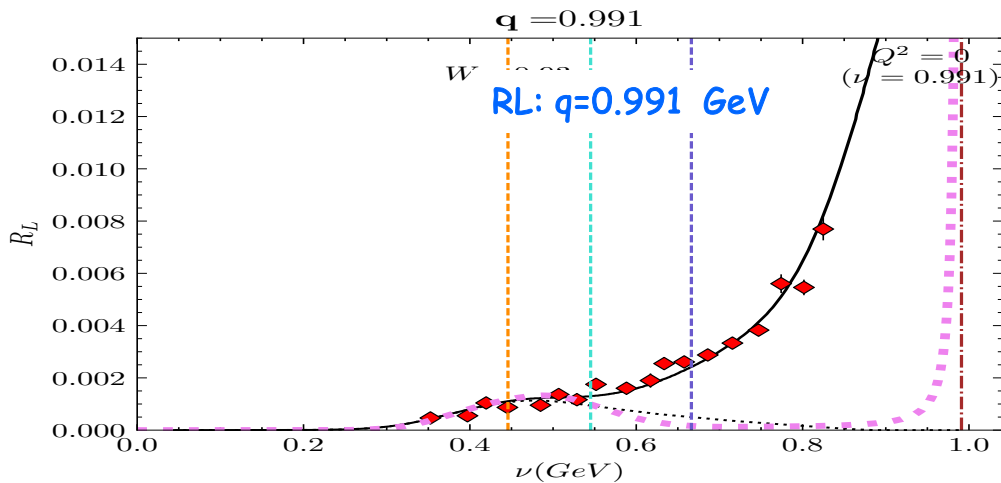
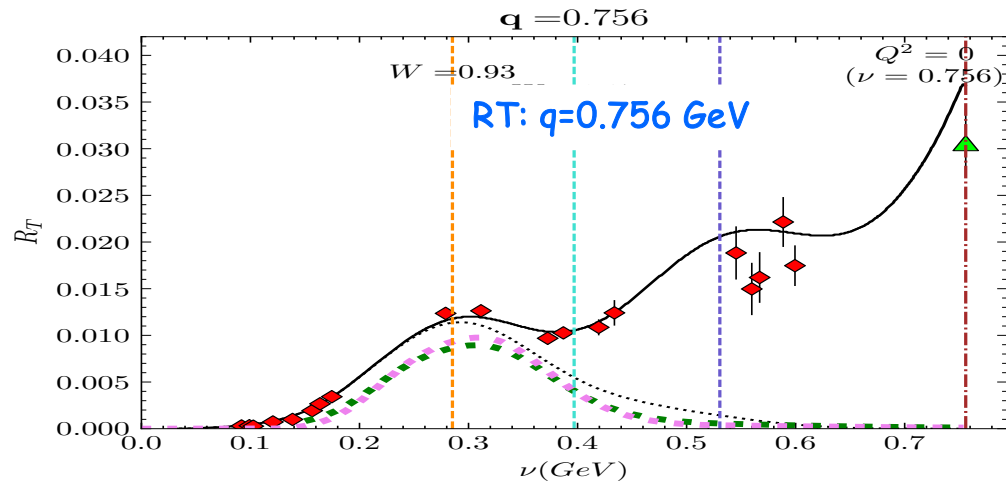
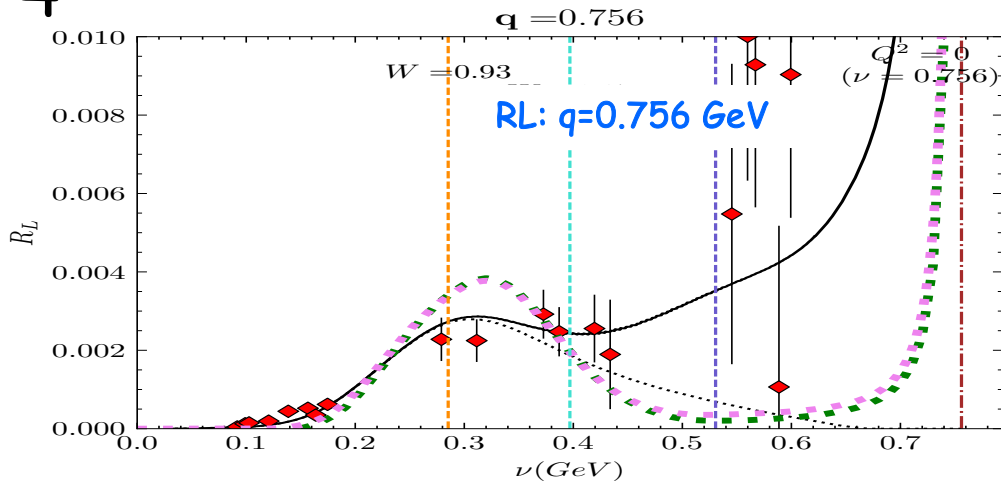
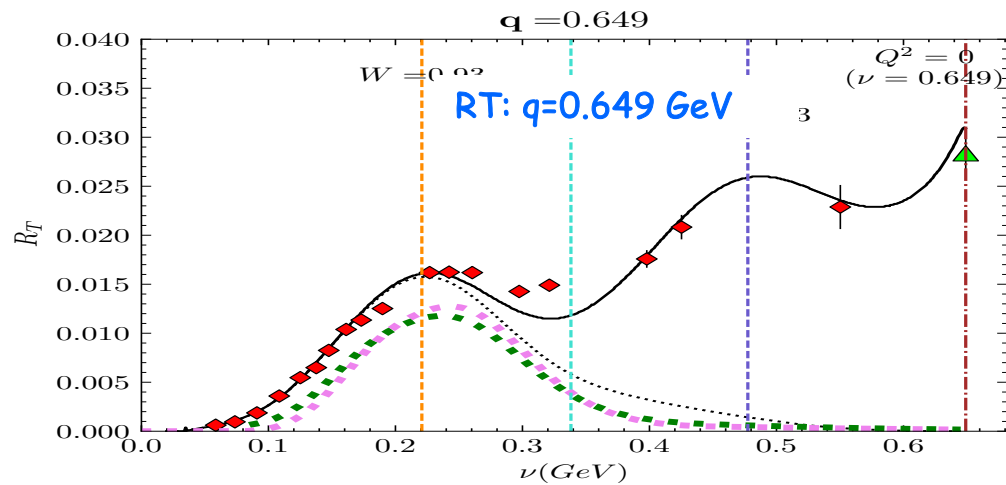
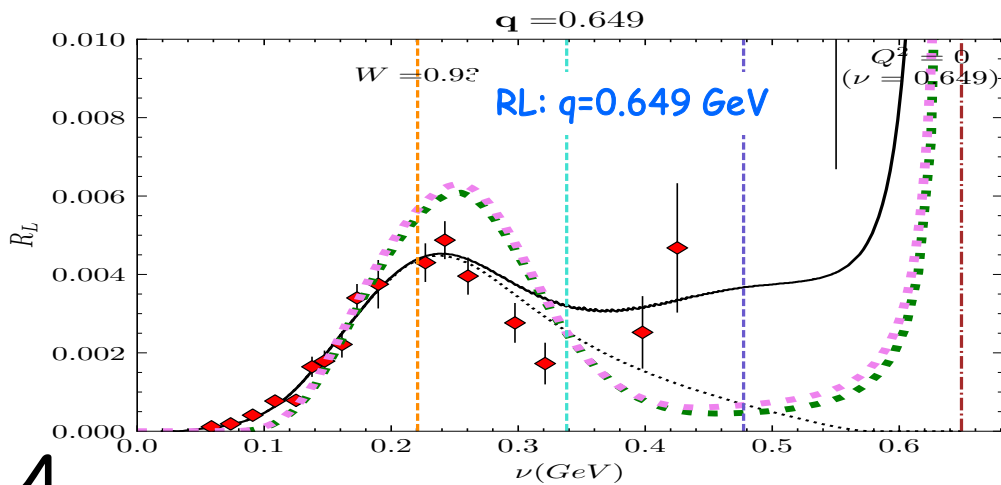
The Δ (1.23 GeV) peak is only in R_T (since the Δ is mostly Transverse)



— RL(total), RT(total) Christy-Bodek Fit
 ⋯ RL(QE), RT(QE+TE) Christy-Bodek Fit

◆ RL, RT this analysis
 ◆ RL(QE), RT(QE) NuWRo-SF-FSI

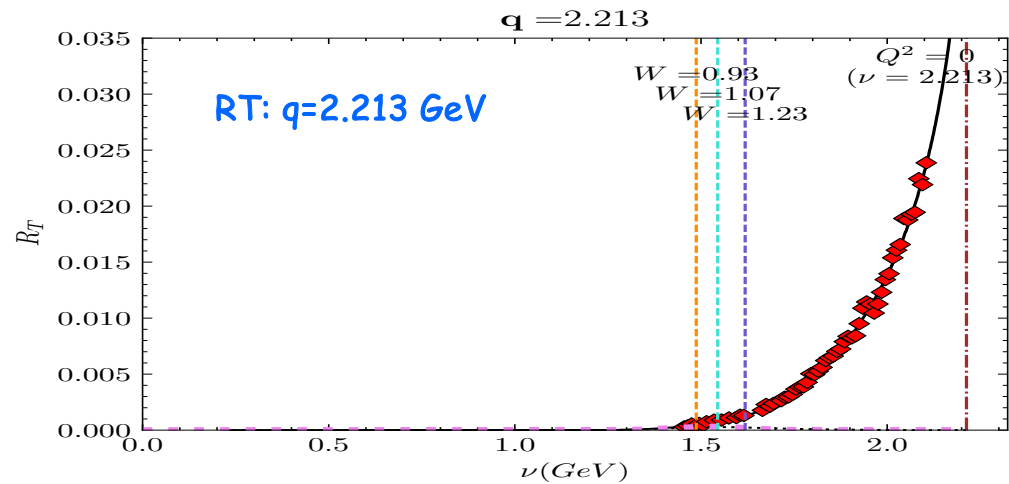
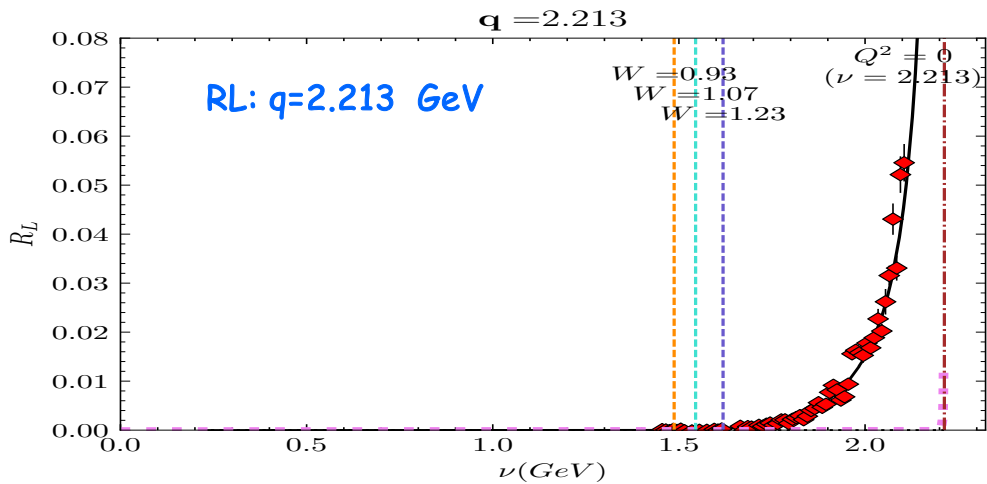
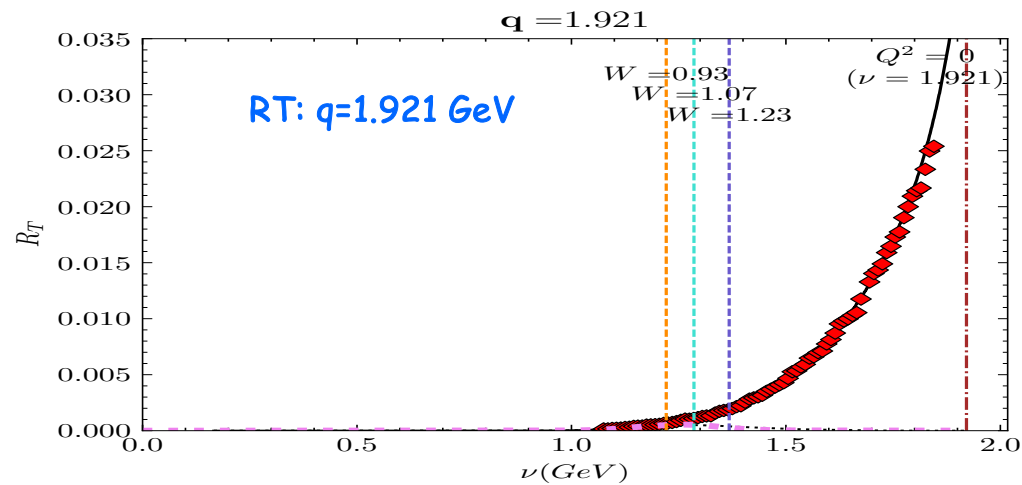
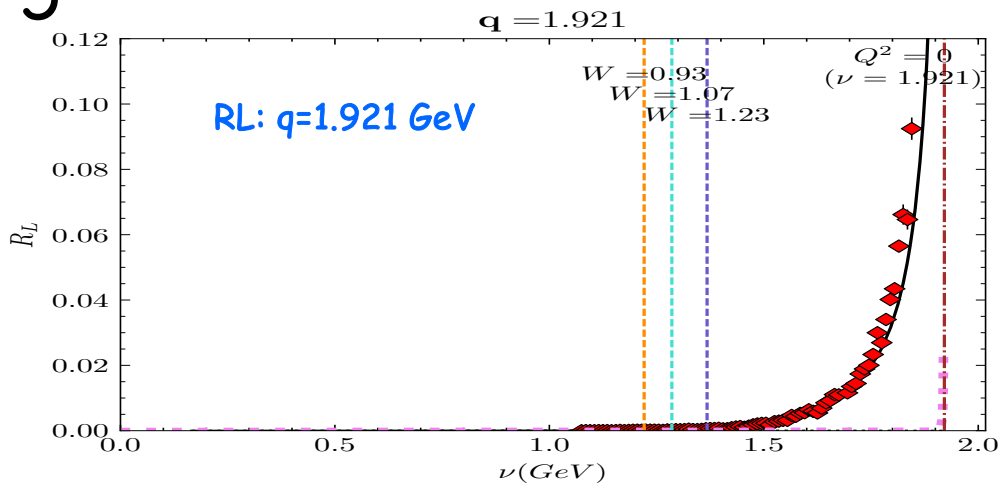
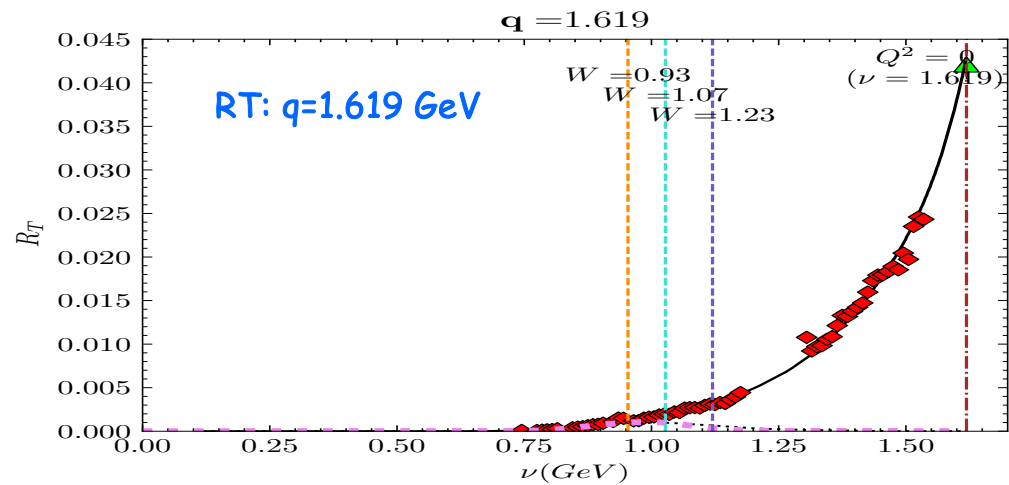
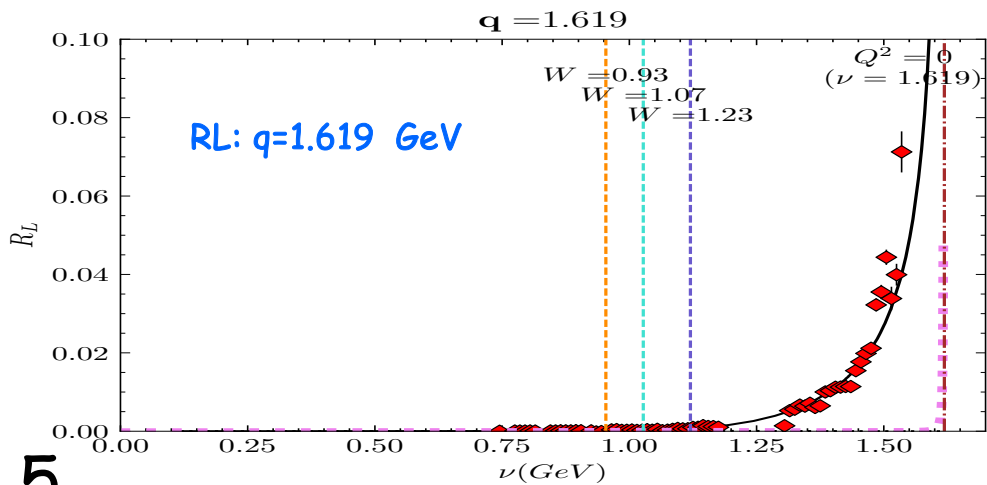
▲ RT Photo-production
 ◆ RL(QE), RT(QE) NuWRo-SF



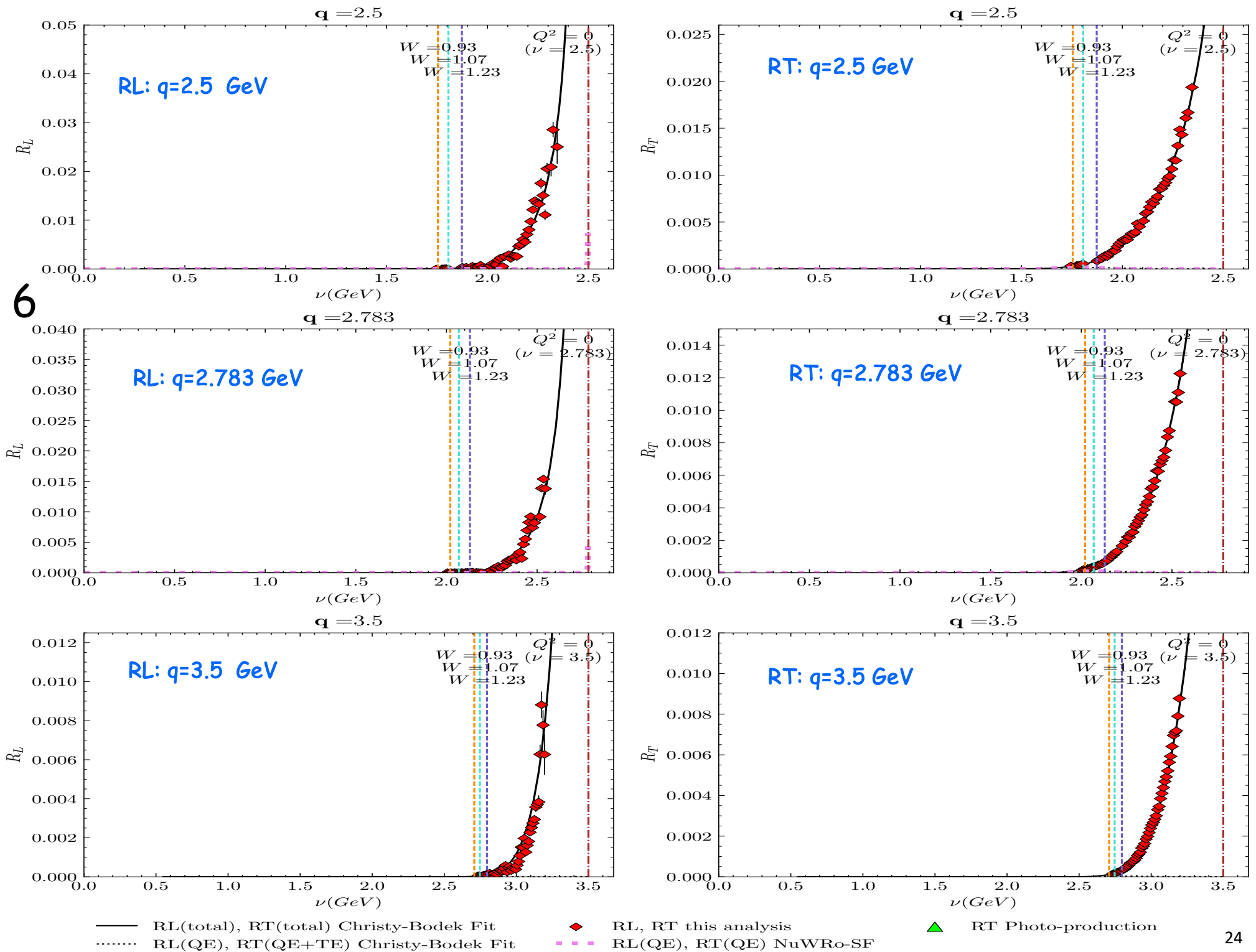
— RL(total), RT(total) Christy-Bodek Fit
 RL(QE), RT(QE+TE) Christy-Bodek Fit

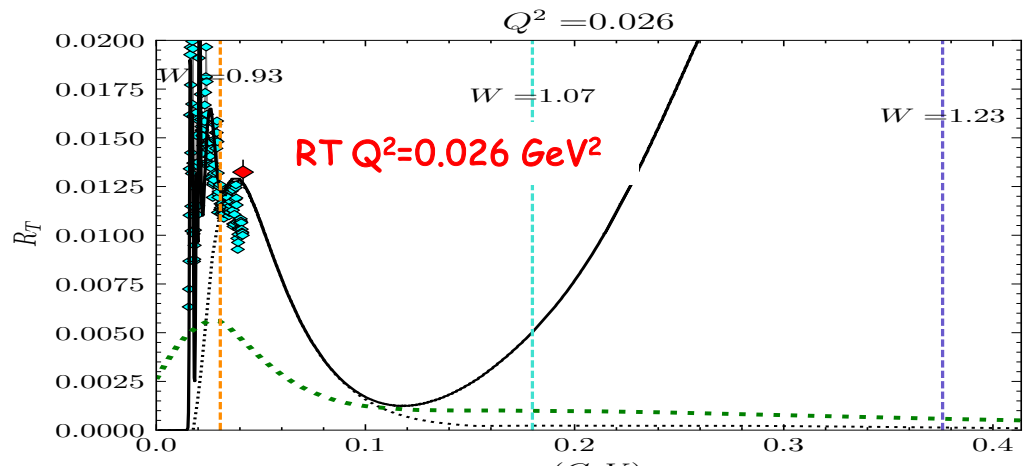
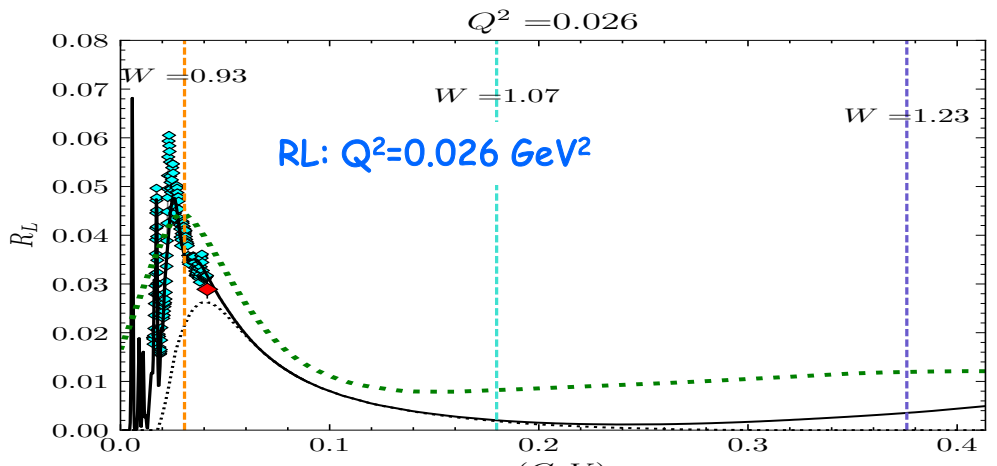
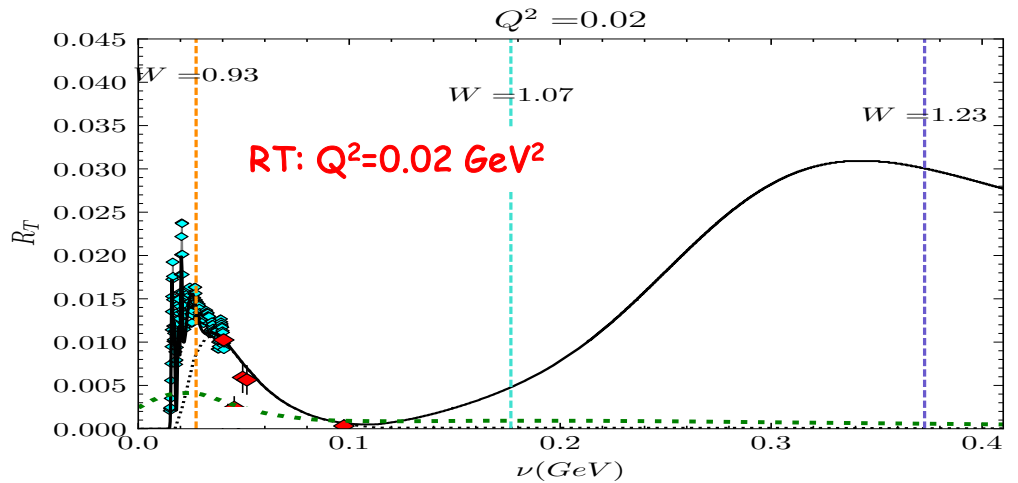
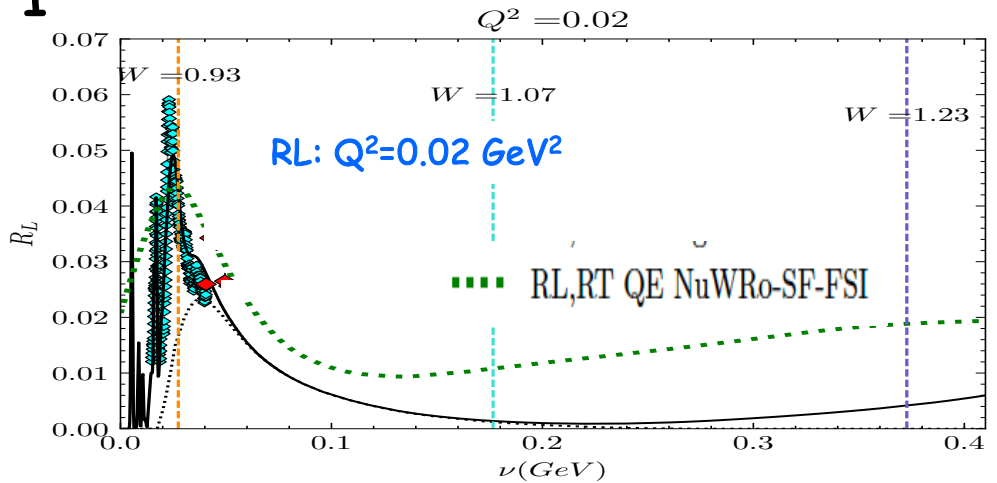
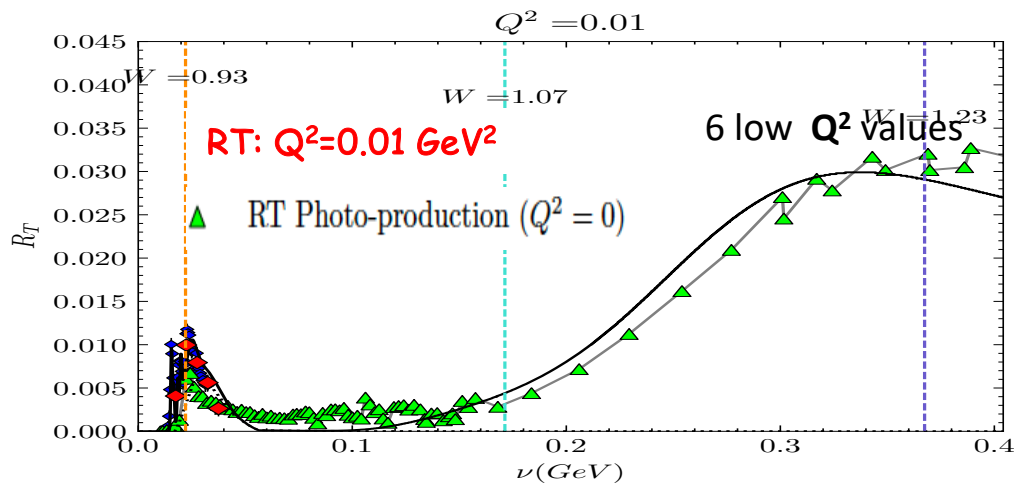
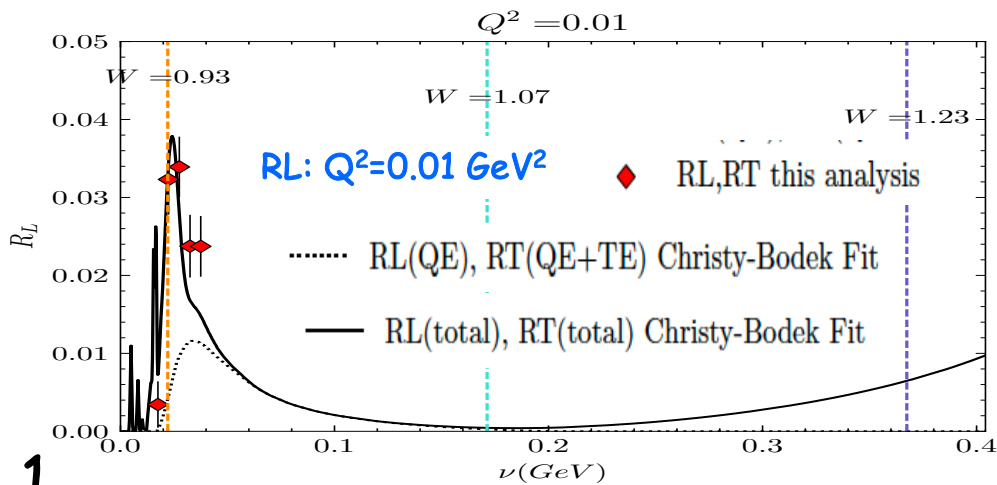
◆ RL, RT this analysis
 ■ RL(QE), RT(QE) NuWRo-SF-FSI

■ RL(QE), RT(QE) NuWRo-SF
 ▲ RT Photo-production

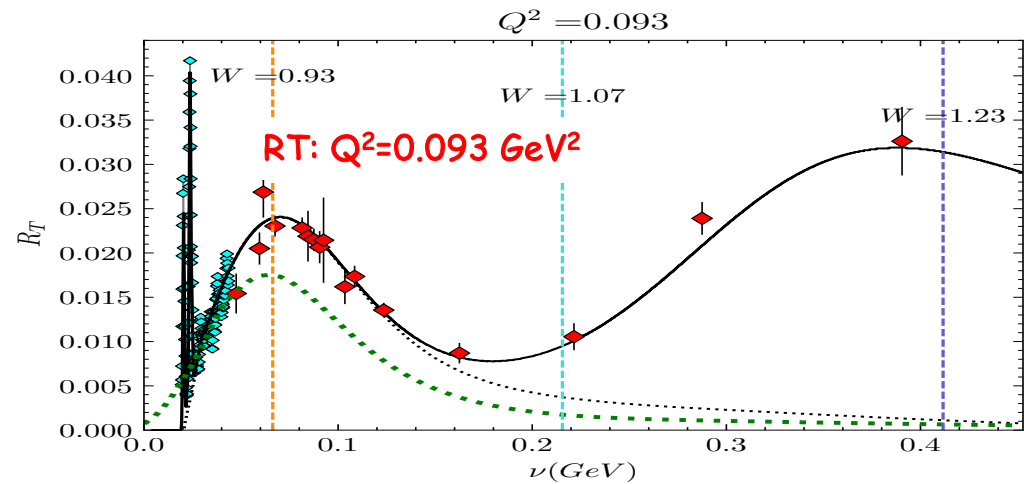
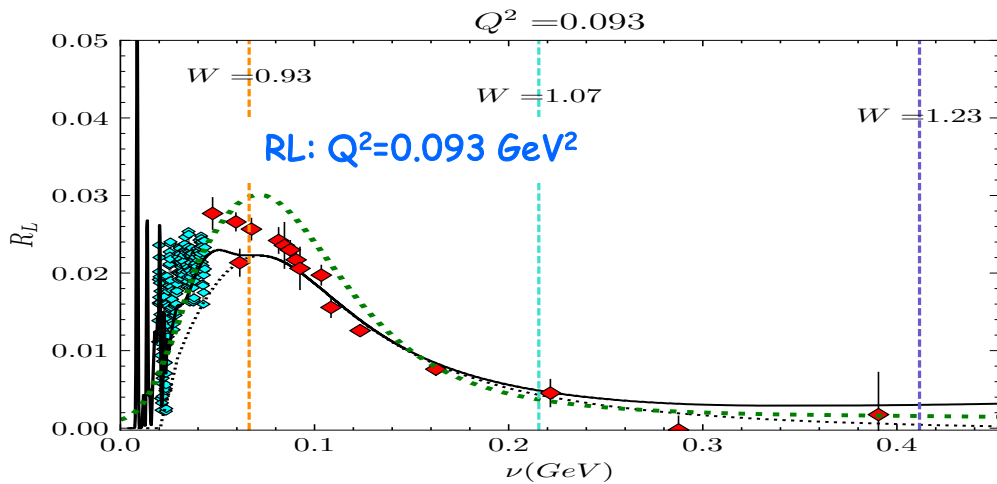
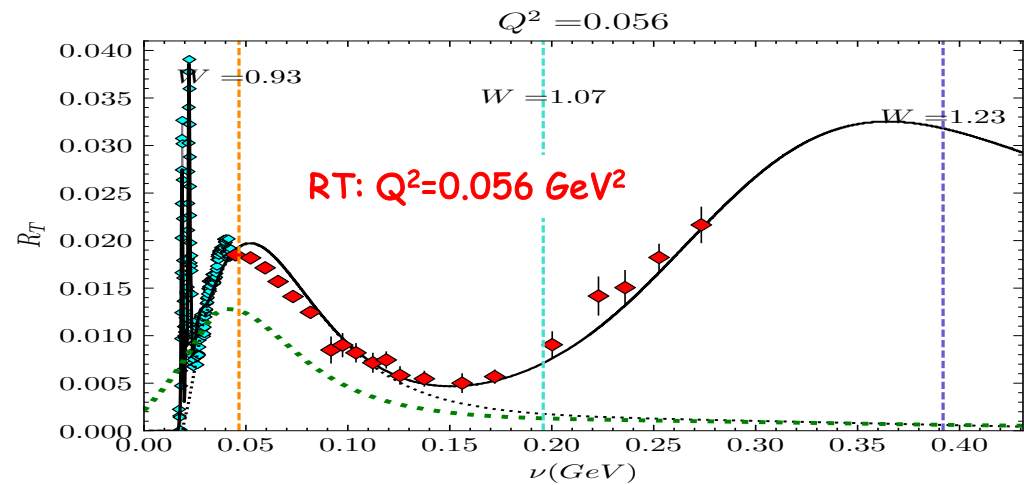
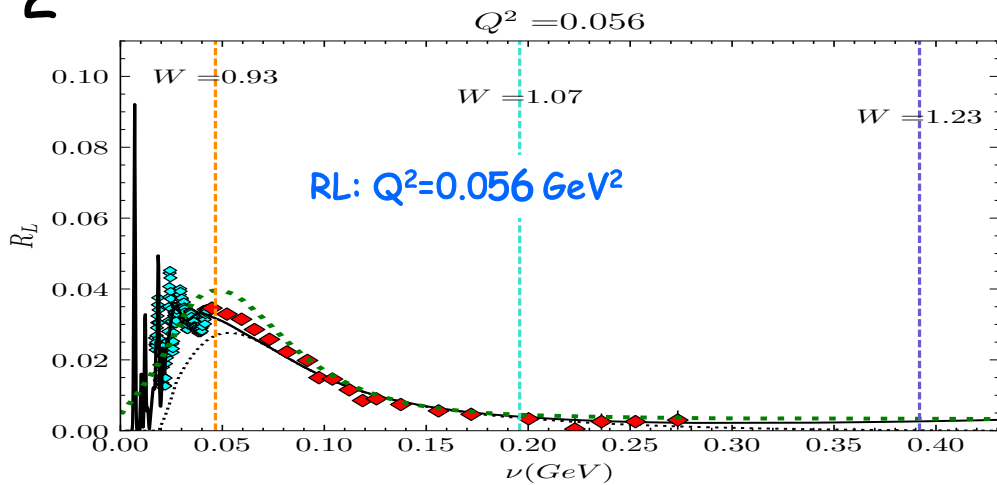
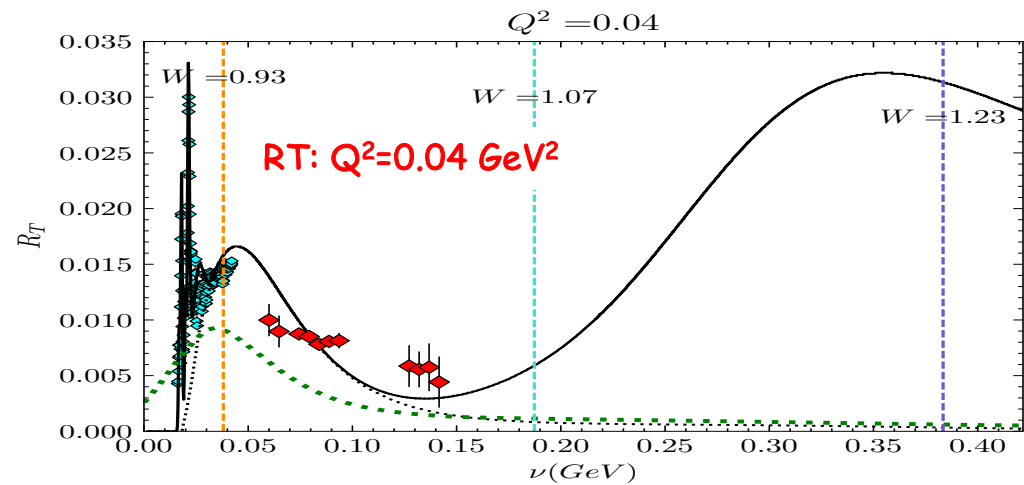
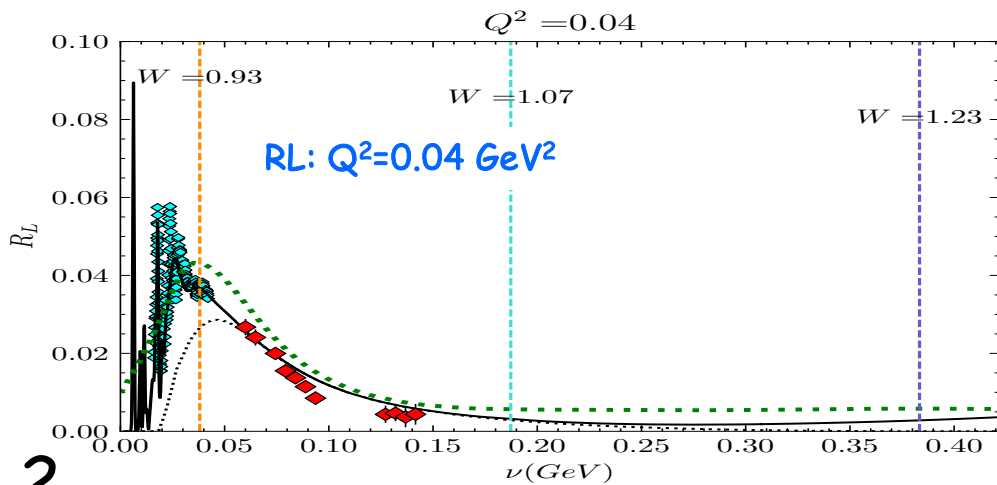


— RL(total), RT(total) Christy-Bodek Fit ◆ RL, RT this analysis ▲ RT Photo-production
 RL(QE), RT(QE+TE) Christy-Bodek Fit - - - - - RL(QE), RT(QE) NuWRo-SF





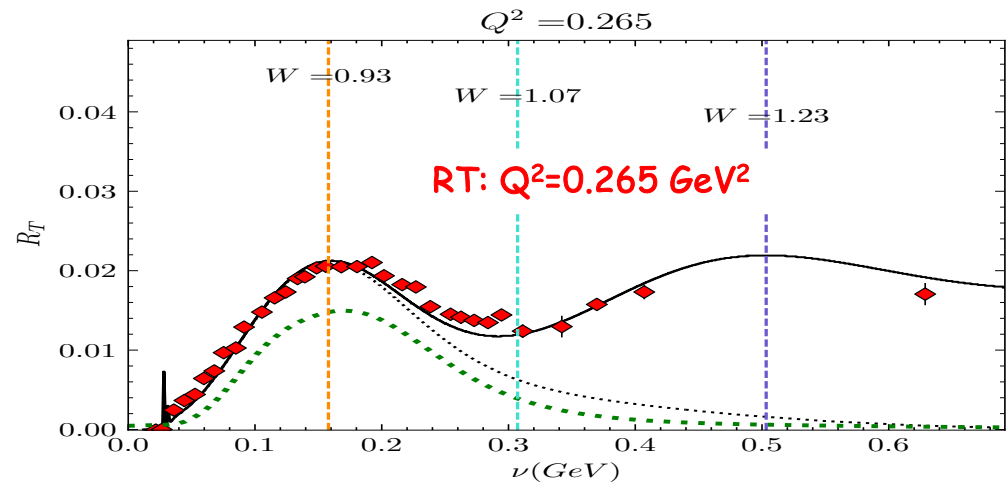
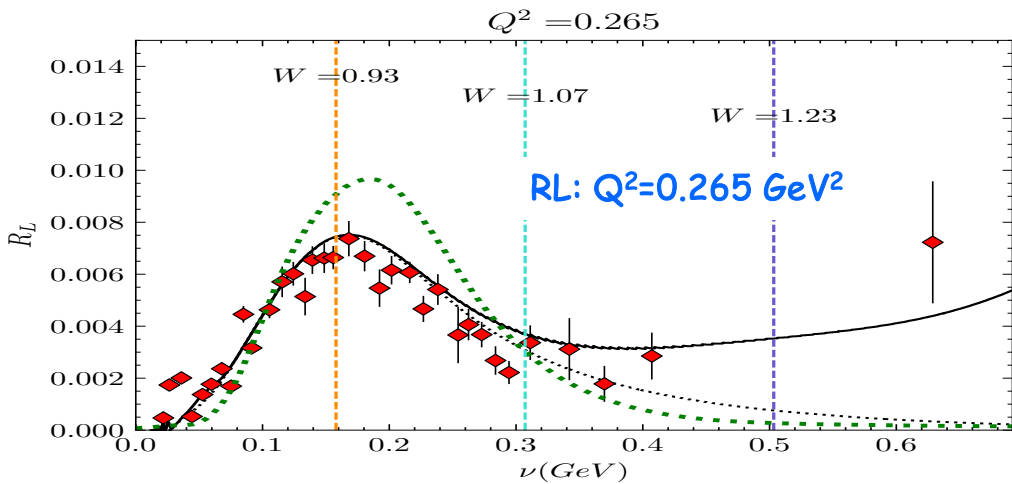
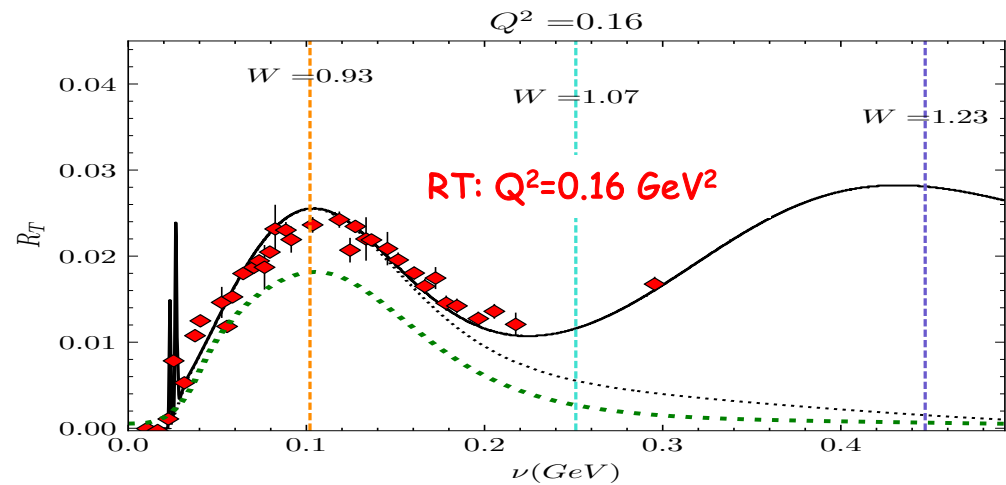
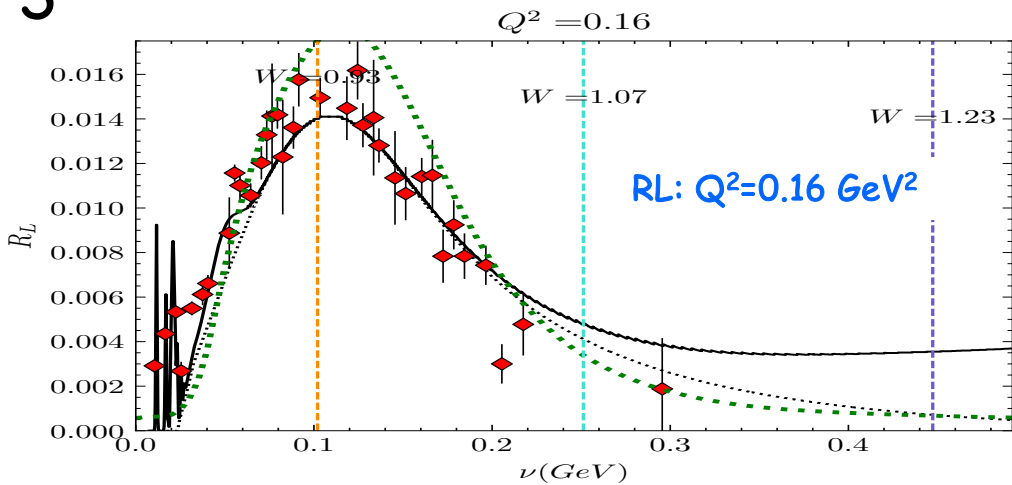
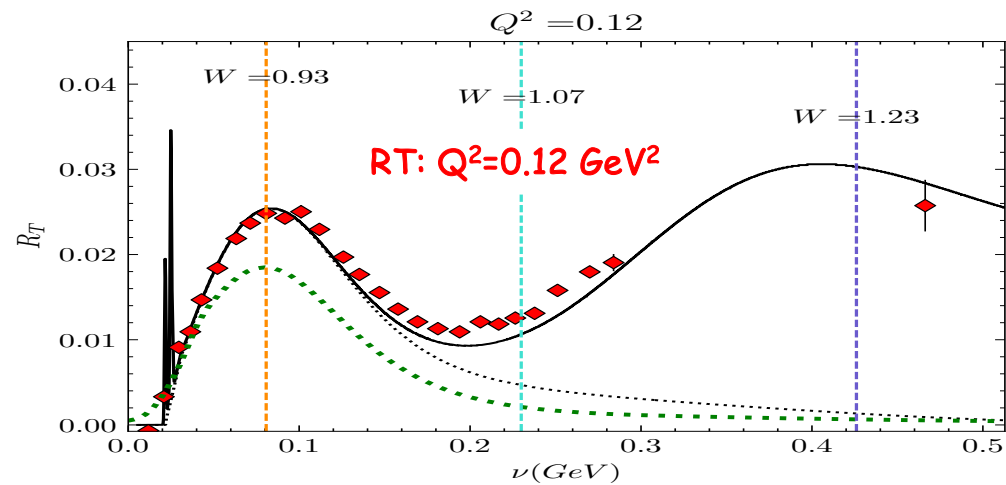
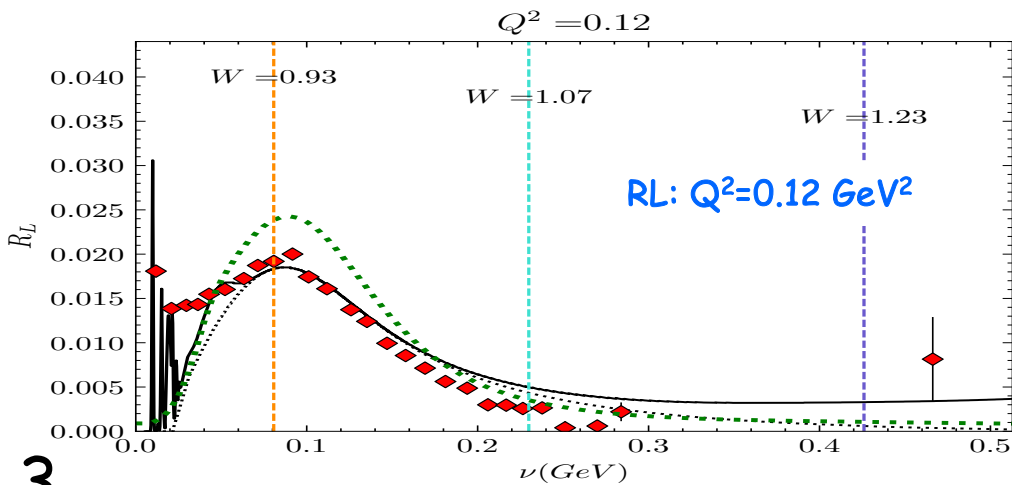
- | | | |
|--|--|--|
| — RL(total), RT(total) Christy-Bodek Fit | ◆ RT Goldemberg $q=0.1$ | ◆ RL,RT Yamaguchi |
| RL(QE), RT(QE+TE) Christy-Bodek Fit | ▲ RT Photo-production ($Q^2 = 0$) | ◆ RL,RT QE NuWro-SF-FSI |
| ◆ RL,RT this analysis | | |



— RL(total), RT(total) Christy-Bodek Fit
 RL(QE), RT(QE+TE) Christy-Bodek Fit

◆ RL,RT Yamaguchi
 ◆ RL,RT this analysis

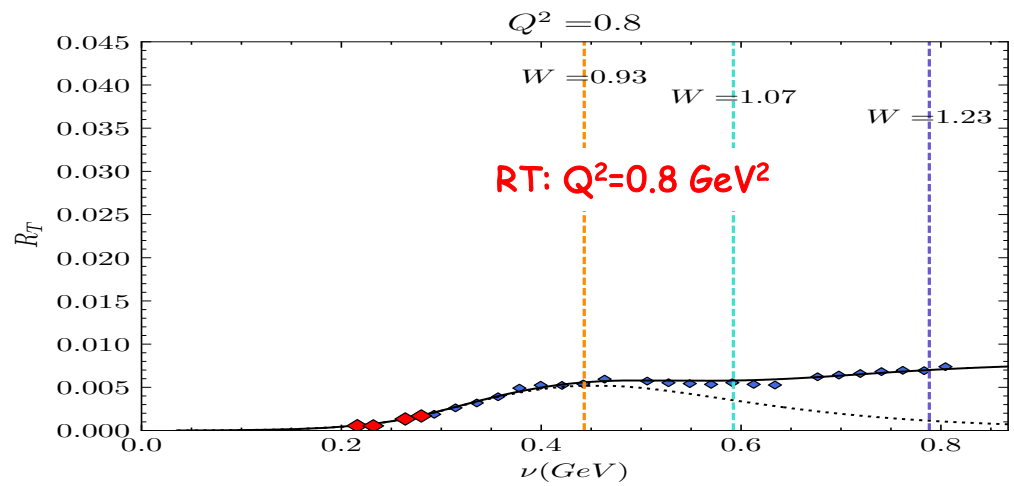
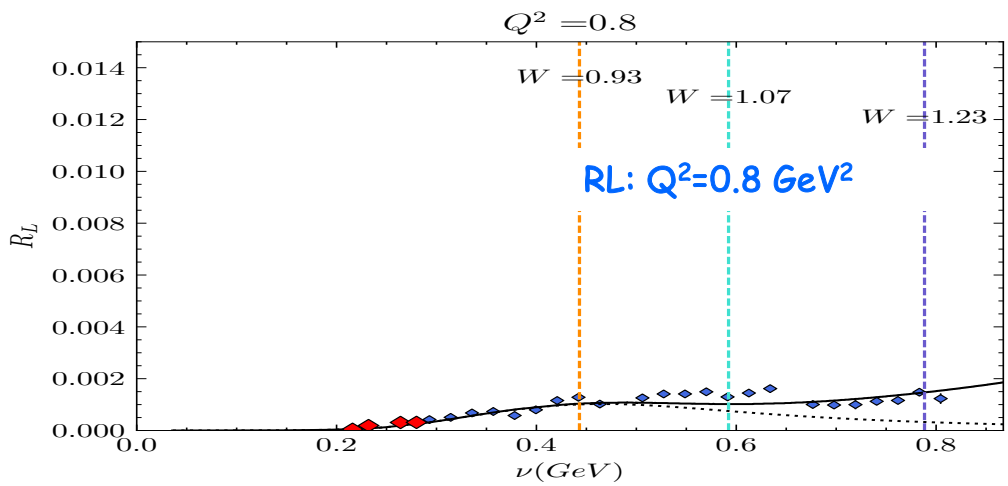
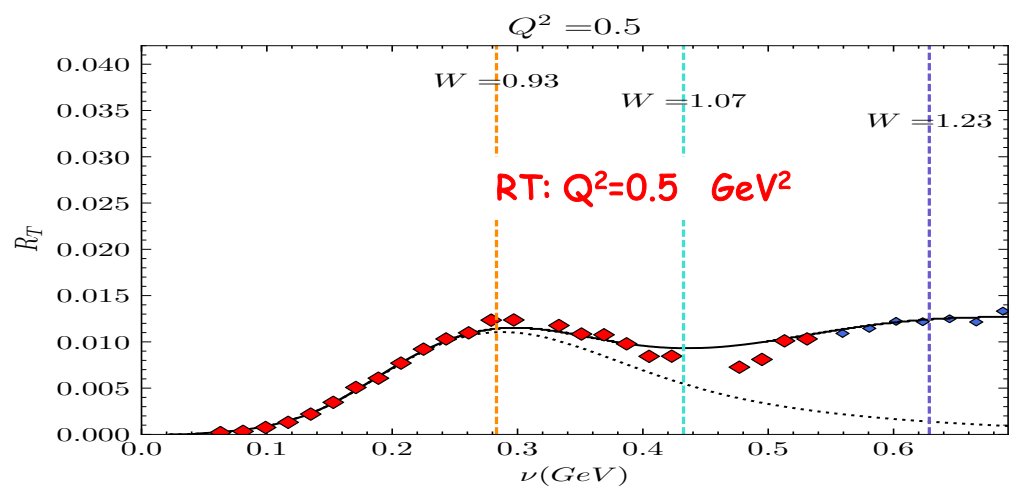
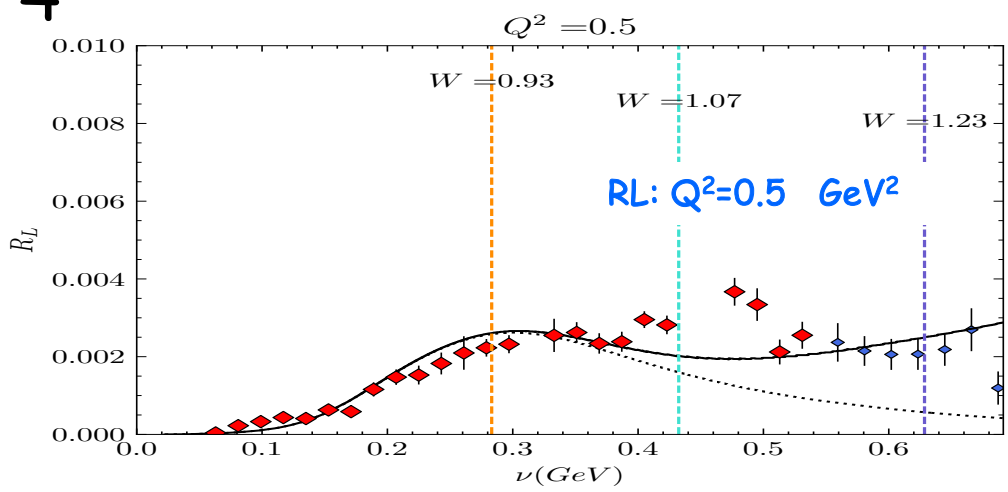
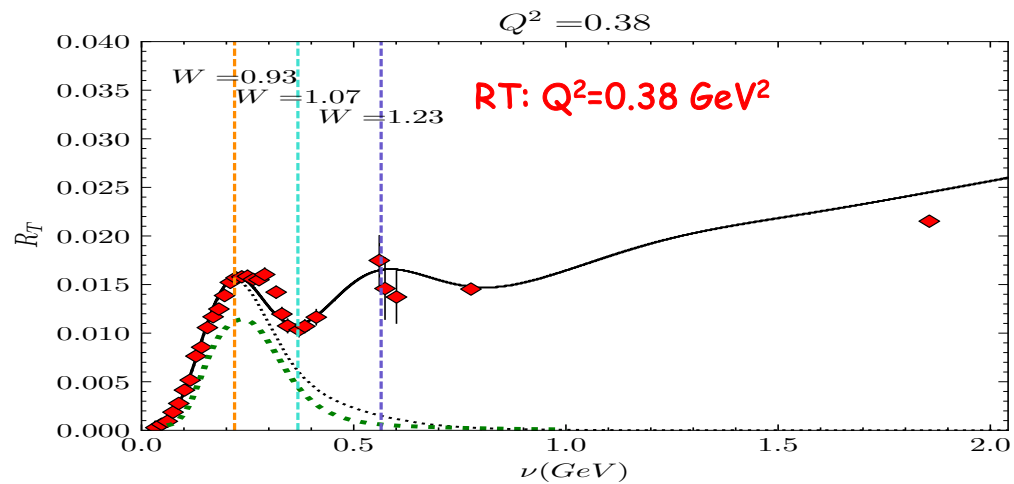
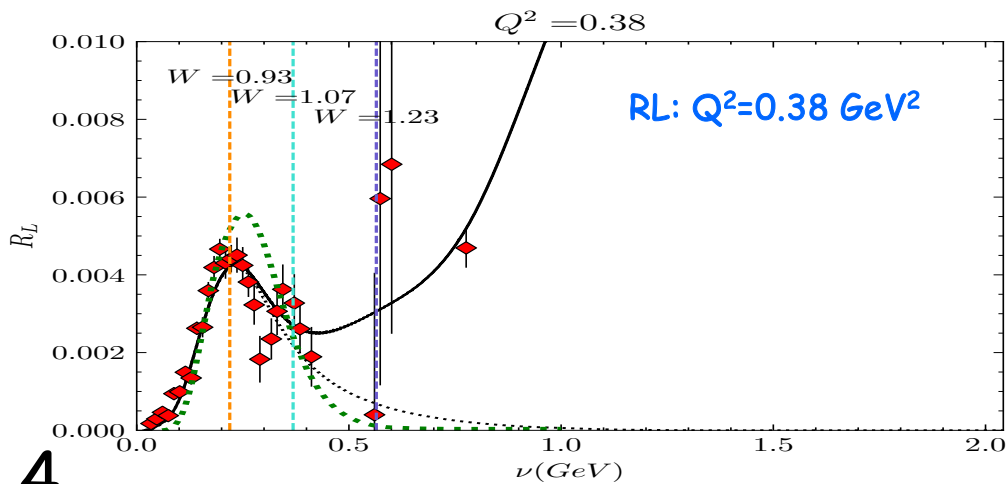
..... RL,RT QE NuWRo-SF-FSI



— RL(total), RT(total) Christy-Bodek Fit
 RL(QE), RT(QE+TE) Christy-Bodek Fit

◆ RL,RT this analysis

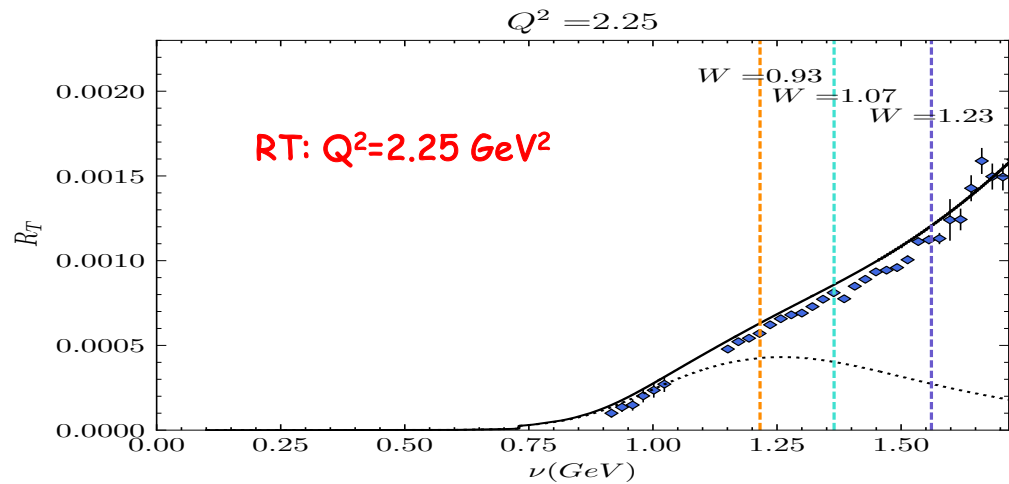
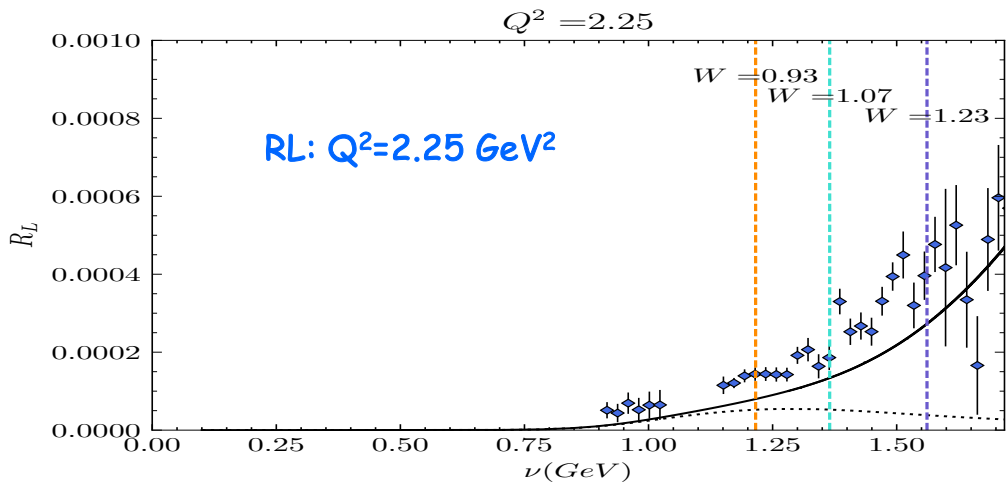
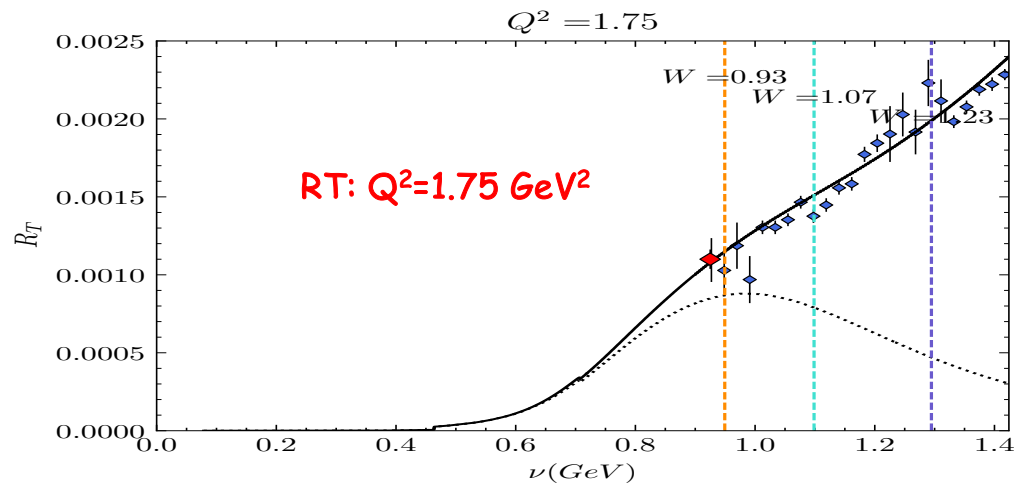
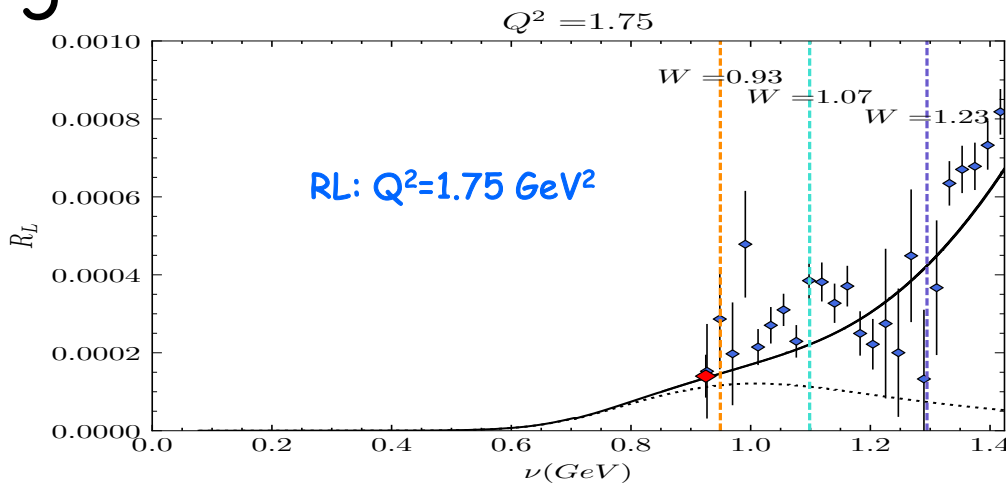
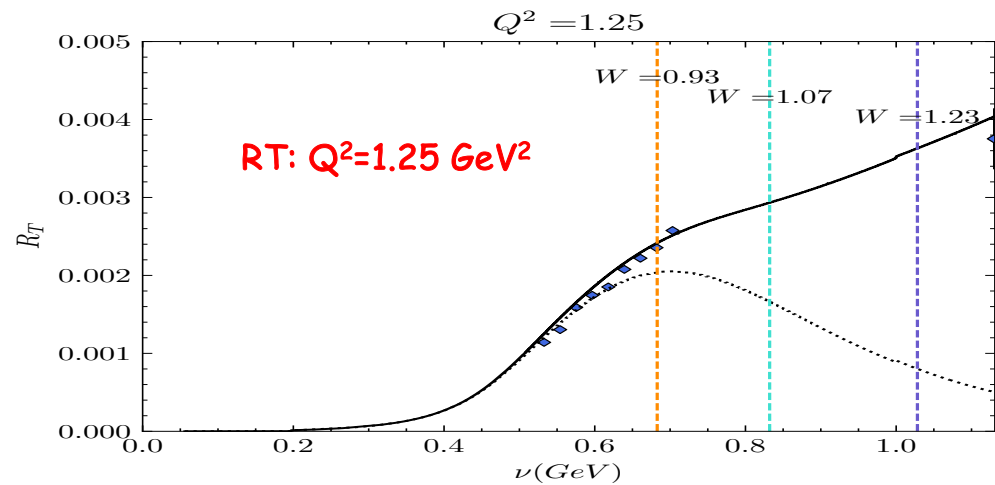
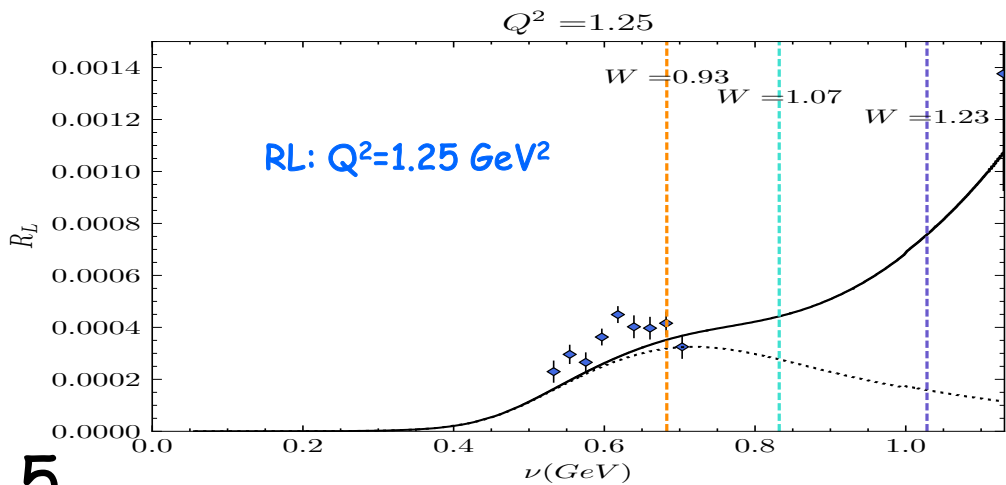
..... RL,RT QE NuWRo-SF-FSI



— RL(total), RT(total) Christy-Bodek Fit
 RL(QE), RT(QE+TE) Christy-Bodek Fit

◆ RL,RT this analysis
 ◆ RL,RT QE NuWRo-SF-FSI

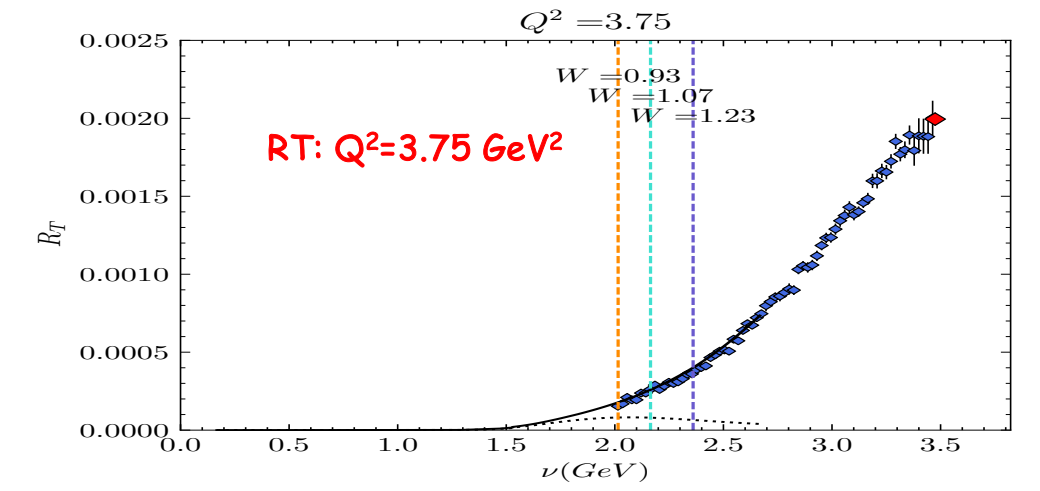
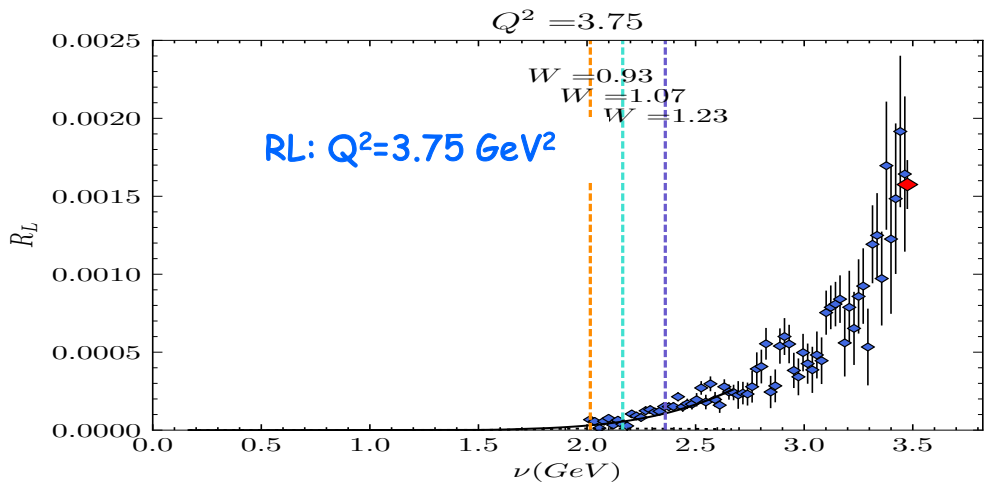
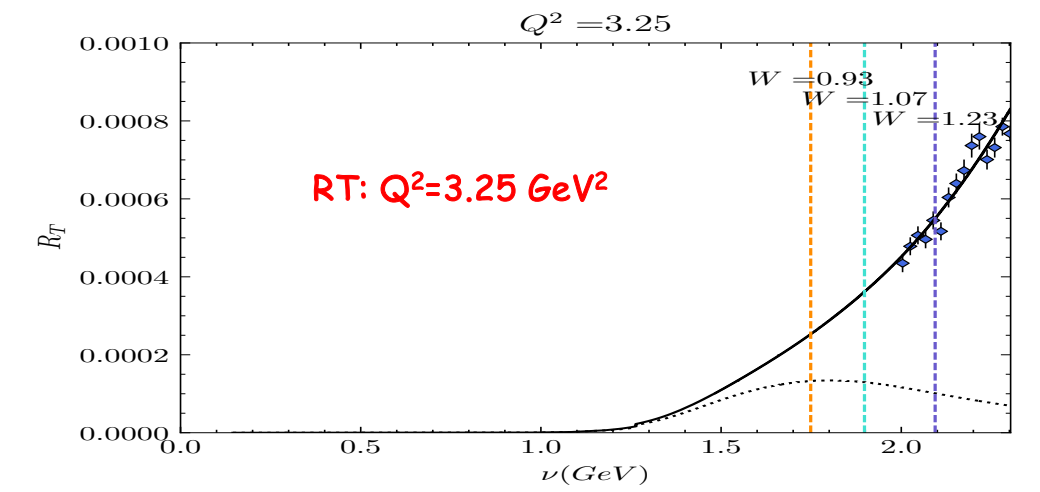
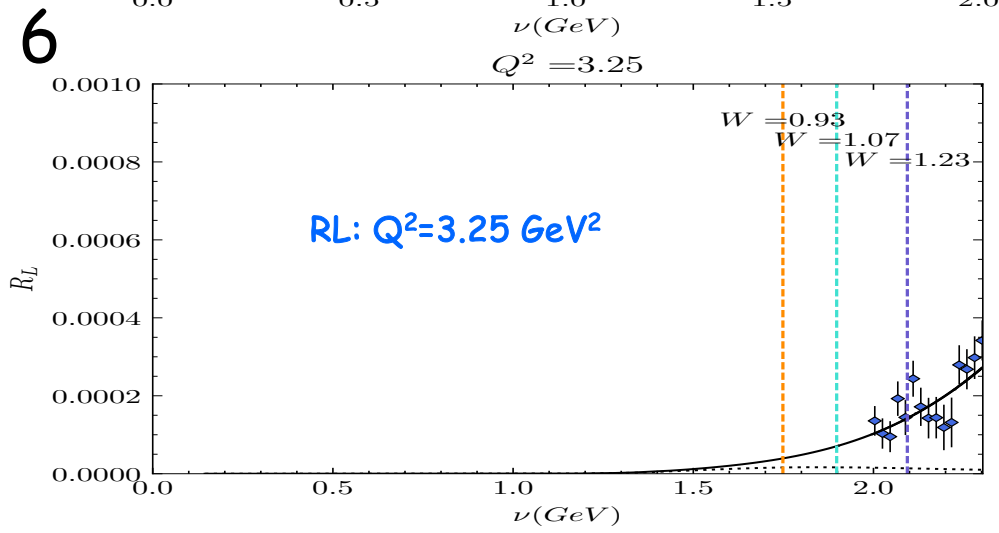
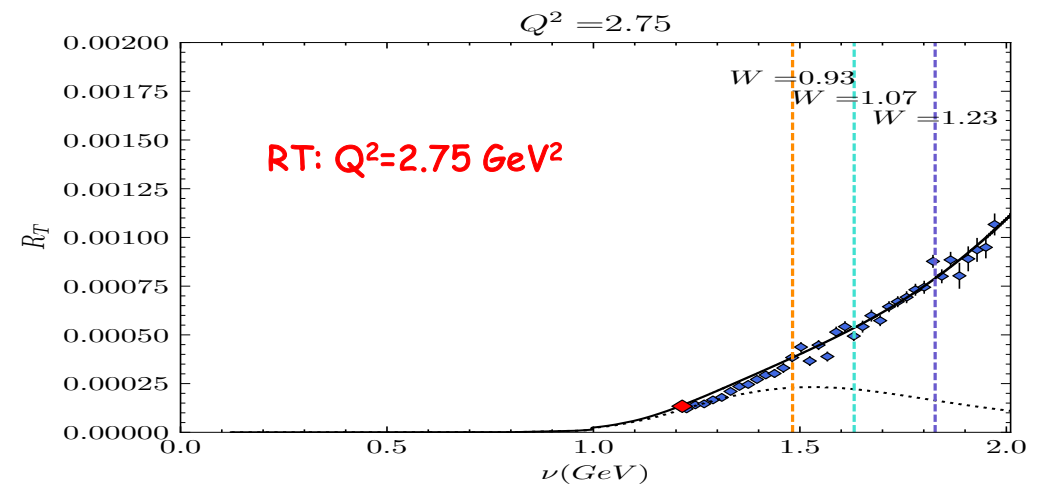
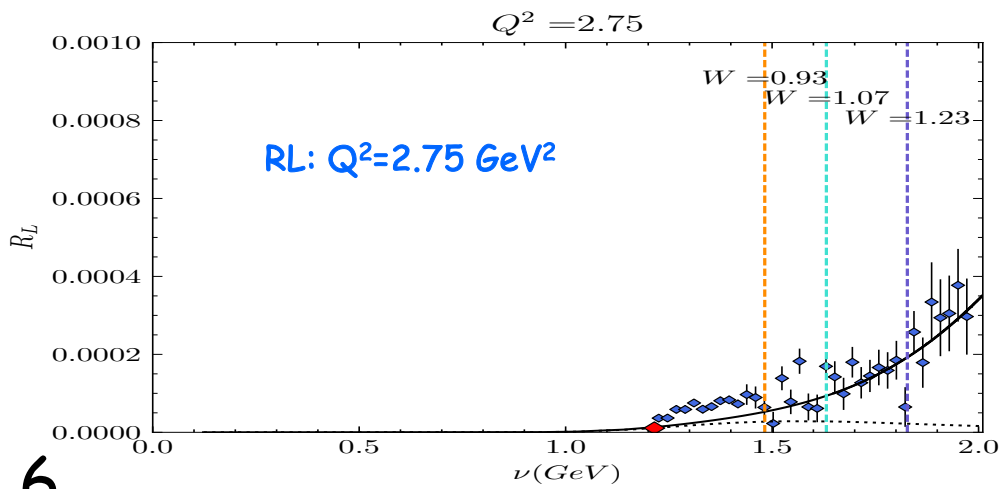
◆ RL,RT JLab e04-001



— RL(total), RT(total) Christy-Bodek Fit
 RL(QE), RT(QE+TE) Christy-Bodek Fit

◆ RL,RT JLab e04-001

◆ RL,RT this analysis



— RL(total), RT(total) Christy-Bodek Fit
 RL(QE), RT(QE+TE) Christy-Bodek Fit

◆ RL,RT JLab e04-001 ◆ RL,RT this analysis

*

Comments

- The extracted R_L and R_T are in good agreement with the Christy-Bodek Universal Fit. The fit can be used for validation of MC generators over a more extended range of q and ν .
- The Christy-Bodek fit includes **Longitudinal Quenching** at low q and **Transverse Enhancement** at intermediate q . It also includes nuclear excitations
- The Δ (1.23 GeV) peak is only seen in R_T (since the Δ is mostly Transverse).
- For fixed q the maximum value of ν is $\nu=q$ (where R_T can also be extracted from photoproduction data).
- Validation of Neutrino Generator NuWRo in electron model: NuWRo only models QE with a spectral function, and adds Final State Interaction (FSI)
- NuWRo (spectral function) requires FSI for better agreement with the data.
- However, even with FSI for $q < 0.3$ GeV NuWRo overestimates R_L (**requires Longitudinal Quenching**)
- NuWRo underestimates R_T (in neutrino mode it has Transverse Enhancement/MEC, but not in electron mode).
- Nuclear excitations are not included
- Comparisons with GENIE (in electron mode) will be available shortly.
- Ca40 data next.

*

Conclusions

- The 18 R_L and R_T extractions cover a large kinematic range. The values are in excellent agreement with the Christy-Bodek Universal fit to all cross section values. The universal fit covers an even larger kinematic range.
- The R_L and R_T measurements as well as the universal fit provide a simple way to validate electron and neutrino MC generators over the entire kinematic range of interest.
- Good agreement in the QE region with nuclear theory for 3 values of q . Predictions for all other values of q not yet available.

In Supplemental Materials we will provide

Tables of the extracted values of R_L and R_T

- Tables of the Christy-Bodek Universal fit values for R_L and R_T for the 18 q and Q^2 values. The contributions of nuclear excitations, QE, transverse enhancement and inelastic scattering will be listed separately.
- Code for the Christy-Bodek Universal fit

Backup