# Neutrino-induced pion production

Alexis Nikolakopoulos
$2^{nd}$ short-baseline Experiment-Theory Workshop
4 April 2024

**“Neutrino-induced pion production and final-state interactions”**

Collaborations with :
R. Gonzalez-Jimenez, J. Garcia-Marcos, J. M Udias (UC Madrid)
N. Jachowicz, M. Hooft, K. Niewczas (Ugent)
J. Sobczyk (NuWro)
N. Steinberg, N. Rocco, J. Isaacson (Fermilab, ACHILLES)

# Outline

- Pion production on the nucleon
  - Electromagnetic
  - Axial currents in the delta region
  - Uncertainties in the higher resonance region

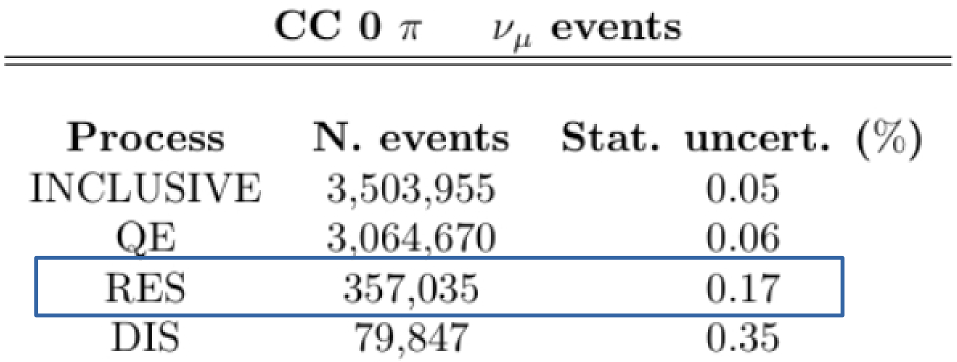
- Nuclear matrix elements and ‘elastic’ final-state interactions

- Pion production in neutrino event generators

# Interactions in the nucleon resonance region

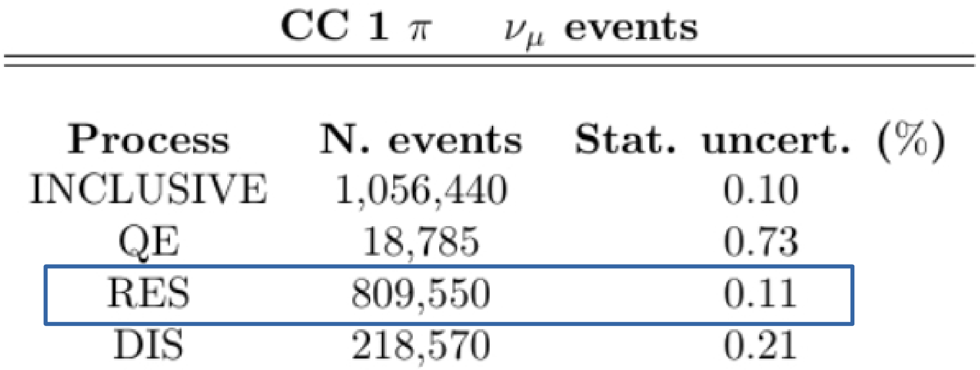
## Motivation:

- **Significant contribution to event rate in ~1 GeV experiments**



CC 0 $\pi$ $\nu_\mu$ events

| Process | N. events | Stat. uncert. (%) |
|---|---|---|
| INCLUSIVE | 3,503,955 | 0.05 |
| QE | 3,064,670 | 0.06 |
| RES | 357,035 | 0.17 |
| DIS | 79,847 | 0.35 |



CC 1 $\pi$ $\nu_\mu$ events

| Process | N. events | Stat. uncert. (%) |
|---|---|---|
| INCLUSIVE | 1,056,440 | 0.10 |
| QE | 18,785 | 0.73 |
| RES | 809,550 | 0.11 |
| DIS | 218,570 | 0.21 |

GENIE estimates for **SBND**:

**~ 23 %** of the signal has a pion

Of which **~80 %** in resonance region

Tables from G. Scanavini
( FERMILAB-MASTERS-2017-05)

# Interactions in the nucleon resonance region

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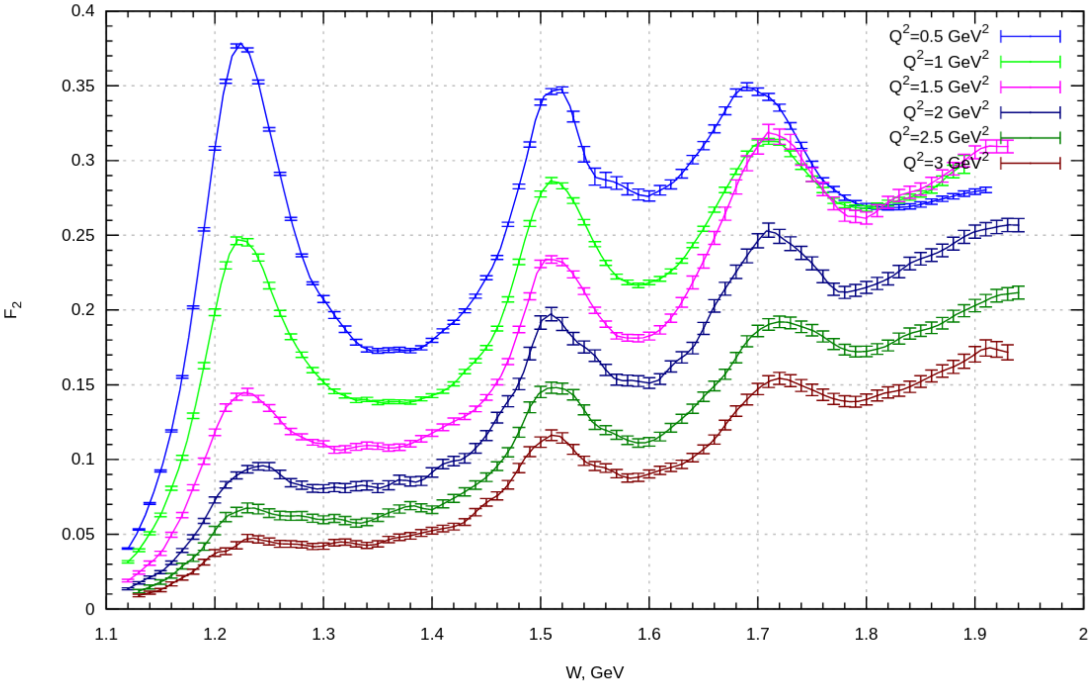


**Proton structure function (e,e') CLAS**

GENIE estimates for **SBND**:

**~ 23 %** of the signal has a pion

Of which **~80 %** in resonance region

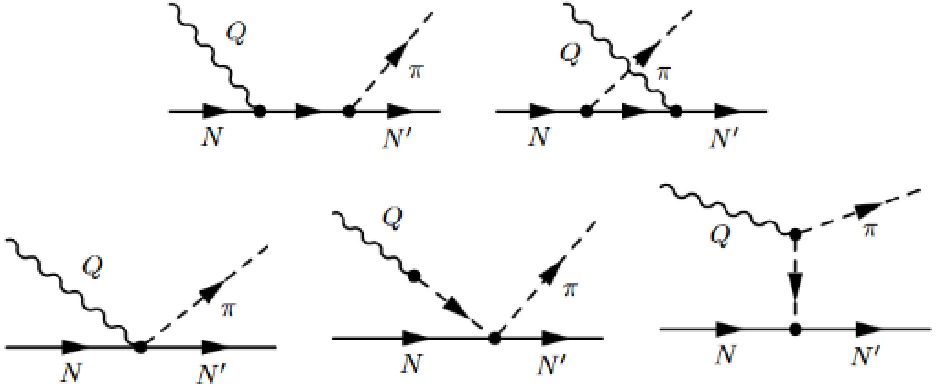
**Non-perturbative, no factorization**
**→ Hadron d.o.f**

# Electro and photoproduction

**Much expertise from electromagnetic interactions with hadrons**

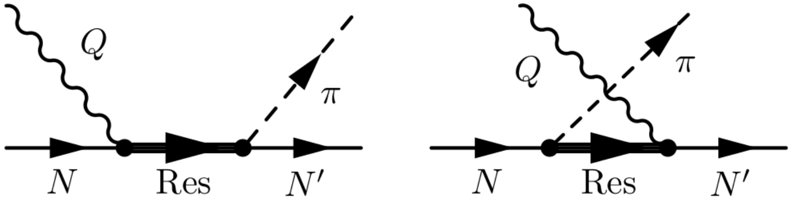
Many approaches available in the literature
- MAID07, CLAS analyses (‘unitary isobar model’)
- Julich-Bonn, ANL-Osaka, ... (Dynamical models)
- ...

Non-resonant



+

Baryon
resonances



# Electro and photoproduction

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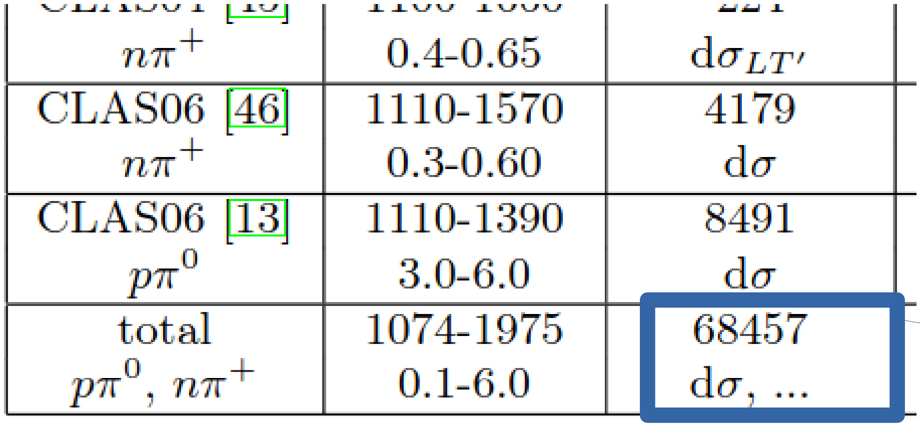
- ...

**Supported by a large amount of data!**

**MAID07**
~18 000 points for photon processes

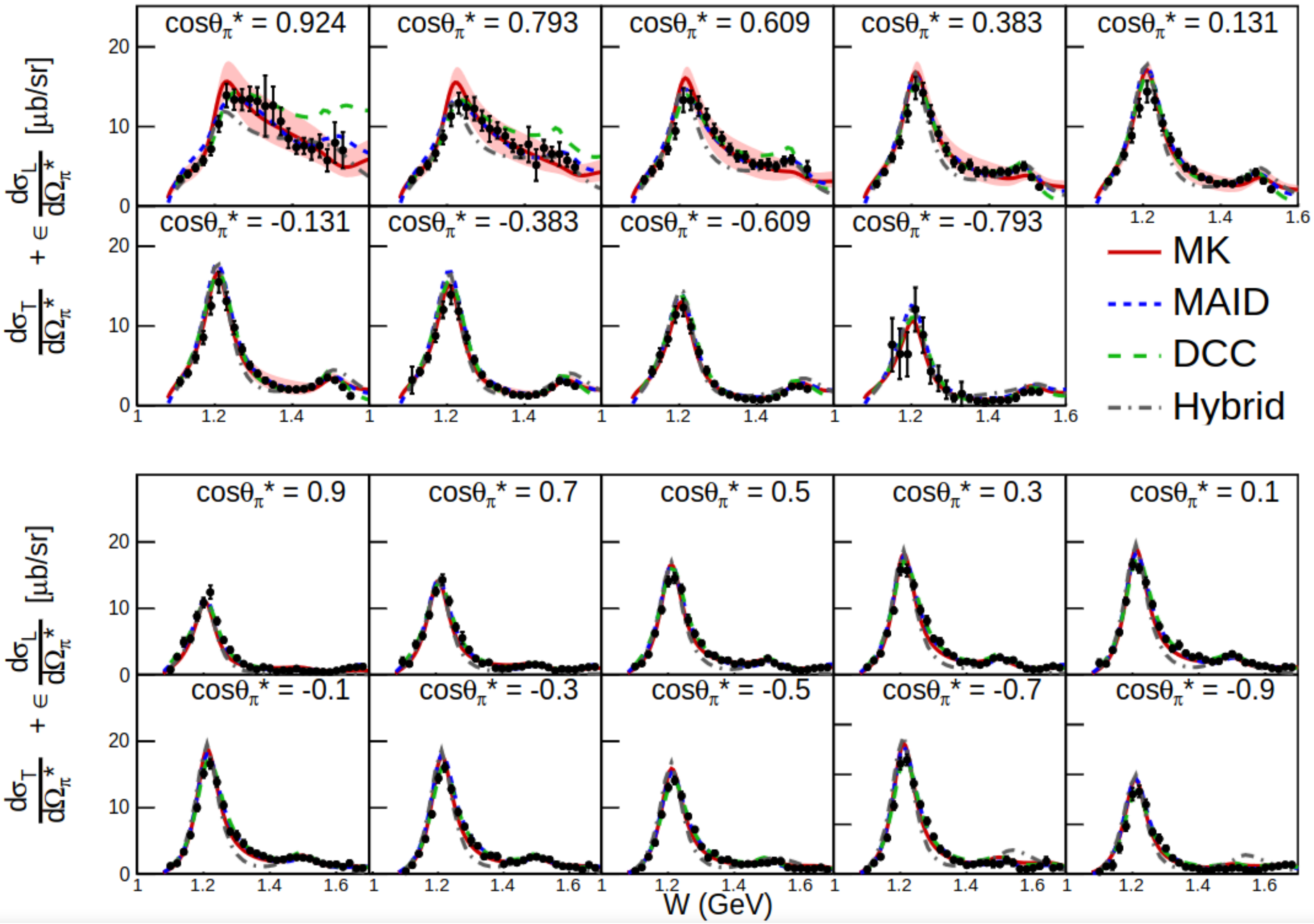


| | | |
|---|---|---|
| $n\pi^+$ | 0.4-0.65 | d$\sigma_{LT'}$ |
| CLAS06 [46]<br>$n\pi^+$ | 1110-1570<br>0.3-0.60 | 4179<br>d$\sigma$ |
| CLAS06 [13]<br>$p\pi^0$ | 1110-1390<br>3.0-6.0 | 8491<br>d$\sigma$ |
| total<br>$p\pi^0$, $n\pi^+$ | 1074-1975<br>0.1-6.0 | 68457<br>d$\sigma$, ... |

Electron-proton data in MAID07

# Electron-induced SPP: high quality proton target data

Figures from M. Kabirnezhad [arxiv:2203.15594]



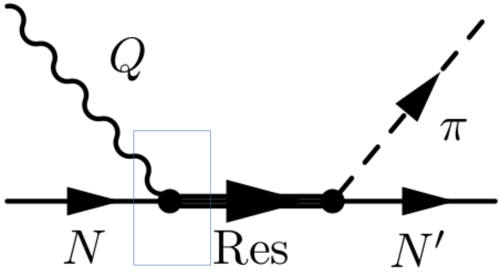
, CLAS data

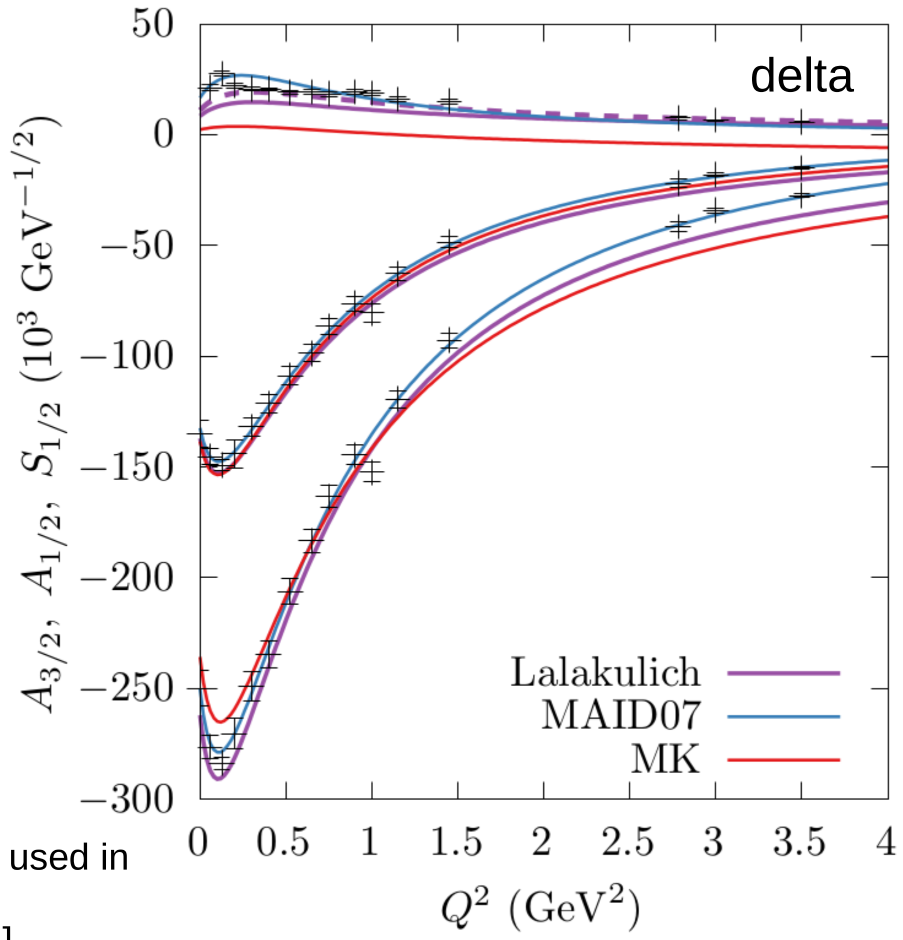
Differential exclusive cross sections are abundant for large kinematic range

# Resonance contributions: proton form factors from helicity amplitudes

**Data analyses produce helicity amplitudes**

Helicity amplitudes can be used to determine γ*p→ R form-factors (model-dependent!)





Helicity amplitudes from CLAS and MAID07 analyses used in Neutrino pion production models :

Lalakulich et al [Phys. Rev. D74, 014009 (2006)]
Hernandez et al. [Phys Rev D 77 053009 (2008)]
Nikolakopoulos et al. [Phys Rev D 107, 05300 (2023)]

# Neutrino-induced SPP: isovector form factors

$$J^{\mu}_{EM} = V^{\mu}_{s} + \mathbf{V}^{\mu} \qquad\qquad J^{\mu}_{CC\pm} = \mathbf{V}^{\mu} - \mathbf{A}^{\mu}$$

$$\langle \pi^{+} n | J^{\mu}_{EM} | p \rangle = V^{\mu}_{3/2} - \sqrt{2}\,\boxed{\left(V^{\mu}_{1/2} + S^{\mu}\right)}\ \mathrm{F}_{\mathrm{p}}$$

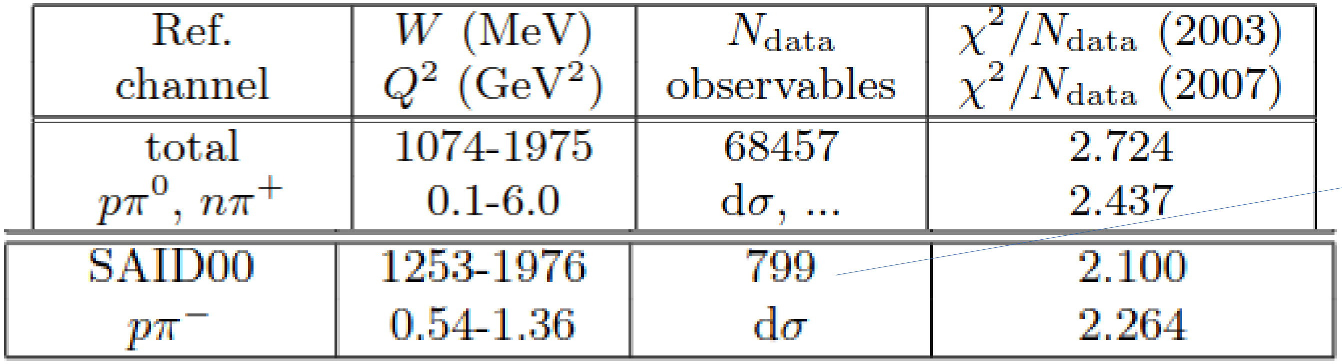
$$\langle \pi^{-} p | J^{\mu}_{EM} | n \rangle = V^{\mu}_{3/2} - \sqrt{2}\,\boxed{\left(V^{\mu}_{1/2} - S^{\mu}\right)}\ \mathrm{F}_{\mathrm{n}}$$

$$\langle \pi^{+} p |\, V^{\mu}_{+}\, | p \rangle = 3V^{\mu}_{3/2},$$

$$\langle \pi^{+} n |\, V^{\mu}_{+}\, | n \rangle = V^{\mu}_{3/2} + 2\sqrt{2}V^{\mu}_{1/2},$$

$$\langle \pi^{0} p |\, V^{\mu}_{+}\, | n \rangle = -\sqrt{2}V^{\mu}_{3/2} + 2V^{\mu}_{1/2}.$$

Isolating the isovector couplings requires analysis of proton and neutron target !



| Ref.<br>channel | $W$ (MeV)<br>$Q^2$ (GeV$^2$) | $N_{\rm data}$<br>observables | $\chi^2/N_{\rm data}$ (2003)<br>$\chi^2/N_{\rm data}$ (2007) |
|---|---|---|---|
| total<br>$p\pi^0$, $n\pi^+$ | 1074-1975<br>0.1-6.0 | 68457<br>$d\sigma$, ... | 2.724<br>2.437 |
| SAID00<br>$p\pi^-$ | 1253-1976<br>0.54-1.36 | 799<br>$d\sigma$ | 2.100<br>2.264 |

Way less abundant!

**Deuteron targets**

# Analyses of electropion production on deuteron

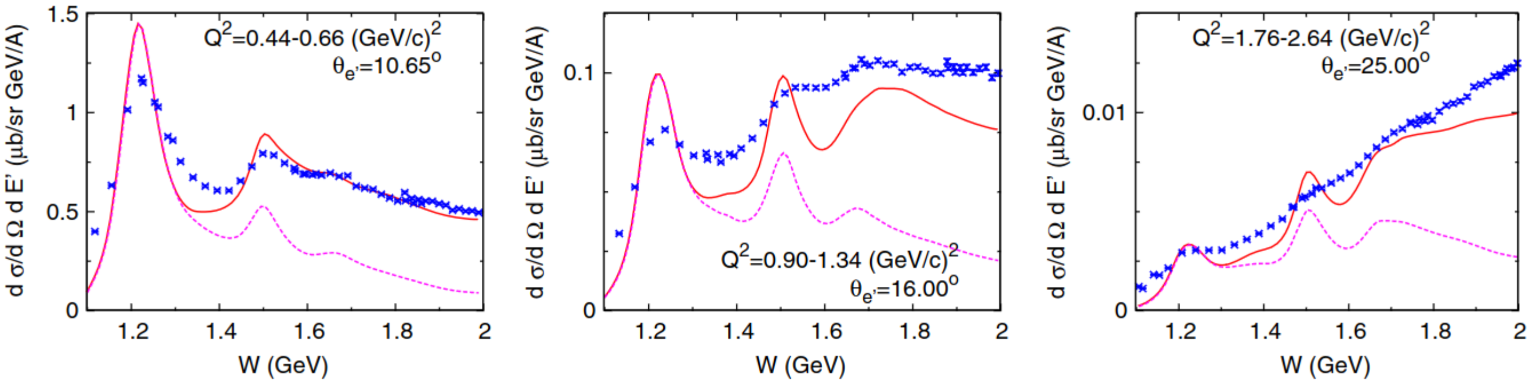
(only the ones we use for comparison)

**MAID07 [Eur.Phys.J.A34:69-97,2007]**

→ Fit ‘neutron’ target exclusive data (deuteron)

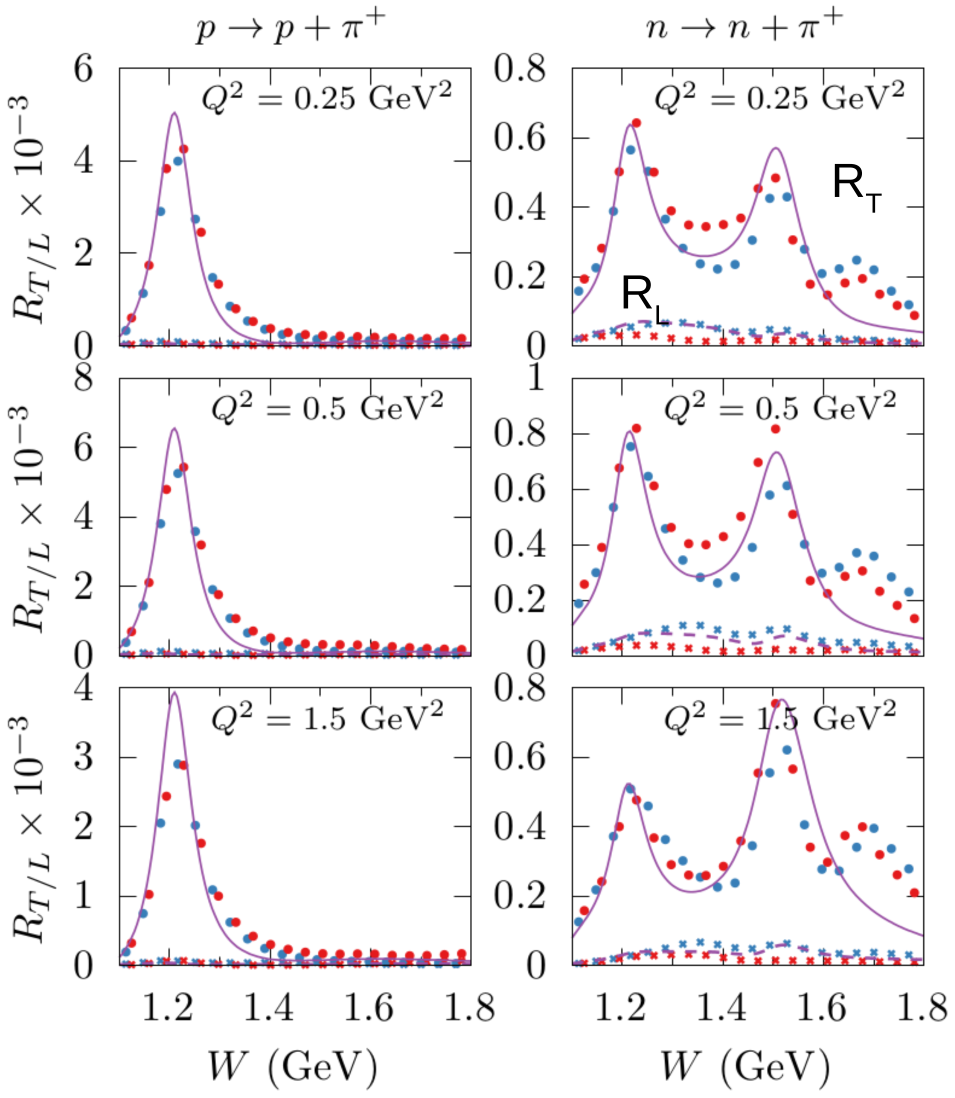
**ANL-Osaka Dynamic Coupled Channels (DCC) [PRD 92, 074024 (2015)]**

→Fit ‘neutron’ target exclusive data (deuteron)
→Fit inclusive structure function on deuteron



Calculation: sum of free nucleon CS, no smearing from deuteron
→ Progress on deuteron FSI in [PRD 99 031301]
+ New CLAS data [Phys. Rev. C 107, 015201 (2023)] over large W-region

# Isovector contribution to charged pion production



For high-E, the VV cross section is

$$\frac{d^2\sigma^{VV}}{dW\,dQ^2} = \frac{G_F^2 \cos\theta_c}{2E^2\,(2\pi)^3}\,\frac{k_W}{1-\epsilon}\,\left(R_T^{VV} + \epsilon R_L^{VV}\right)$$

● ANL-Osaka DCC model

Nakamura, Kamano and Sato,
Phys. Rev. D92, 074024 (2015)

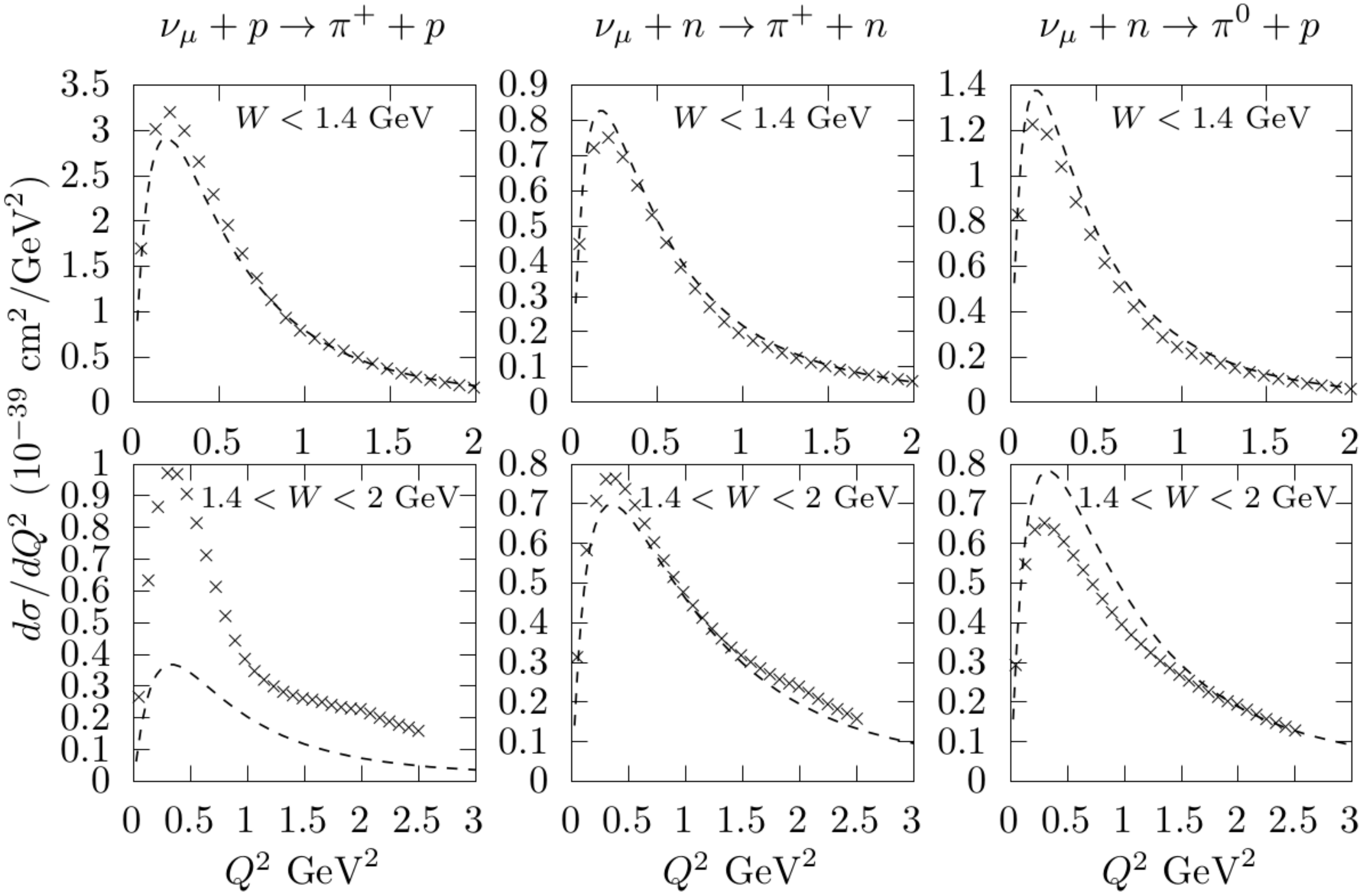
● MAID07

Drechsel, Kamalov and Tiator,
Eur. Phys. J. A34, 69-97 (2007)

● Hybrid model

R. Gonzalez-Jimenez et al.
Phys. Rev. D 95, 113007 (2017)

# Isovector contribution to neutrino pion production: flux-averaged



BEBC flux-folded

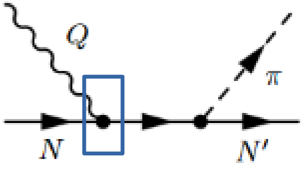
$$\int dE_\nu \Phi(E_\nu)\frac{d\sigma(E_\nu)}{dQ^2}$$

$\langle E_\nu \rangle \approx 20$ GeV

XX = ANL-Osaka DCC
– - = Hybrid model

**How to assign uncertainty to isovector ?**

# Axial couplings to isobars

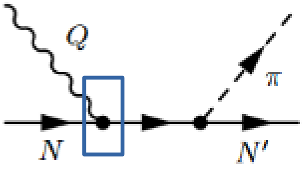


**Spin ½** : $\Gamma^{\mu}_{QRN,A} = G_A \gamma^{\mu}\gamma^5 + \frac{G_P}{M_N} Q^{\mu}\gamma^5$

Naive PCAC and pion-pole dominance inform both couplings at low-$Q^2$

$$F_A(0) = f_\pi \frac{\sqrt{2} f_{\pi NR}}{m_\pi} \qquad G_P = 2M_N(M_R \pm M_N)\frac{F_A(Q^2)}{Q^2 + m_\pi^2}$$

# Axial couplings to isobars



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---

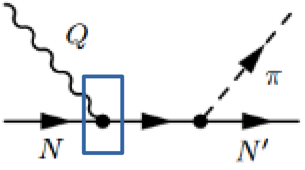
**Spin 3/2** :
$$\Gamma^{\beta\mu}_{A} = \frac{C_3^A}{M}\left(g^{\beta\mu}\not{Q} - Q^{\beta}\gamma^{\mu}\right) + \frac{C_4^A}{M^2}\left(g^{\beta\mu} Q\cdot k_R - Q^{\beta}k_R^{\mu}\right)$$
$$+ \boxed{C_5^A g^{\beta\mu} + \frac{C_6^A}{M^2} Q^{\beta}Q^{\mu},}$$

PCAC and π-pole dominance:

$$C_A^5(0) = f_\pi I_{iso} \frac{\sqrt{2} f_{\pi NR}}{m_\pi \sqrt{3}}$$

$$C_A^6(Q^2) = -M_N^2 \frac{C_A^5(Q^2)}{Q^2 + m_\pi}$$

# Axial couplings to isobars



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$$\Gamma^{\beta\mu}_A = \frac{C_3^A}{M}\left(g^{\beta\mu}\not{Q} - Q^\beta \gamma^\mu\right) + \frac{C_4^A}{M^2}\left(g^{\beta\mu} Q \cdot k_R - Q^\beta k_R^\mu\right) + \boxed{C_5^A g^{\beta\mu} + \frac{C_6^A}{M^2} Q^\beta Q^\mu,}$$

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**No constraints for $Q^2$ dependence**

**No constraint on $C_3$ or $C_4$**

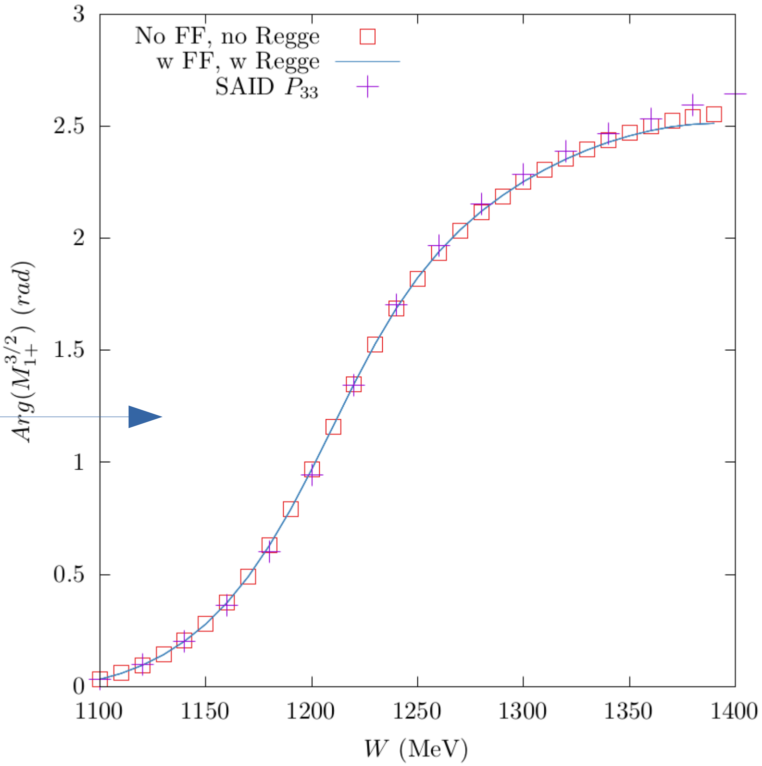
# Fit bubble chamber data in the delta region: *partial* unitarity

L. Alvarez-Ruso, E. Hernández, J. Nieves, M.J. Vicente Vacas
[Phys. Rev. D93, 014016 (2016)]

## Fit of axial Delta form factor(s)

- ANL data
- Adler model for Delta
  → $C_4 = -C_5/4$ , $C_3 = 0$
  → $C_4$ and $C_3$ contributions small

- **Watson's theorem in *Δ* dominated Vector and Axial multipoles**



$$J^\mu = J_V^\mu(s,t,Q^2)\Phi_V(Q^2,s) - J_A^\mu(s,t,Q^2)\Phi_A(Q^2,s)$$

The Olsson phases are fixed by
Requiring that the dominant $P_{33}$ amplitudes
Have the πN scattering phase-shift

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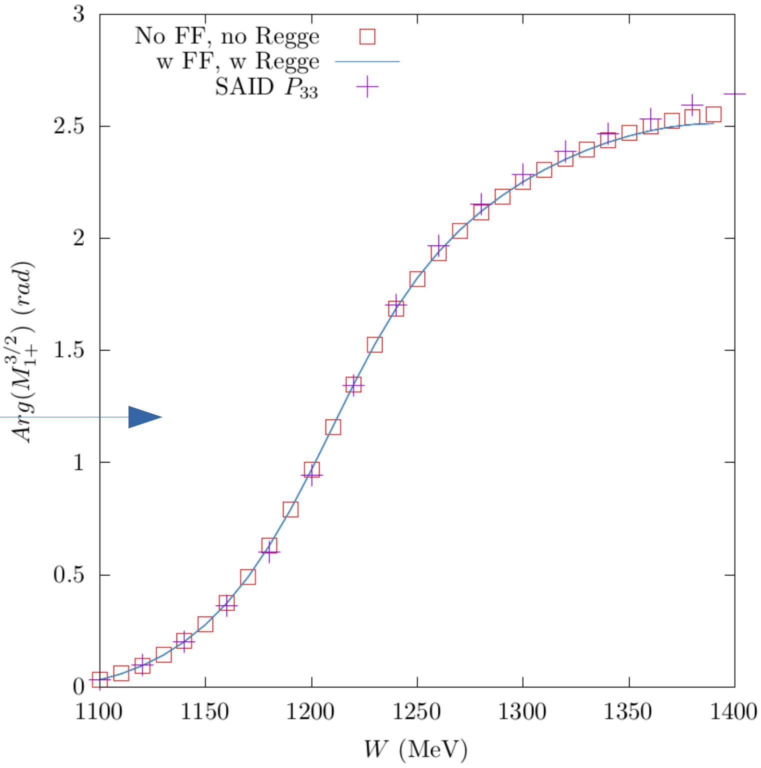

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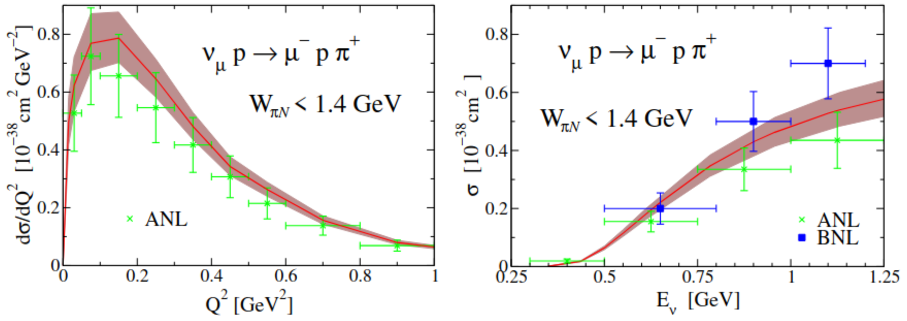
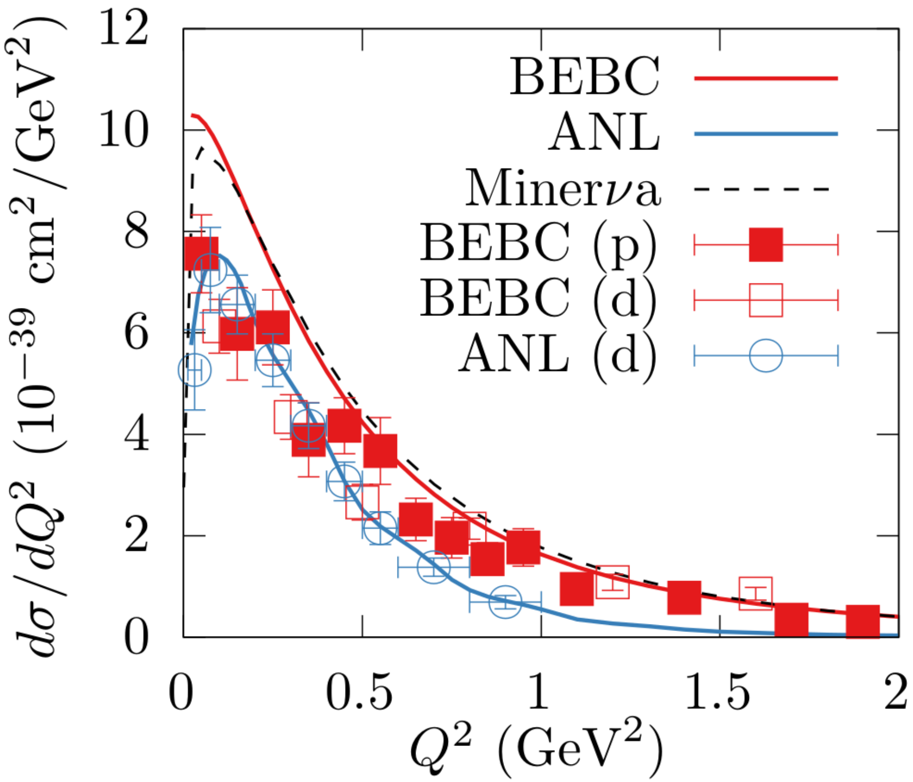


Find $C_5(0)$ consistent with PCAC
parametrize $C_5(Q^2)$

# Comparison to bubble chamber data

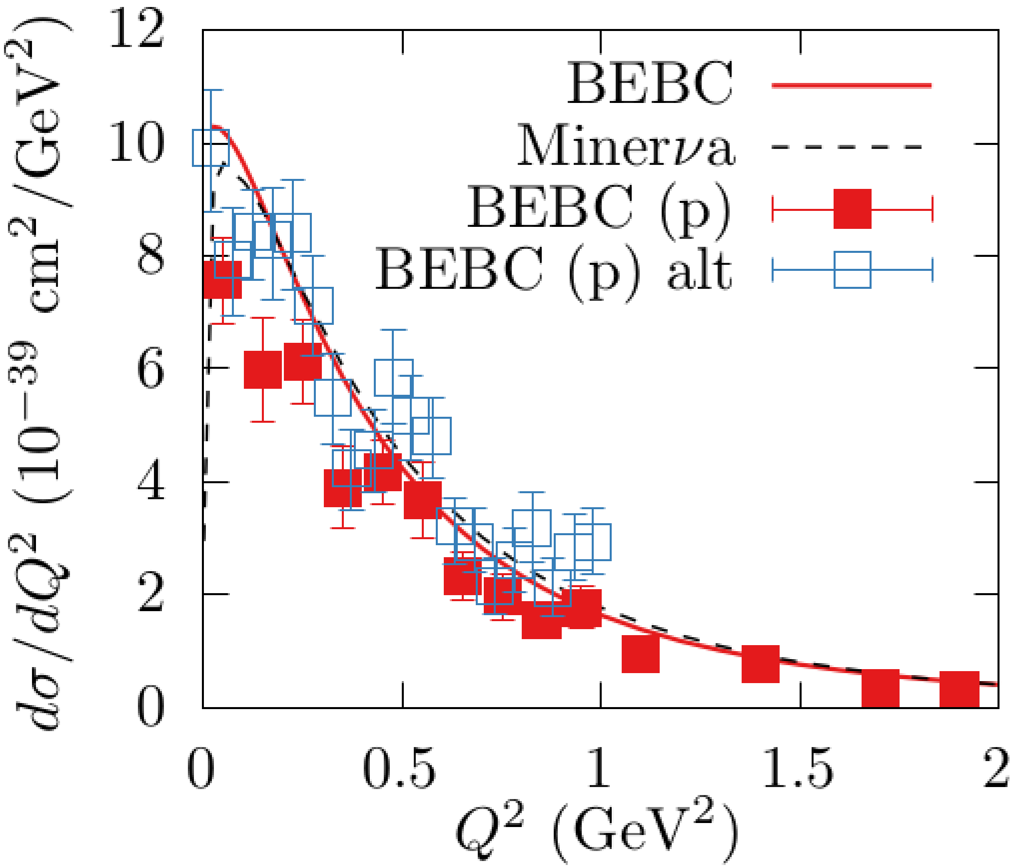
**[A.N. et al. PRD 107 (2023) 5, 053007]**



Data and calculations for W < 1.4 GeV ~ Δ dominated

**The fit to ANL data does not reproduce the BEBC data**
**Even in the $\pi^+$ channel**

# BEBC flux-folded: alternative dataset



Data from BEBC:

[Z. Phys. C 43, 527–540 (1989)]

[Nucl.Phys.B 176 (1980) 269]

- related to low-$p_N$ efficiency
- Unclear how correction is done
- Errors likely underestimated?

**Will revisit data with unitary model (M. Hooft, UGent)**

**Need new data on proton (deuteron)!**

# Axial couplings to higher-mass resonances

For Δ we’re relatively ‘safe’:

- $C_3$ and $C_4$ give small contribution
- Bubble chamber data to constrain $Q^2$-dependence
- → **Need more data for precision goals**

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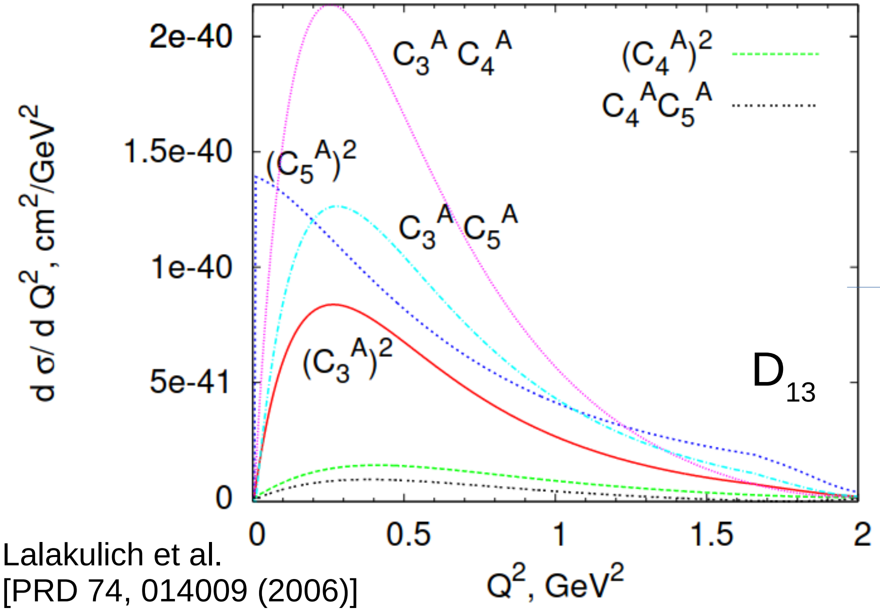
**Higher mass resonances:**

- The contribution from $C_3$ and $C_4$ can be large!



Lalakulich et al.
[PRD 74, 014009 (2006)]

→ 100% uncertainty in axial-axial contribution ?!!

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$C_3$ and $C_4$ give small contribution

Bubble chamber data to constrain $Q^2$-dependence

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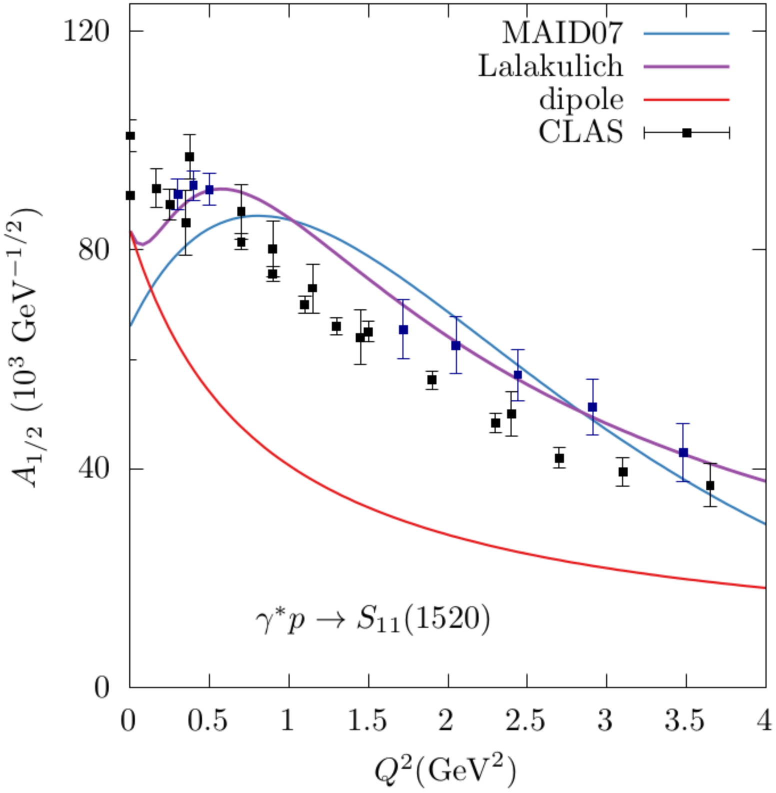
**Higher mass resonances:**

- The contribution from $C_3$ and $C_4$ can be large!
- The form factors can be very far from dipoles!



# Recap: pion production on the nucleon

- Electron and photoproduction data and pion-nucleon scattering
  → Many theoretical approaches and analyses available

- Require neutron measurements for isovector-isoscalar separation
  → Deuteron target data: need to describe FSI
  → New experimental efforts : [CLAS, PRC 107, 015201]
  → How to assess uncertainty due to isovector current ?

**Axial current source of mayor uncertainties:**

- Delta resonance mostly dominated by 1 axial FF
  → Can constrain with bubble chamber data
  → Inconsistencies between datasets, need deuteron

- Higher mass resonances not well constrained

# Pion production on the nucleus

Approximate treatment of production of state X

$$|\mathcal{M}|^2 \approx |\sum_\alpha \langle \Psi_0 | T_{1b} | \psi_\alpha \rangle \langle \psi_\alpha | X \rangle |^2, \longrightarrow \text{One-body transition}$$

$$\approx \sum_\alpha |\langle \Psi_0 | T_{1b} | \psi_\alpha \rangle|^2 |\langle \psi_\alpha | X \rangle|^2 \longrightarrow \text{Classical approximation}$$

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Approximate treatment of production of state X

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$$\approx \sum_\alpha \underline{|\langle\Psi_0|T_{1b}|\psi_\alpha\rangle|^2}\,\underline{|\langle\psi_\alpha|X\rangle|^2} \longrightarrow \text{Classical approximation}$$

Region of validity of these approximations is not fully established!

- Nuclear matrix elements
- ‘final-state interactions’ (intranuclear cascade)

‘split up’ between these effects is not exactly the case

→ ‘Elastic FSI’
→ ‘Inelastic FSI’

# Local RDWIA for pion production on the nucleus: elastic FSI

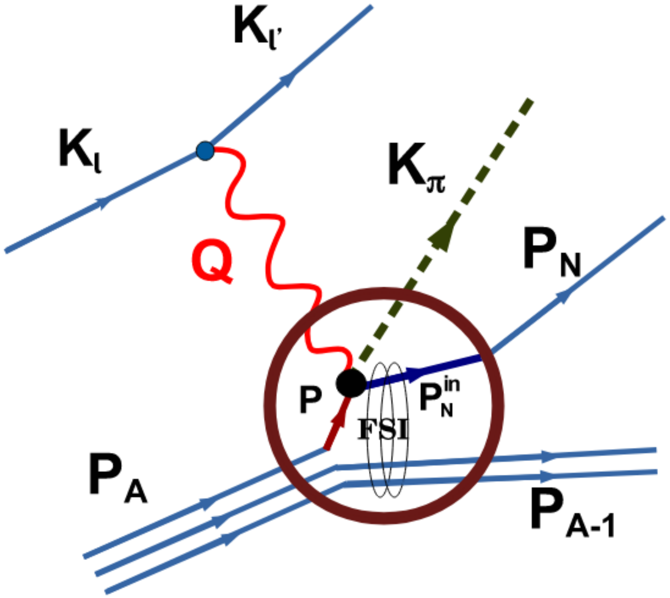
Single-nucleon current in IA:

$$\langle \Psi_0 | T_{1b} | \psi_\alpha \rangle = \frac{1}{(2\pi)^{3/2}} \int d\mathbf{p}'_N \int d\mathbf{p}'_\pi \underline{\overline{\psi}^{s_N} \left(\mathbf{p}'_N, \mathbf{k}_N\right) \phi^* \left(\mathbf{p}'_\pi, \mathbf{k}_\pi\right)}$$

$$\mathcal{O}^\nu \left(q^\mu, p'_N, p'_\pi, p'_m\right) \psi_\kappa^{m_j} \left(\mathbf{p}'_m = \mathbf{p}'_N + \mathbf{p}'_\pi - \mathbf{q}\right) . \quad (13)$$

Wavefunctions in nuclear medium experience FSI

= ‘Elastic FSI’ = exchange of momentum with medium



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$$\mathcal{O}^\nu \left(q^\mu, p'_N, p'_\pi, p'_m\right) \psi_\kappa^{m_j} \left(\mathbf{p}'_m = \mathbf{p}'_N + \mathbf{p}'_\pi - \mathbf{q}\right). \quad (13)$$

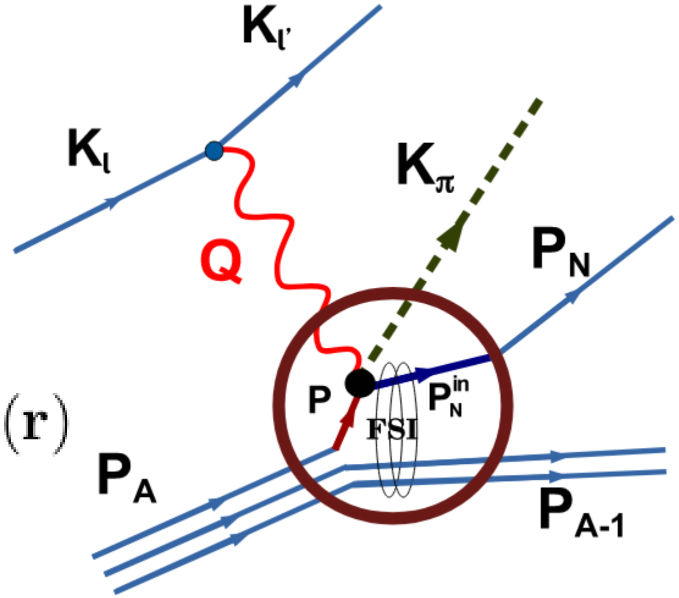
**Local/asymptotic approximation**

$$\mathcal{O}^\mu \left(q, p'_m, p'_N, p'_\pi\right) \rightarrow \mathcal{O}^\mu \left(q, p_m, k_N, k_\pi\right)$$

$$= \int d\mathbf{r} e^{i\mathbf{q}\cdot\mathbf{r}} \phi^*(\mathbf{r}, \mathbf{k}_\pi) \, \overline{\psi}^{s_N} \left(\mathbf{r}, \mathbf{k}_N\right) \mathcal{O}^\nu \psi_\kappa^{m_j} \left(\mathbf{r}\right)$$

**Plane Wave**

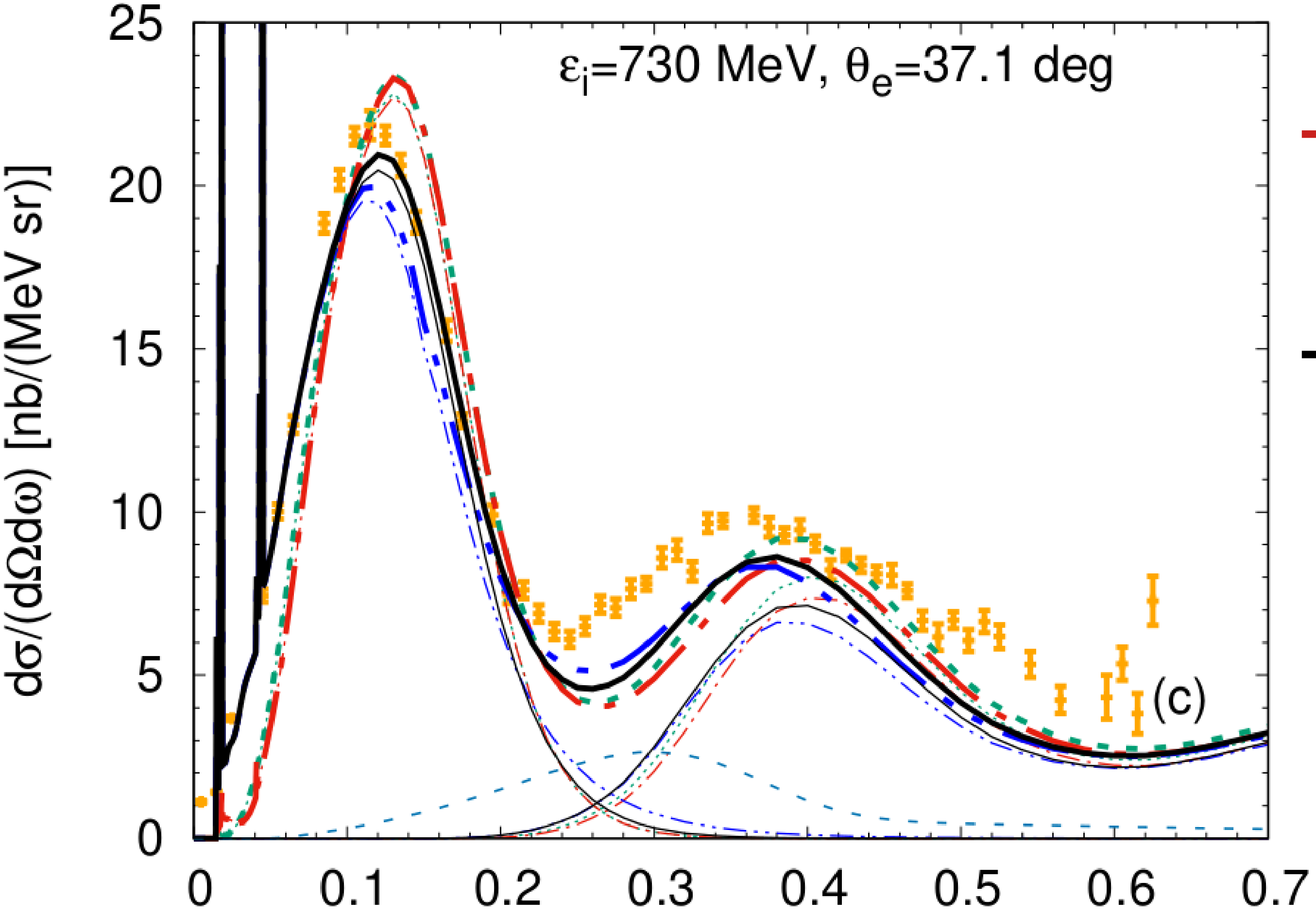
**Distorted wave**



# RDWIA for inclusive electron scattering

[R. Gonzalez-Jimenez, A. Nikolakopoulos, N. Jachowicz, J.M. Udias PRC 100, 045501 (2019)]

Non-negligible at intermediate (SBN) energies:



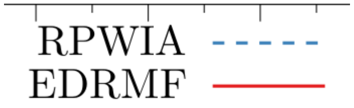
🟥 Plane-waves

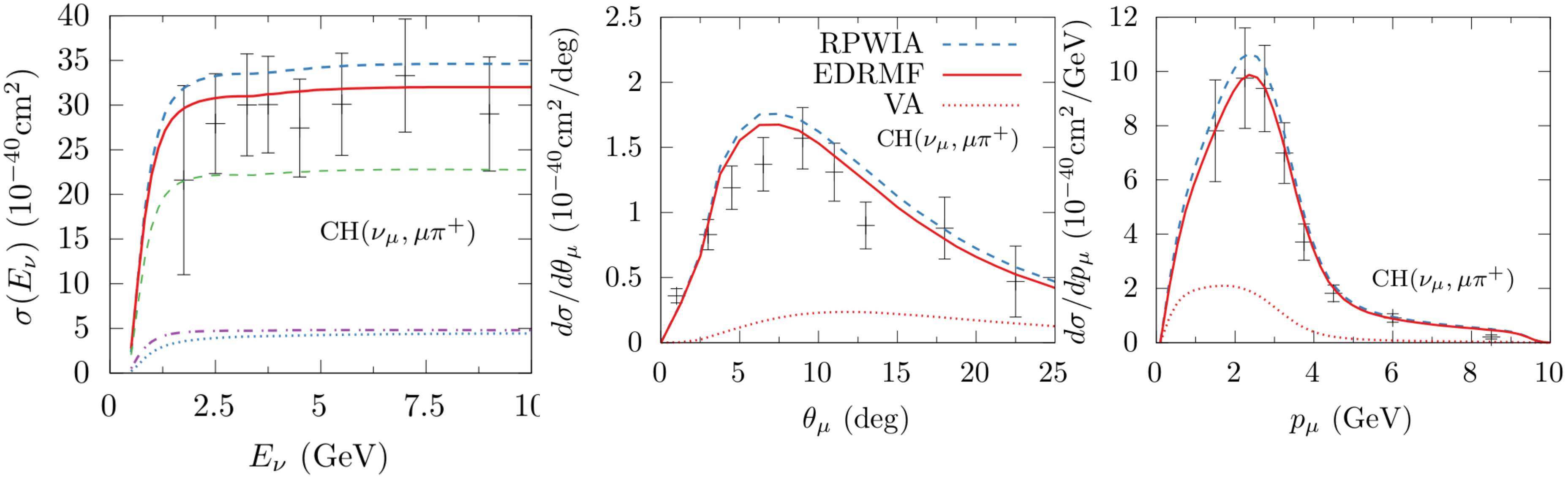
■ Distorted waves

# Neutrinoproduction of charged pions on CH

MINERνA experiment 1$\pi^+$

$W_{(exp)} < 1.4$ GeV





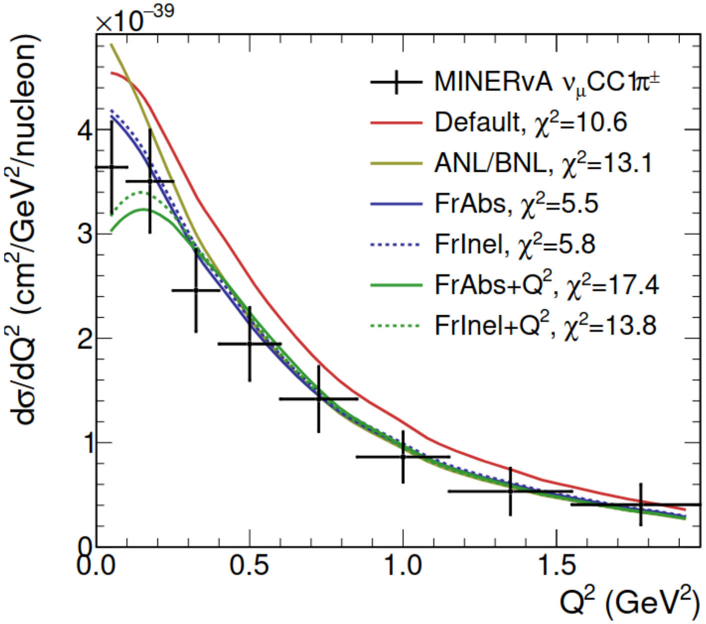
Up to 10% effect for flux-averaged cross section in delta region

# Tensions in the resonance region ?

[MINERvA PRD100, 072005 (2019)]

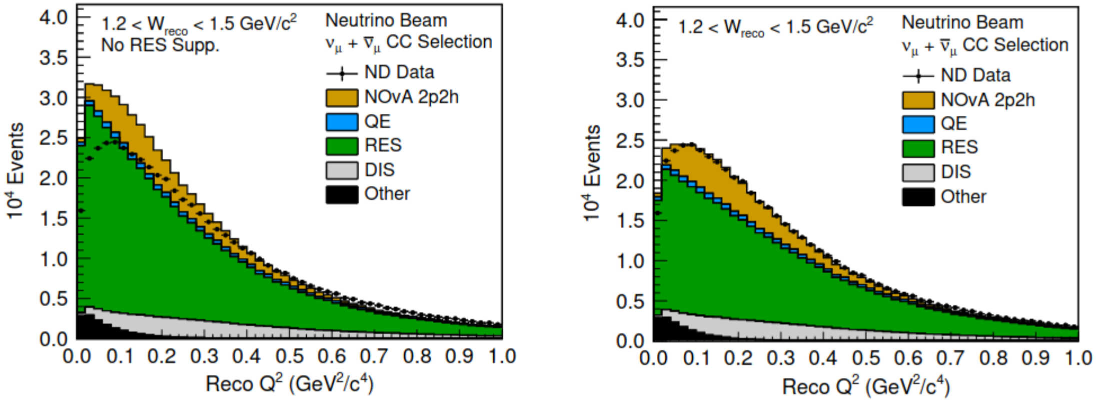
To resolve tension between deuteron / carbon results:

> “An additional *ad hoc* correction for the low-Q2 region, where collective nuclear effects are expected to be large”



[MINERvA PRD100, 072005 (2019)]

Similar correction introduced by NOvA

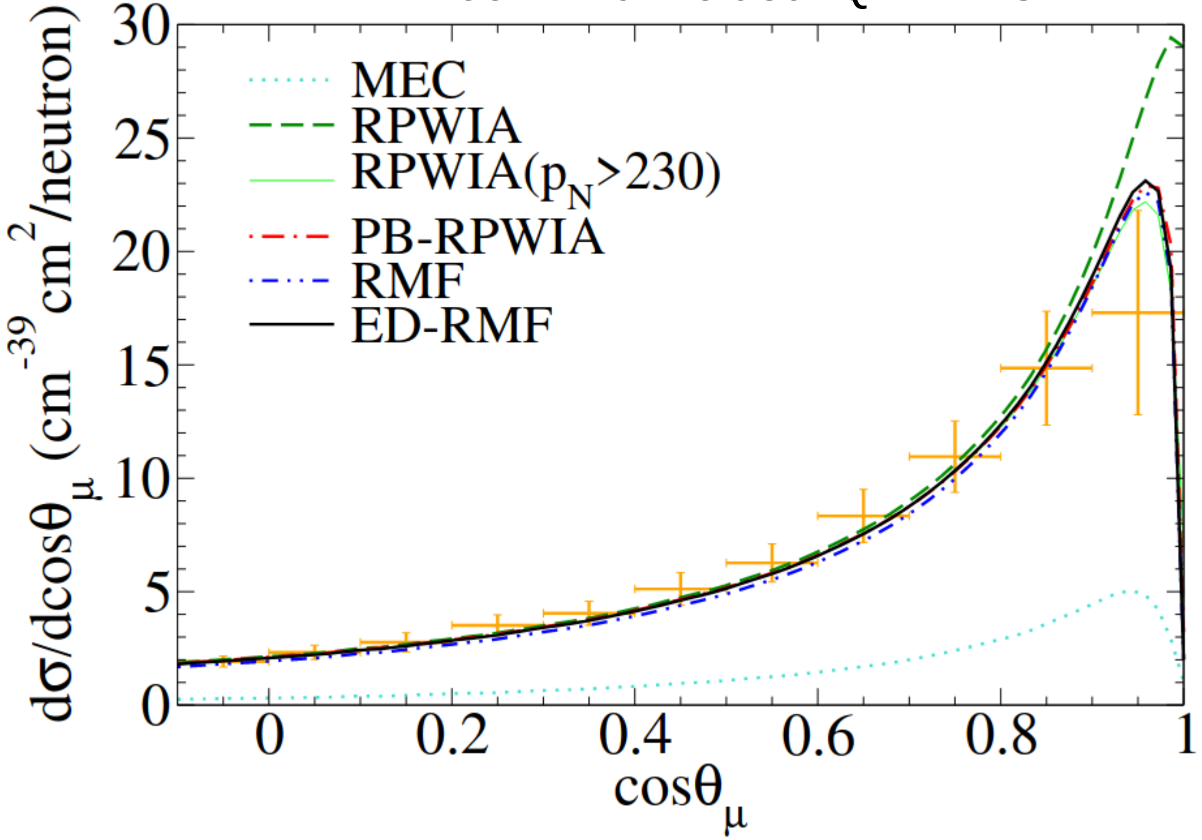


[NOvA Eur. Phys. J. C 80, 1119 (2020)]

# Low-$Q^2$ suppression in the CCQE region: RDWIA

[R. Gonzalez-Jimenez, A. Nikolakopoulos, N. Jachowicz, J.M. Udias PRC 100, 045501 (2019)]

MiniBooNE flux-folded: QE+MEC



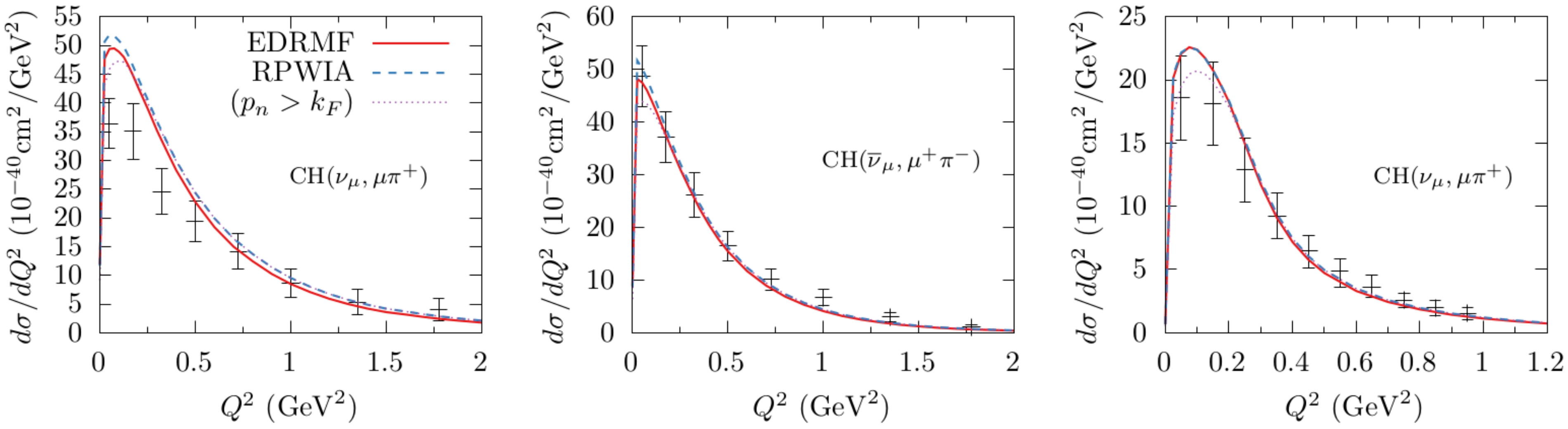
The RDWIA leads to a suppression at small angles (≈low $Q^2$)

# Neutrinoproduction of charged pions on CH: $Q^2$ distributions

MINERνA $1\pi^+$
$W_{(exp)} < 1.4$ GeV

MINERνA $1\pi^-$
$W_{(exp)} < 1.8$ GeV

T2K $1\pi^+$



Overprediction of low-$Q^2$ region in EDRMF and RPWIA

- Many caveats in interpretation of data-theory comparison!

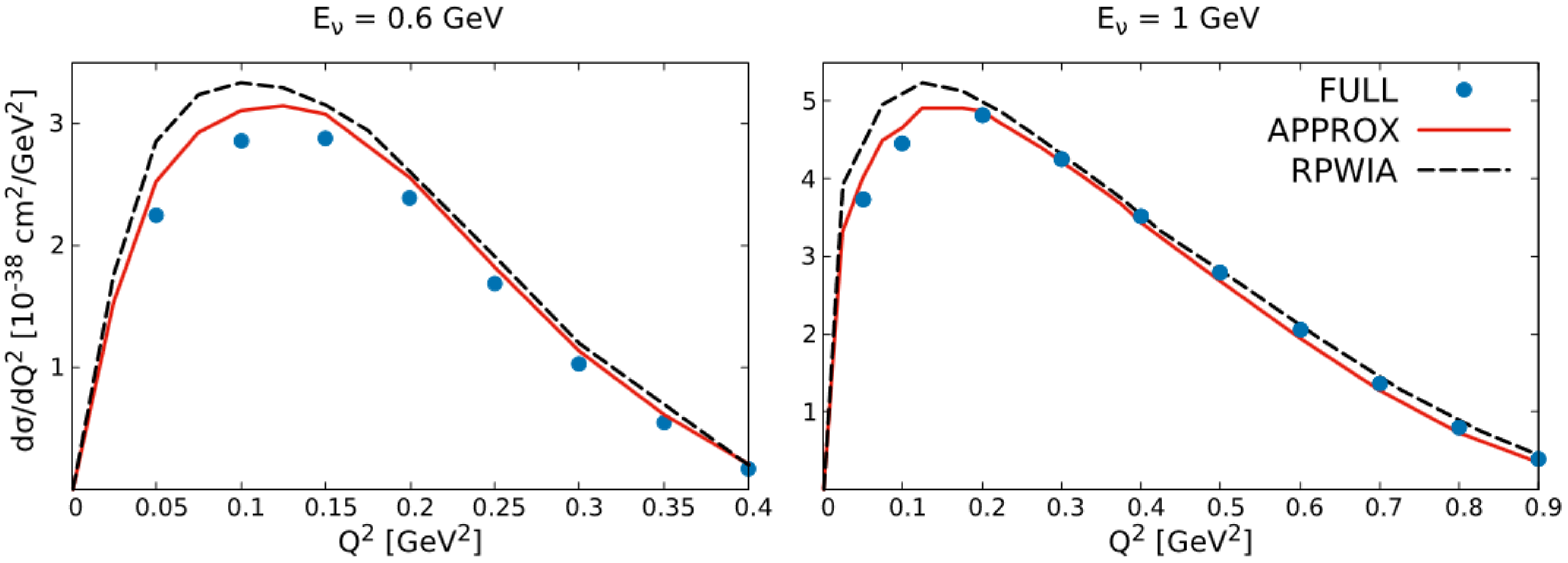
But certainly:
Nucleon FSI does not reproduce a significant reduction in the low-$Q^2$ region!

# Beyond the local approximation

Recent work by J. Garcia Marcos et al. [Phys Rev C 109 024608]

**~~Local/asymptotic approximation~~**

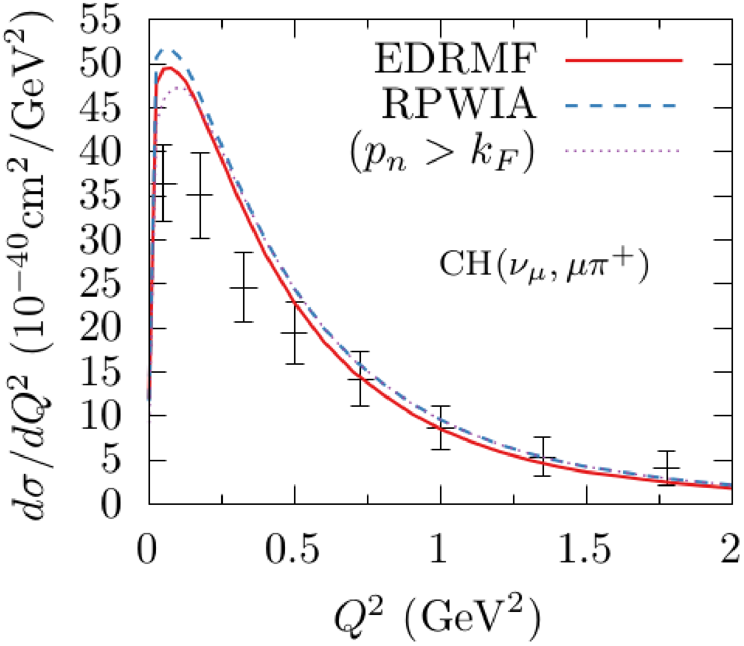
$$\mathcal{O}^\mu\left(q, p'_m, p'_N, p'_\pi\right) \rightarrow \cancel{\mathcal{O}^\mu\left(q, p_m, k_N, k_\pi\right)}$$



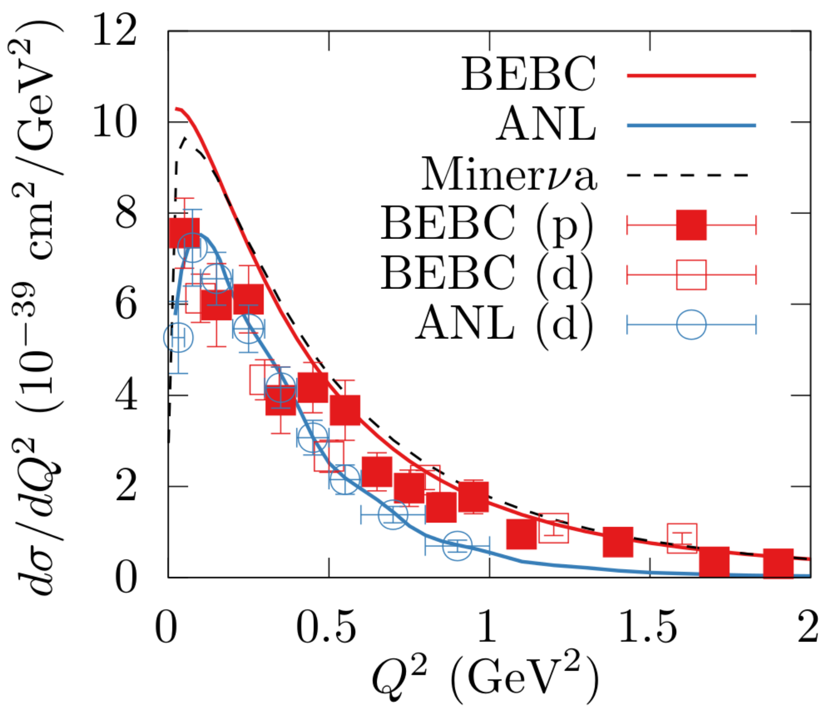
**+Work ongoing to include distorted pion wavefunctions**

# Neutrinoproduction of charged pions on CH: $Q^2$ distributions

MINERνA 1$\pi^+$ on carbon
$W_{(exp)}$ < 1.4 GeV



1$\pi^+$ on proton
$W_{(exp)}$ < 1.4 GeV



Overprediction of low-$Q^2$ region in nuclei **and** nucleons at high-E
→ nucleon level matrix elements not well-constrained ?

# Neutral pion production in MINERvA : a puzzle ?

Isospin-symmetric target : neutrino and antineutrino → sign of VA changes

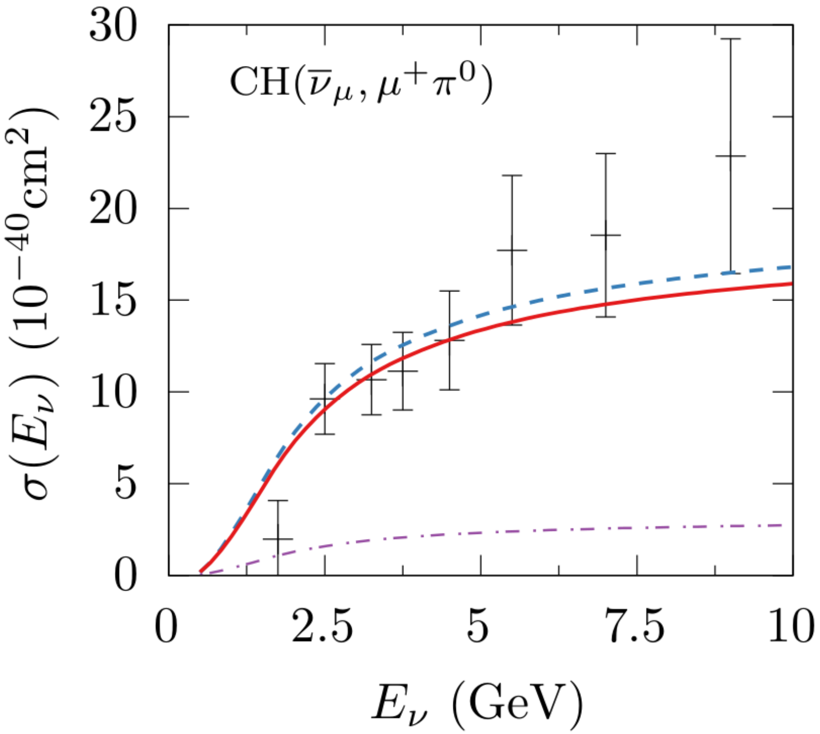
$$|\mathcal{M}|^2 \approx L^{\mu\nu} H_{\mu\nu}^{VV+AA} \pm 2|L^{12}| \ \mathrm{Im}(H_{12}^{VA})$$

# Neutral pion production in MINERvA : a puzzle ?

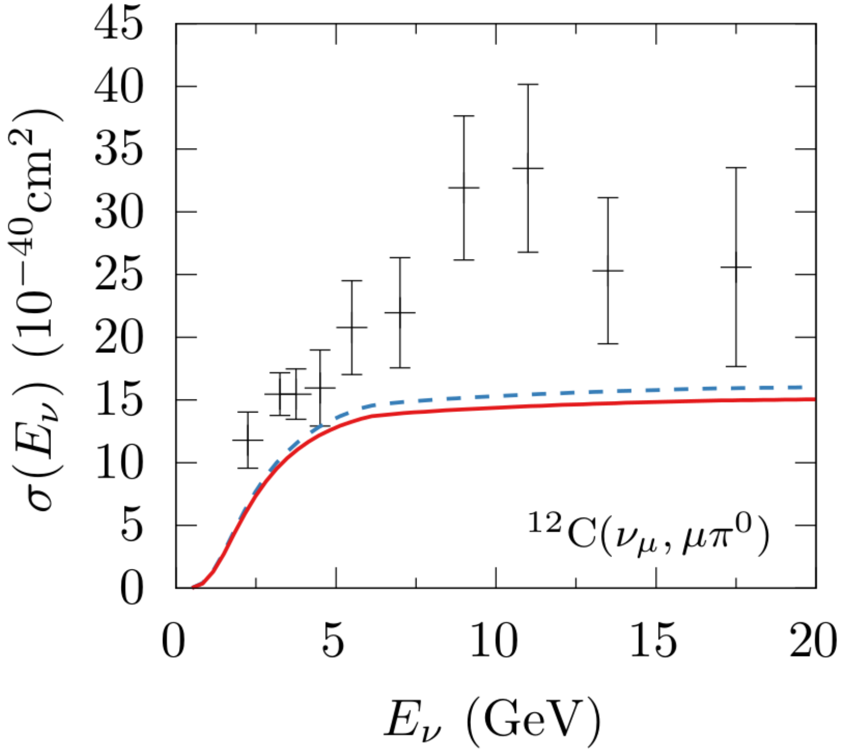
Isospin-symmetric target : neutrino and antineutrino → sign of VA changes

$$|\mathcal{M}|^2 \approx L^{\mu\nu} H^{VV+AA}_{\mu\nu} \pm 2|L^{12}|\ \mathrm{Im}(H^{VA}_{12})$$

antineutrino

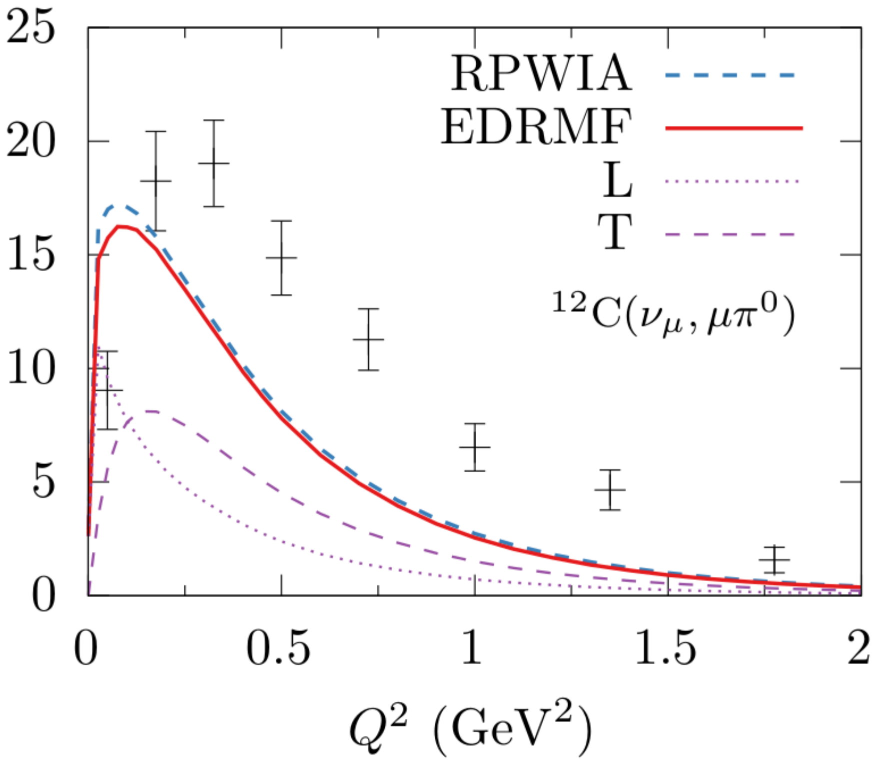


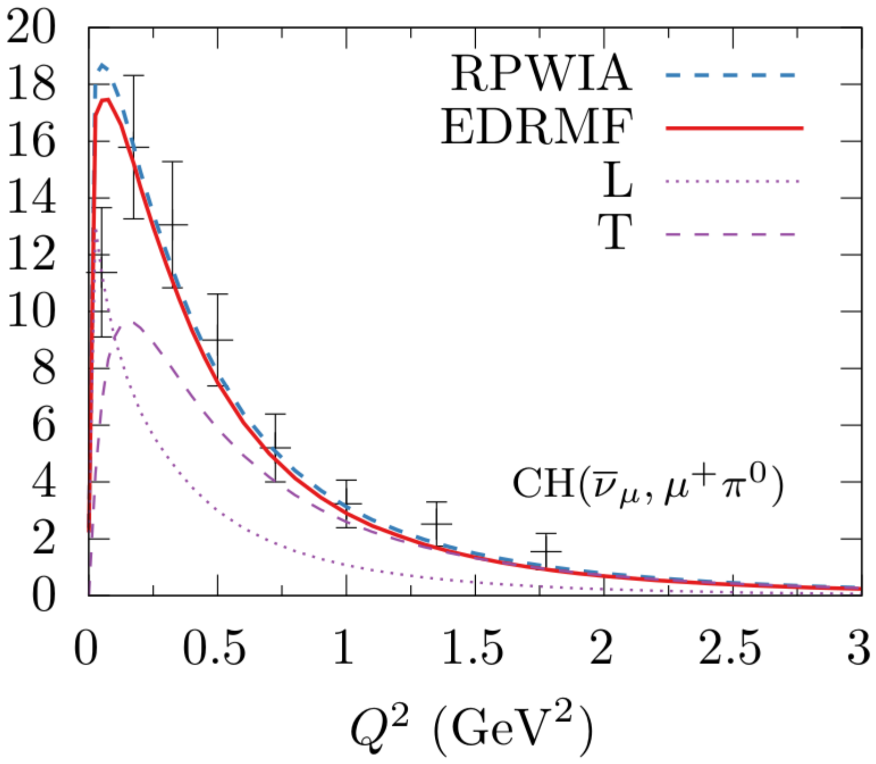
neutrino



# Neutral pion production in MINERvA : a puzzle ?

Isospin-symmetric target : neutrino and antineutrino → sign of VA changes


neutrino


antineutrino

Sensitive to unconstrained second resonance region!

# Pion production on the nucleus: cascade models and generators

Approximate treatment of production of state X

$$\sum_{\alpha} \underline{|\langle \Psi_0 | T_{1b} | \psi_\alpha \rangle|^2} \, \underline{|\langle \psi_\alpha | X \rangle|^2}$$

- Nuclear matrix elements
- ‘Inelastic final-state interactions’ → intranuclear cascade (INC)

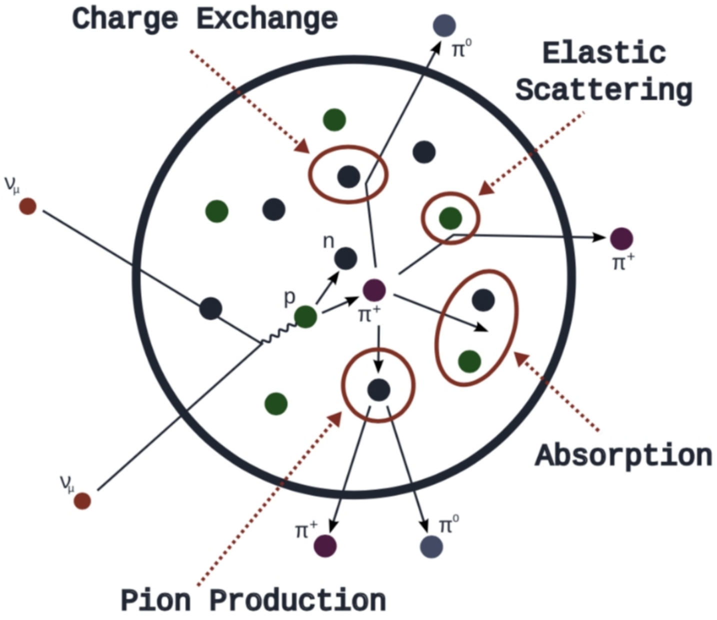


*Fig: T. Golan*

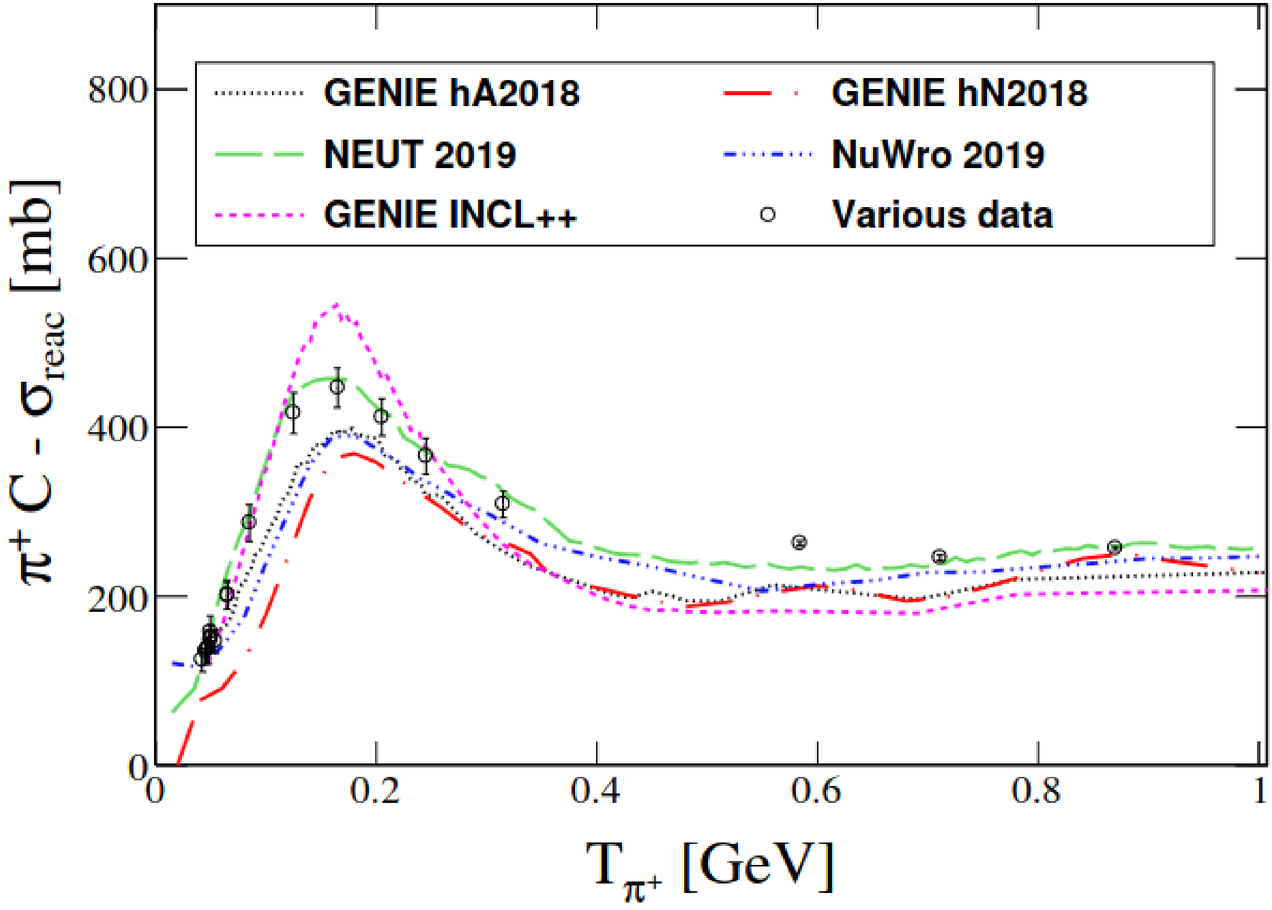
- Classical treatment of coupled-channel problem

- Momentum states propagated

- Constrained by hadron-nucleus interactions

# Pion production on the nucleus: cascade models and generators

Several INC implementations in different generators:

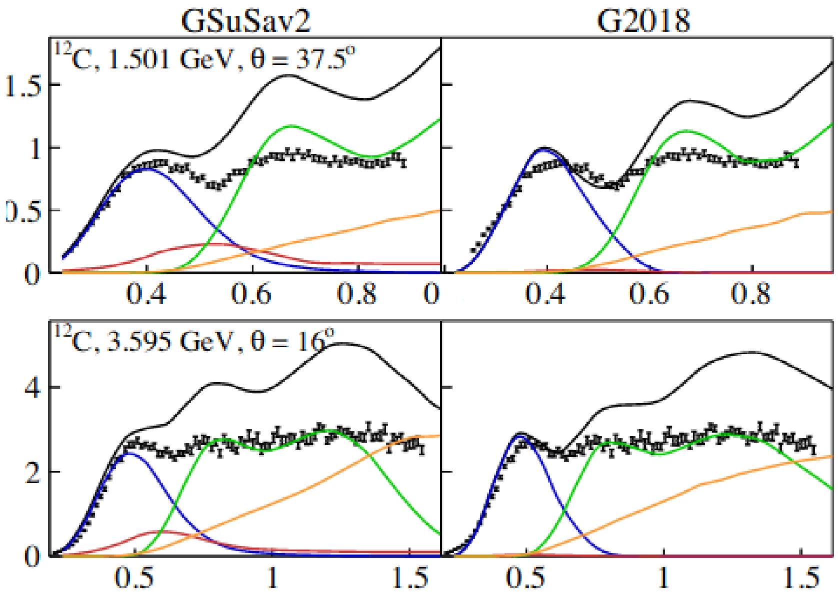


[Dytman et al. Phys. Rev. D 104, 053006 (2021)]

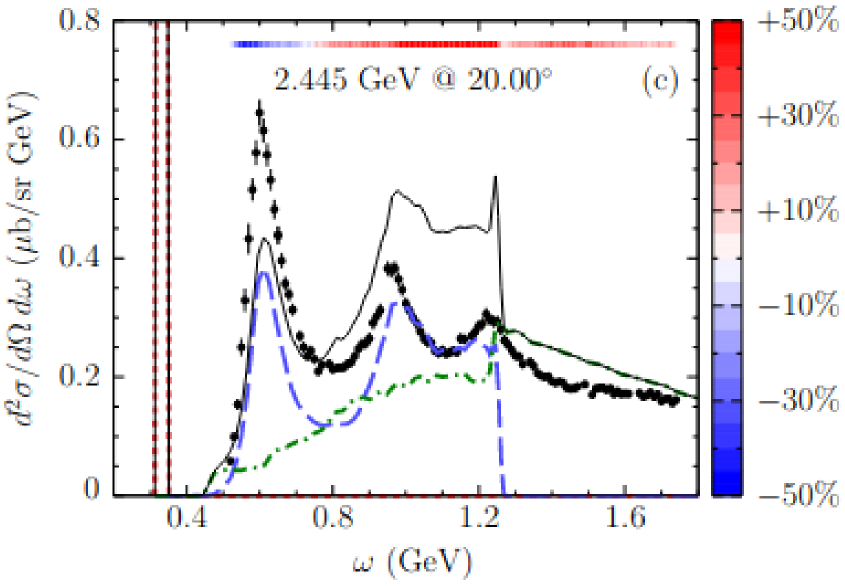
# Pion production on the nucleon in generators

Hard to judge data comparisons when **input to the generator** is not well established

Examples from GENIE with electron-scattering data:



E4nu collaboration
[PRD 103, 113003 (2021)]



Ankowski & Friedland
[PRD 102, 053001 (2020)]

# Pion production on the nucleon in generators

**GIBUU:**

-Using MAID07 for vector current

- Axial from Lalakulich and Leitner e.g [Phys.Rev.C79:034601,2009]

**GENIE:**

-Ongoing efforts to improve electron scattering resonance models and upcoming analysis of CLAS pion production data.

- Large scale fits of neutrino data [J. Tena Vidal et al. PRD 104, 072009]

**NEUT:**

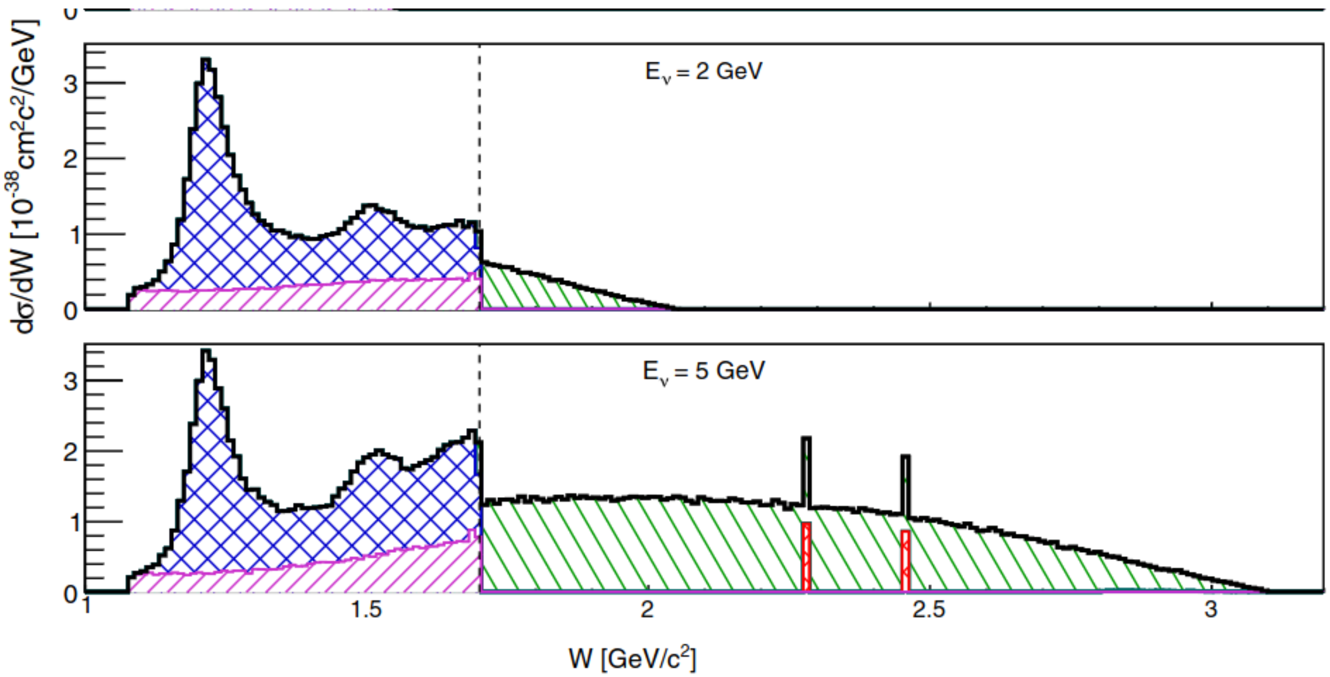
- Implementation M. Kabirnezhad amplitudes [Phys. Rev. D 97, 013002]

→ Recent high-quality fits of ep data: [Phys. Rev. C 107, 025502]

Will discuss some ongoing/recent efforts with
**NuWro and ACHILLES**

# Pion production in generators typically

- Incoherent sum of resonances
- Angular distributions $\pi$N isotropic in CMS
- ‘Non-resonant background’ extrapolated from DIS

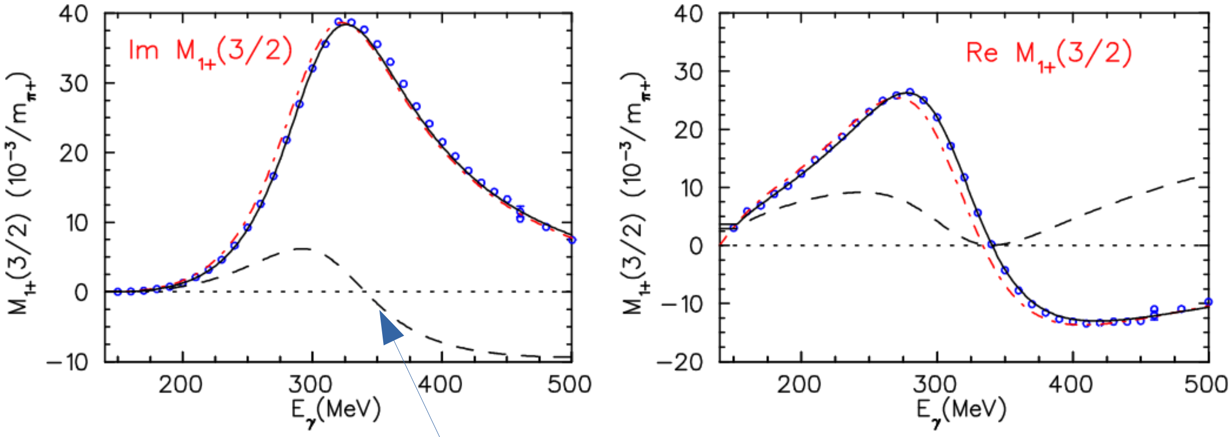


[J. T. Vidal et al. Phys. Rev. D 104.072009]

# Pion production in generators typically

- **Incoherent sum of resonance contributions**

A ‘resonance’ is a structure in specific πN partial waves: (I, J, P) e.g. Δ = $P_{33}$



**Inclusive cross sections** are incoherent sums of squared partial waves

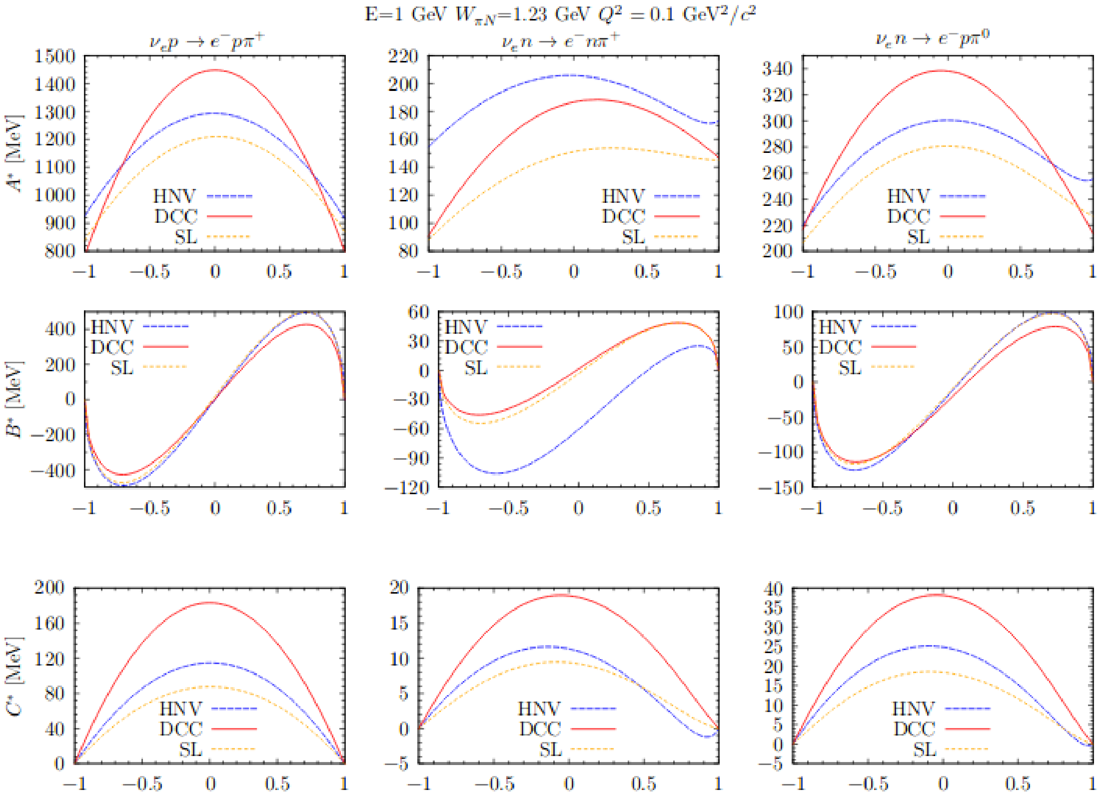
→ Can get away with incoherent sums of resonance contributions

→ Cannot incoherently add ‘Background’ contribution

→ Lose information on hadron kinematics

# Pion production in generators: hadron observables

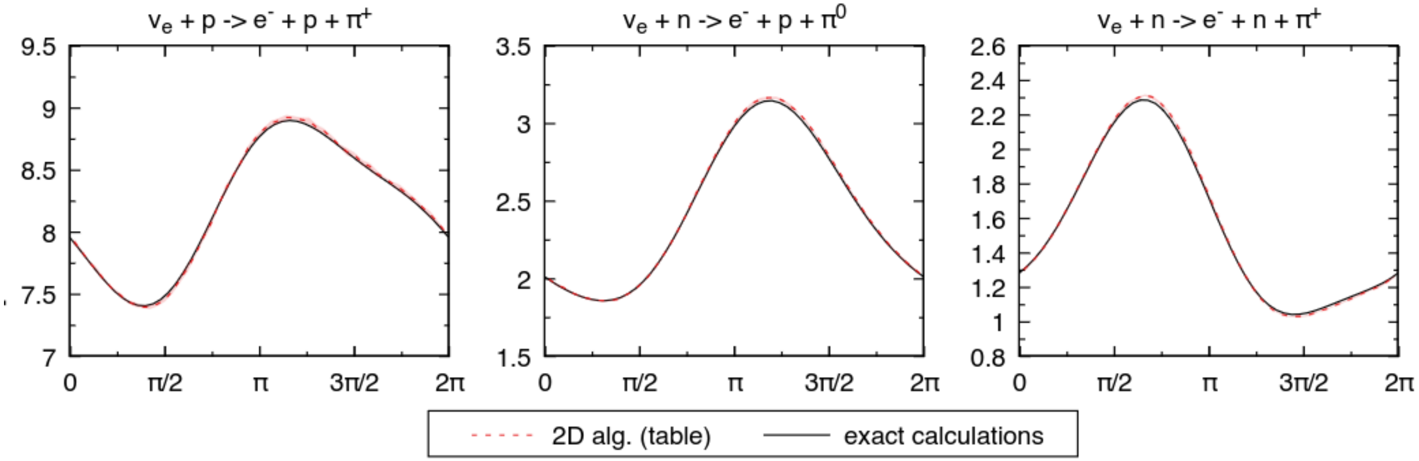
- **Angular distributions πN** often taken as isotropic in CMS
- In reality **they are not:** $$\frac{d\sigma}{dQ^2 dW d\Omega^*_\pi} = \frac{\mathcal{F}^2}{(2\pi)^4}\frac{k^*_\pi}{k_l^2}\times\left[A + B\cos(\phi^*)\, C\cos(2\phi^*) + D\sin(\phi^*) + E\sin(2\phi^*)\right]$$



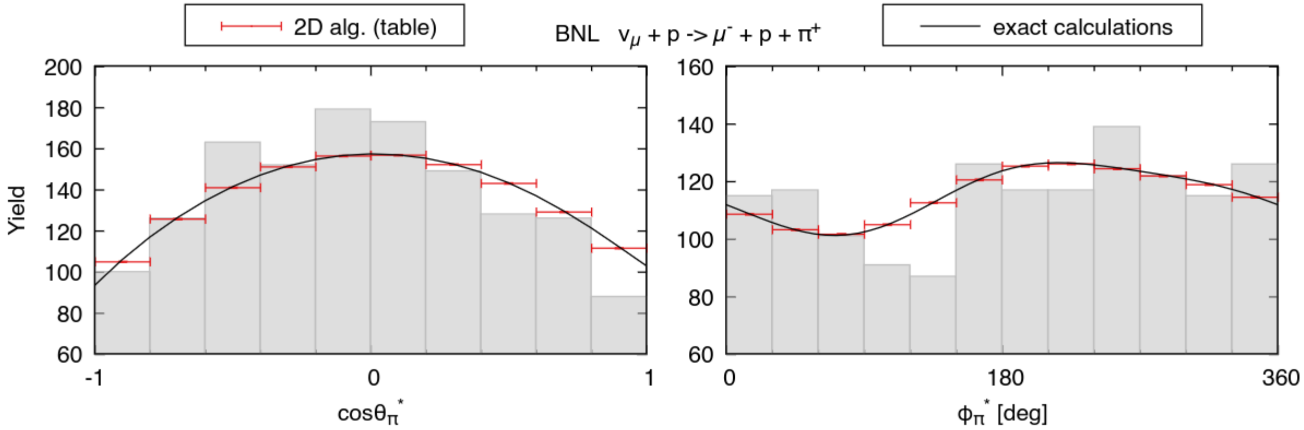
[J. E. Sobczyk et al. Phys. Rev. D 98, 073001 (2018)]

# Structure functions for neutrinos in NuWro

Full implementation of cross section with all interference in NuWro : [Niewczas, PRD 103, 053003 (2021)]
**Completely general: works for every model**



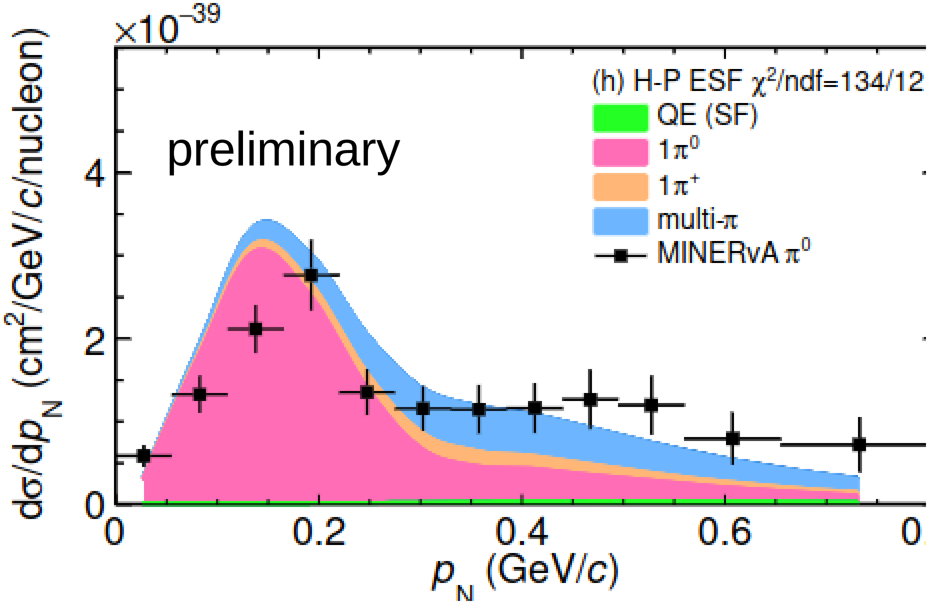
Hydrogen/deuteron bubble chambers:

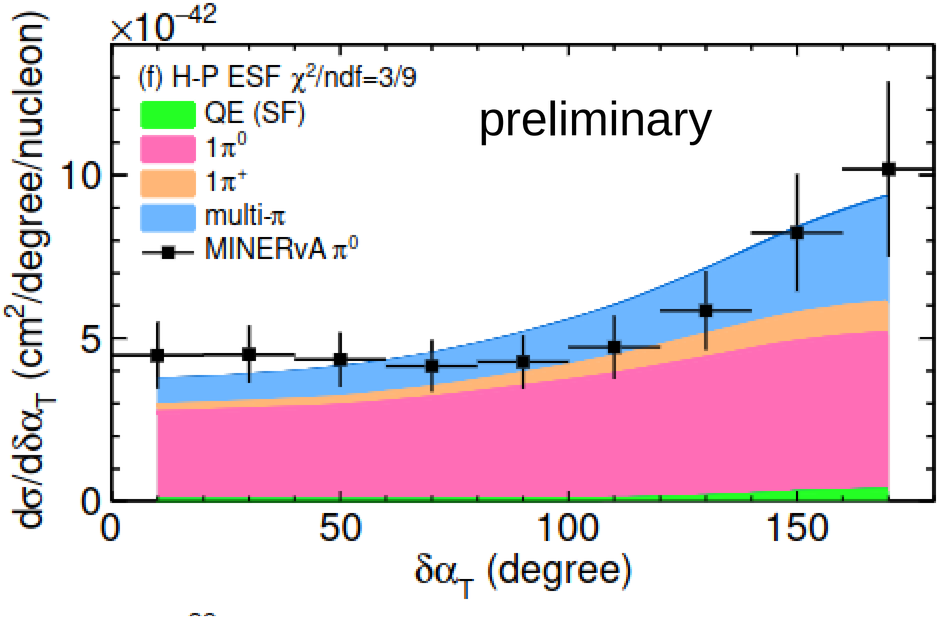


**Will see results for flux-averaged data in near future**

# Pion production in NuWro

**Coming soon:**
Hybrid model fully implemented and coupled to the NuWro INC

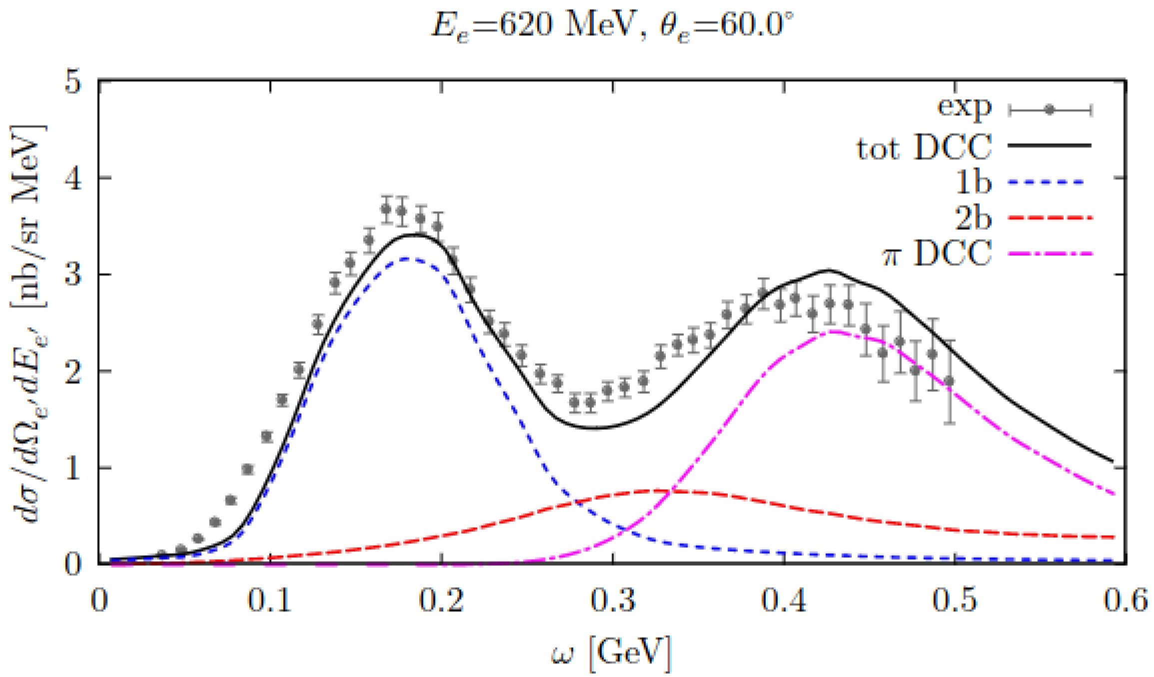




In collaboration with **Qiyu Yan** , Lu Xianguo, J. Sobczyk, K. Niewczas, N. Jachowicz and R. Gonzalez-Jimenez

# Pion production in ACHILLES

**Ongoing:**
Fully exclusive implementation of ANL-Osaka DCC amplitudes for **electron and neutrino interactions** [Nakamura et al. Phys. Rev. D 92, 074024 (2015)]

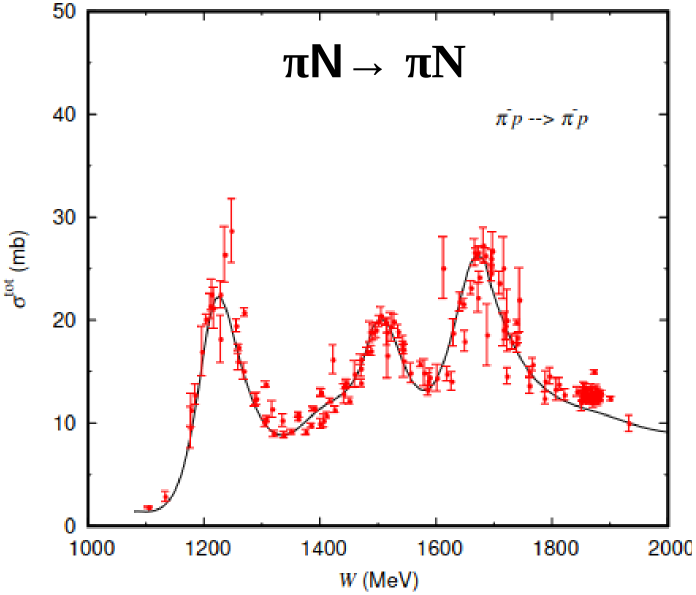


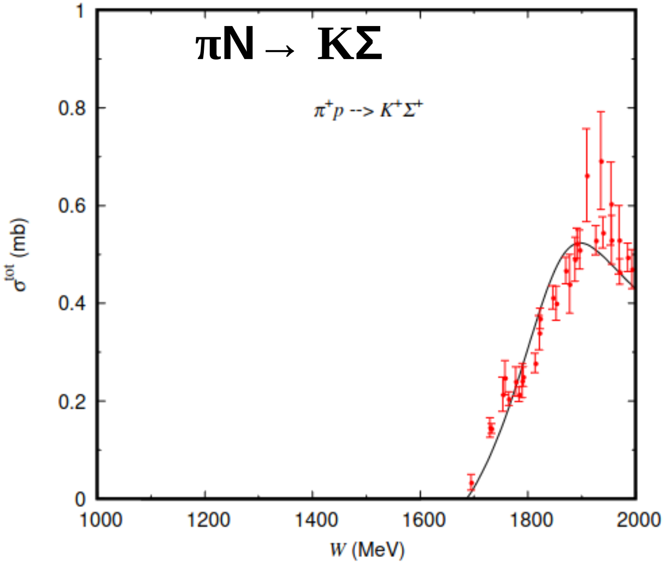
[N. Rocco et al. Phys. Rev. C 100, 045503 (2019)]

Work with Noah Steinberg, N. Rocco, J. Isaacson

# Pion production in ACHILLES: Intranuclear cascade

- Include in ACHILLES INC [Isaacson et al. Phys. Rev. C 103, 015502]
  **Meson-baryon interactions for all octet mesons:**
  **πN, ηN, KΛ, KΣ,** from ANL-Osaka DCC analysis





- Including interactions in the framework at the **amplitude level**
  → Access to fundamental parameters
  → Allows to include nuclear medium modifications

# Concluding remarks:

Can (partly) study and validate nuclear matrix elements and FSI with electrons

→ Nucleon-level amplitudes are relatively well-known from experiment

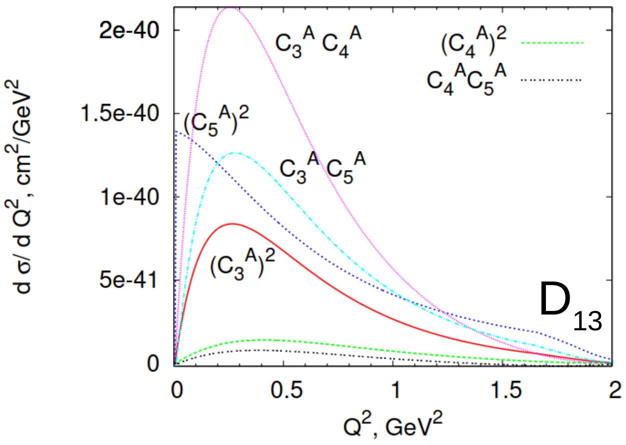
**Mayor uncertainties in nucleon-level amplitudes for neutrinos!**

→ **Isovector contribution**
→ **Axial current**

# Where to go from here ? : nucleons

Difficulties in describing data in the delta region
→ This is more severe for higher-mass resonances

Lalakulich et al.
[PRD 74, 014009 (2006)]



Constraints could come from:

- Progress in (l)QCD for axial form-factors
  [L. Barca et al. PoS LATTICE2021, 359 (2022)]

- ChPT calculations with delta d.o.f
  [Yao et al. Phys. Rev. D 98, 076004 (2018)]

- Quark-Hadron duality
  [ T. Sato, Eur. Phys. J. ST 230, 4409 (2021)]

- **Modern experiments on hydrogen & deuterium**
  **L. Alvarez-Ruso et al., (2022), arXiv:2203.11298 [hep-ex]**

# Other stuff

# Resonance models in generators

- **Incoherent sum of resonance contributions**

Can decompose the pion production amplitude in s-channel angular momenta

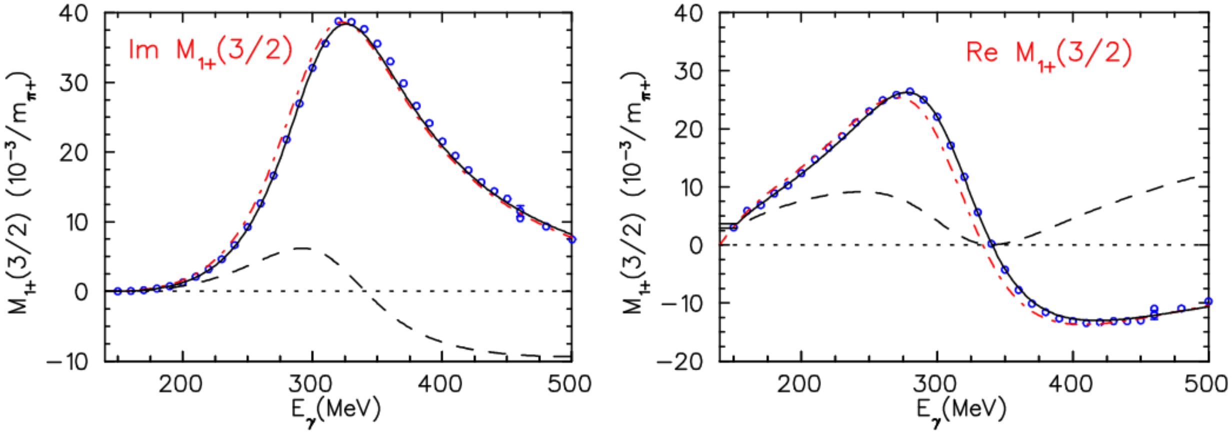
$$H^r_{\lambda',\lambda}(\Omega^*) = \sum_{J=\frac{1}{2}}^{3/2} \left(J+\frac{1}{2}\right)\langle\lambda'|T^J|r,\lambda\rangle D^J_{M,\lambda'}(\Omega^*) \qquad (M=\lambda-r)$$

And definite parity:

$$T^{J,P=\pm} = \langle r-\lambda|T^j|\lambda'\rangle \pm \langle r-\lambda|T^J|-\lambda'\rangle$$

(For EM interactions → 6 independent amplitudes $M_{l\pm}, E_{l\pm}, S_{l\pm} \quad (l = J \pm 1/2)$

A resonant structure is found in specific πN partial waves: (I, J, P) e.g. Δ = $P_{33}$



[MAID07] photoproduction

# Regge model for the high-W background

Background in non-linear σ model

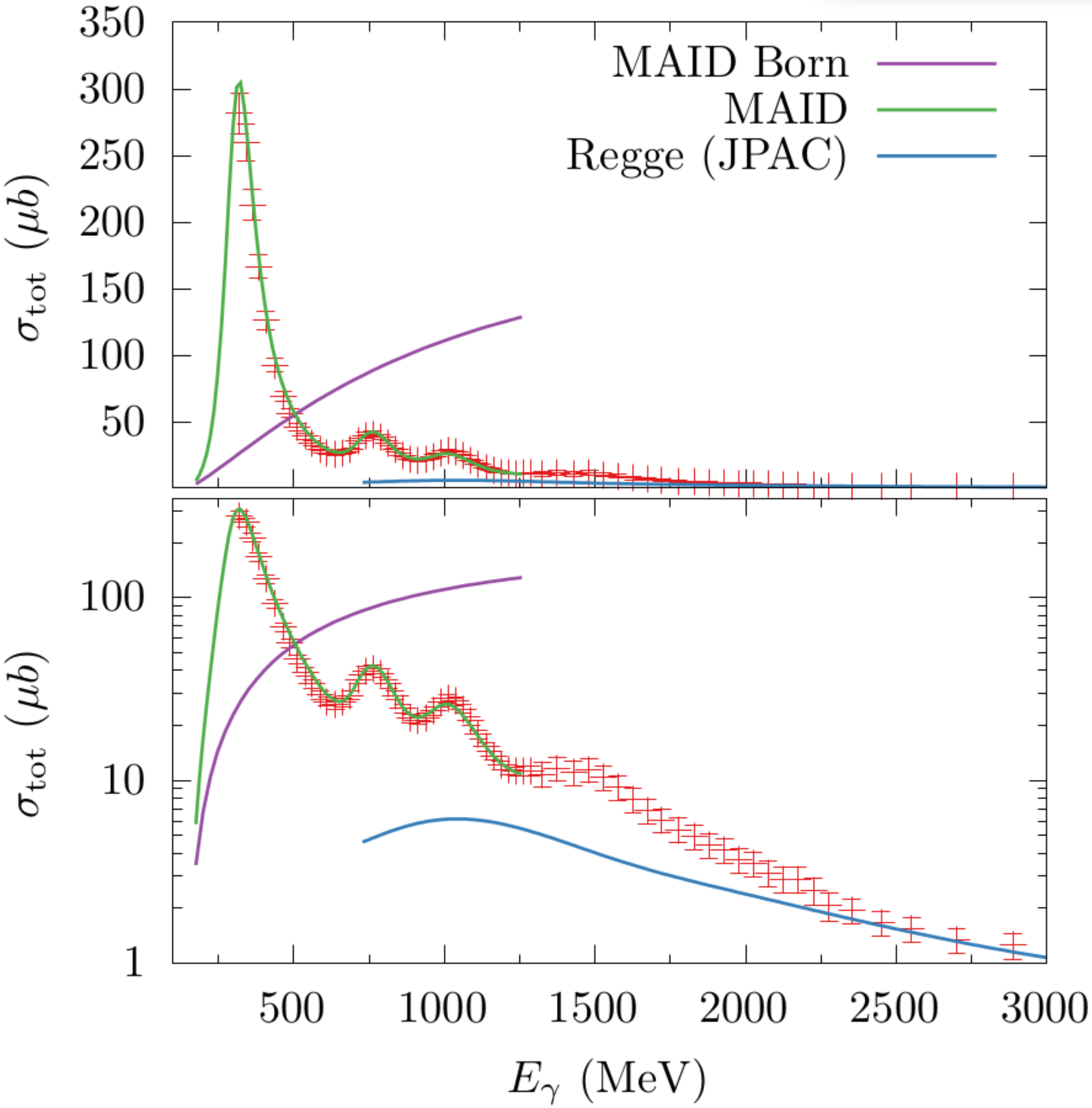
Hernandez, Nieves, Valverde

[Phys.Rev.D76, 033005 (2007)]

The effective tree-level terms are suitable at low-W

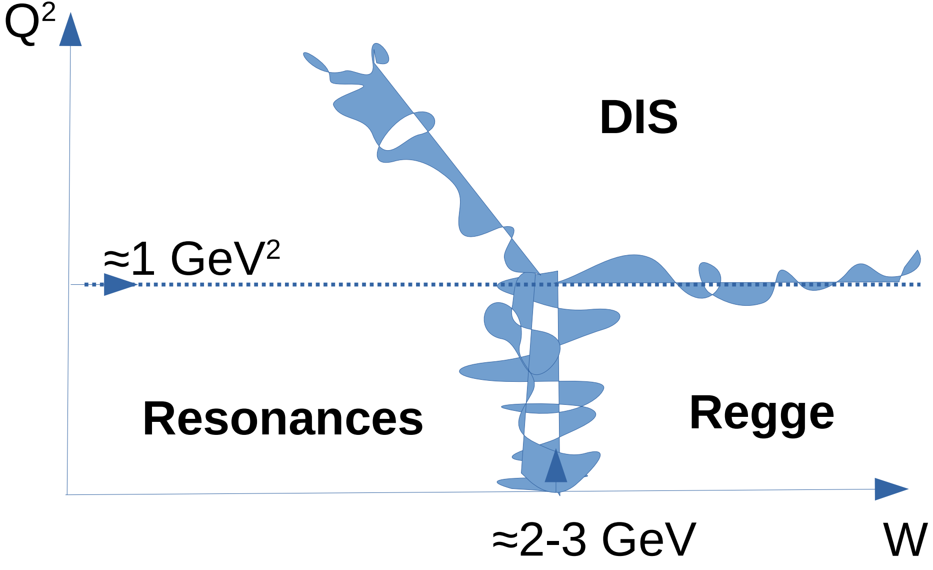
For intermediate W adjusting the phases of resonant contributions is necessary e.g. in MAID

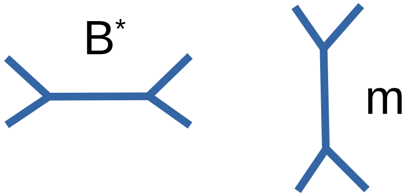
At high-W (low $Q^2$) a Regge approach describes the amplitude



# Regge model (briefly)

R. Gonzalez-Jimenez et al.
[Phys. Rev. D 95, 113007 (2017)]



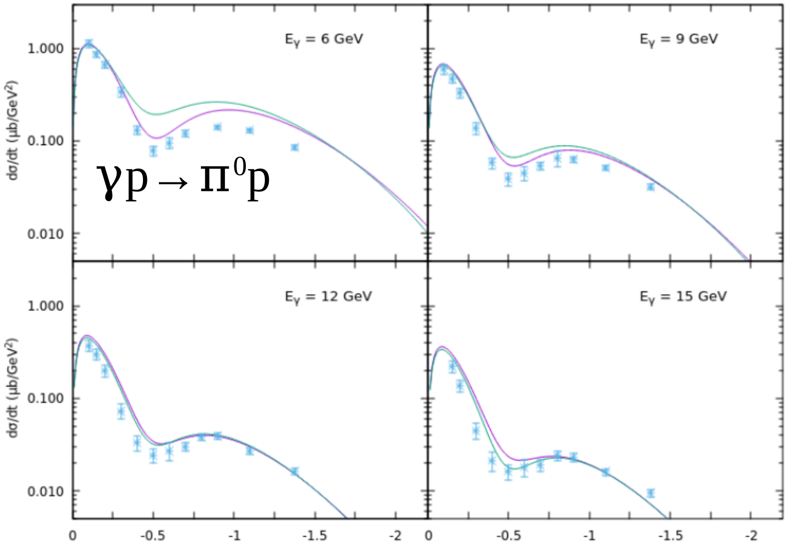


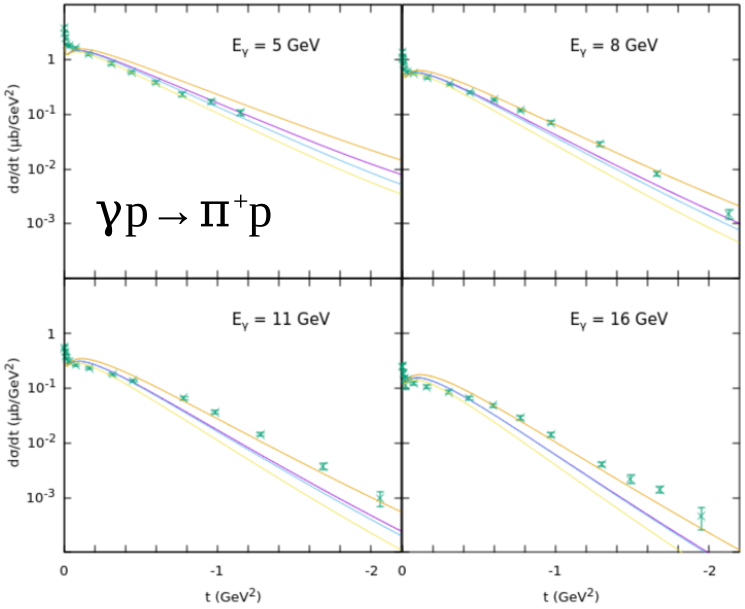
For high-s
Small -t
↓
t-channel meson d.o.f

$$s >> M_{B^*}^2 \approx (1\text{-}2\ \text{GeV})^2$$

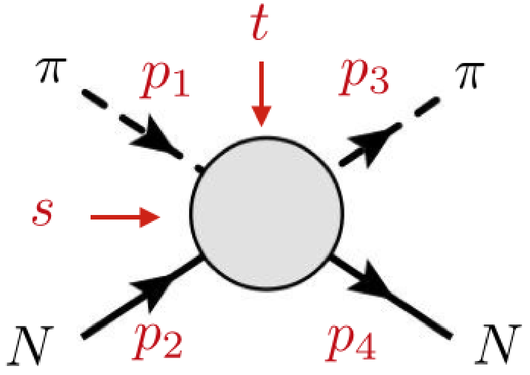
$$-t \approx M_m^2 \approx (0.1\text{-}1\ \text{GeV})^2$$

[H. Damien, Masters thesis, UGent (2020)]

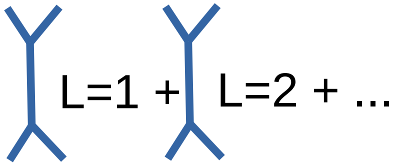




# Regge poles (briefly)



$$A(s,t) = \sum_{l=0}^{\infty} A_l(t) P_l(z_t)$$



$$z_t \equiv \cos\theta_t = 1 + \frac{2s}{t - 4m^2}$$

Partial wave series is a natural description in t-channel does not converge in physical region of s-channel

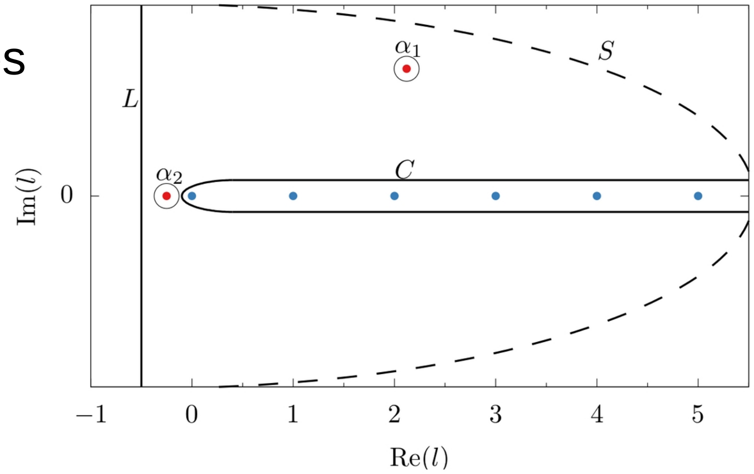
Analytic continuation A(l,t):

$$\sum_{l=0}^{\infty} A_l(t) P_l(z_t) = \frac{-1}{2i} \oint_C A(l,t) \frac{P_l(-z_t)}{\sin(l\pi)}\, dl$$

Assume A(l,t) has isolated singularities

$$A(l,t) \to \frac{\beta(t)}{l - \alpha(t)} \quad \text{for } l \to \alpha(t)$$

α(t) is the position of a
*Regge pole*
With residue β(t)

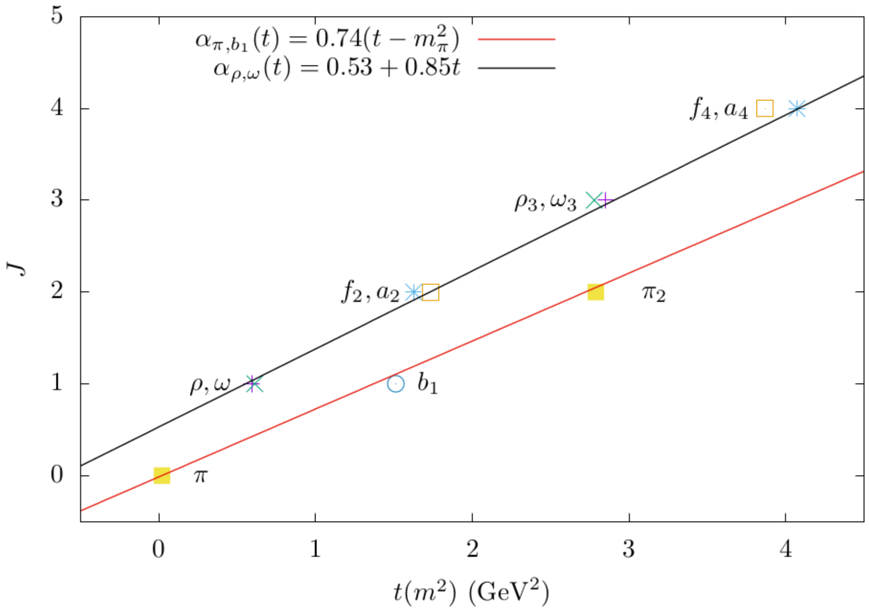


# Regge poles (briefly)

$$A(l,t) \to \frac{\beta(t)}{l-\alpha(t)} \text{ for } l \to \alpha(t)$$

$$A(s,t) = \frac{-1}{2i}\oint_{L+S} A(l,t)\frac{P_l(-z_t)}{\sin l\pi}\,dl - \sum_{i=0}^{n} \pi \frac{\beta_i(t) P_{\alpha_i(t)}(-z_t)}{\sin(\pi\alpha_i(t))}.$$

$$A_{l=n}(t) = A(l=n,t) \quad \alpha(t_0) = n$$



Trajectories $\alpha(t) = \alpha' t + \alpha_0$

For t > 0 $\alpha(t)$ describes the spin-mass relation of exchanged particles

For large s: $A_{pole}(s,t) \to \beta(t)\alpha'\Gamma\left[-\alpha(t)\right](\alpha' s)^{\alpha(t)}$

$$A_{pole}(s,t) \to \frac{\beta(t)}{t-m^2} \qquad \text{for } t \to m^2$$

# Regge poles (briefly)

$$\beta(t)\alpha'\Gamma\left[-\alpha(t)\right](\alpha' s)^{\alpha(t)}$$

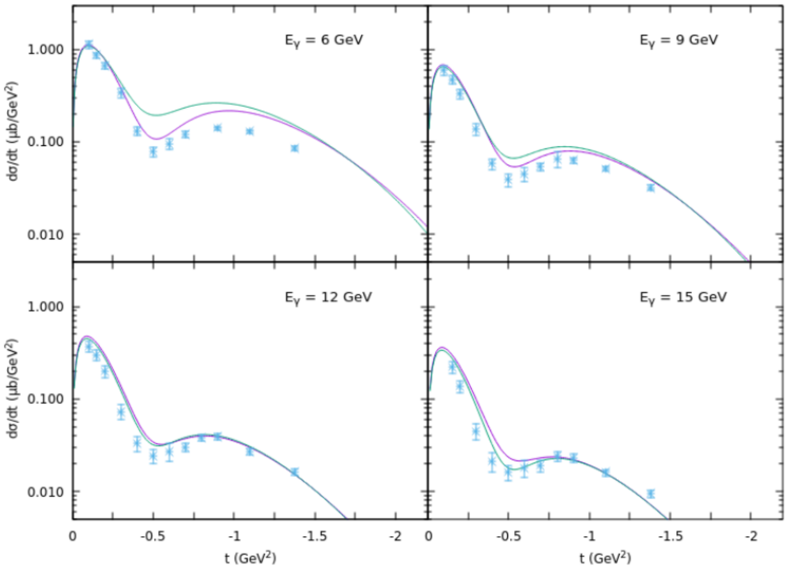
for $t \to m^2$

$$A_{pole}(s,t) \to \frac{\beta(t)}{t-m^2}$$

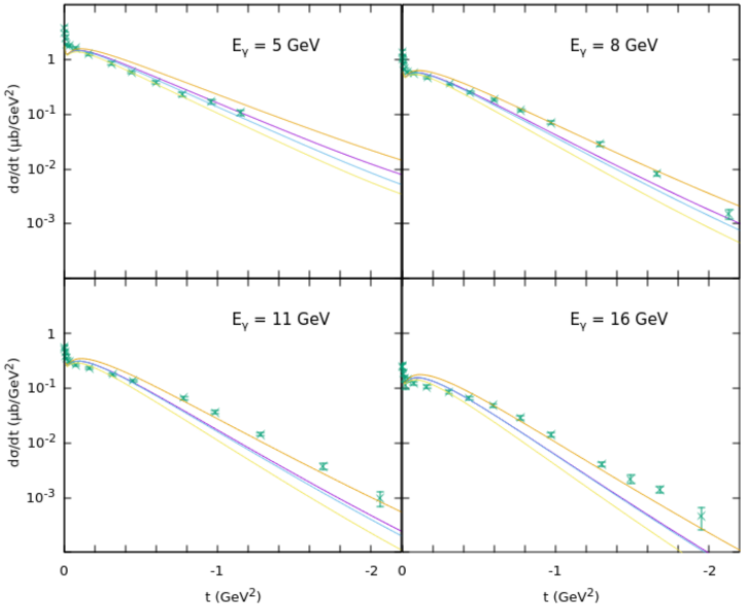
- For small -t, β(t) is approximated as

$$A_{Regge}(s,t,Q^2) \approx A_{tree}(t,Q^2) \times \left(t^2 - m^2\right)\left\{\alpha'\Gamma\left[-\alpha(t)\right](\alpha' s)^{\alpha(t)}\right\}$$

$\gamma p \to \pi^0 p$ dominated by $\omega$ exchange

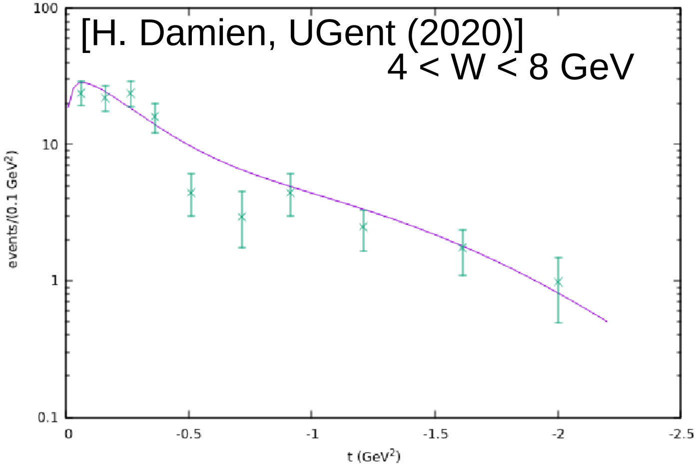


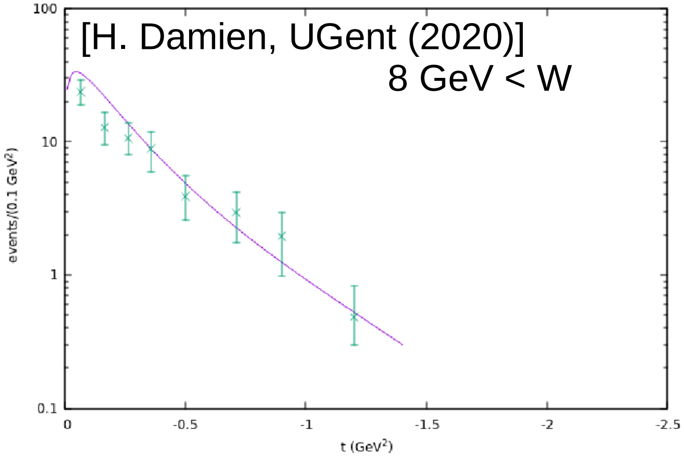
$\gamma p \to \pi^+ p$ from pion-exchange (GLV perscription)

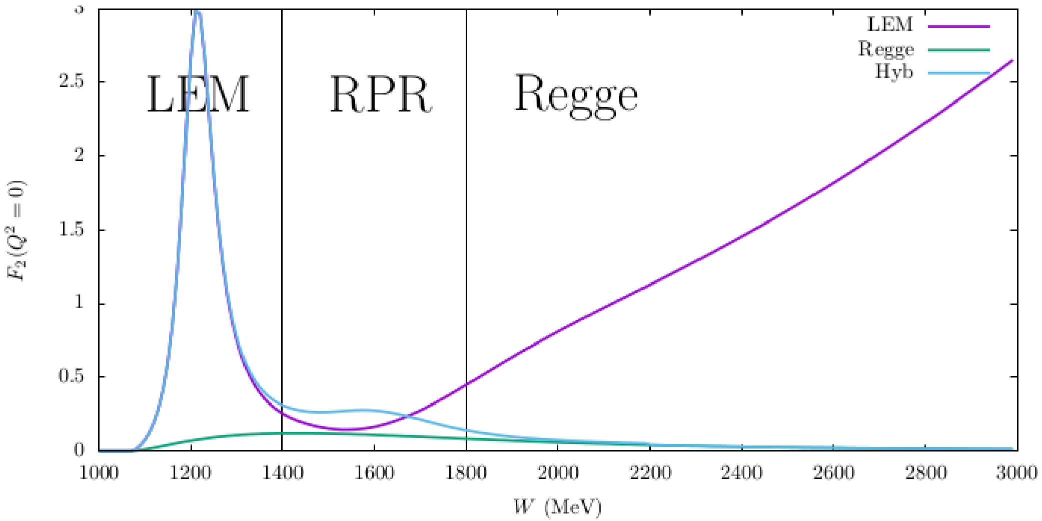


# Hybrid Regge-plus-resonance description

R. Gonzalez-Jimenez et al.
[Phys. Rev. D 95, 113007 (2017)]

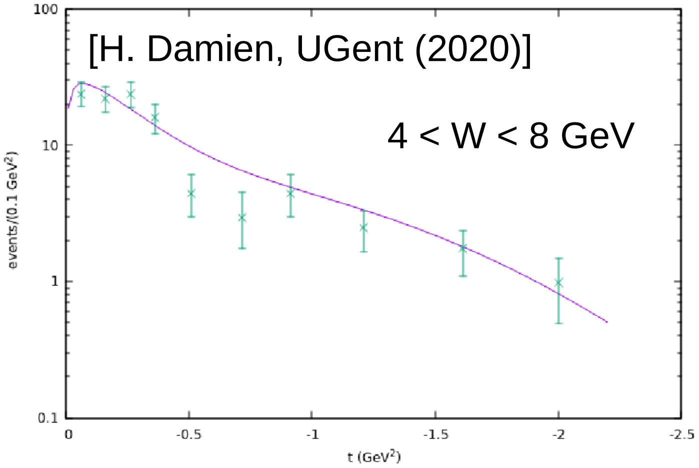




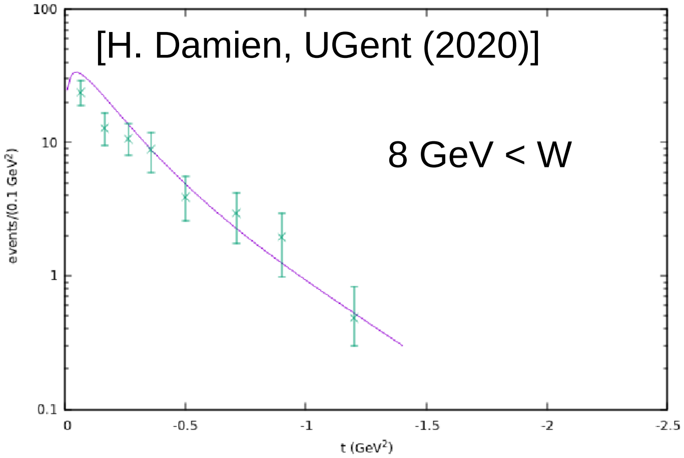


$p \rightarrow p + \pi^+$

# Regge approach for high-energy neutrino-pion production

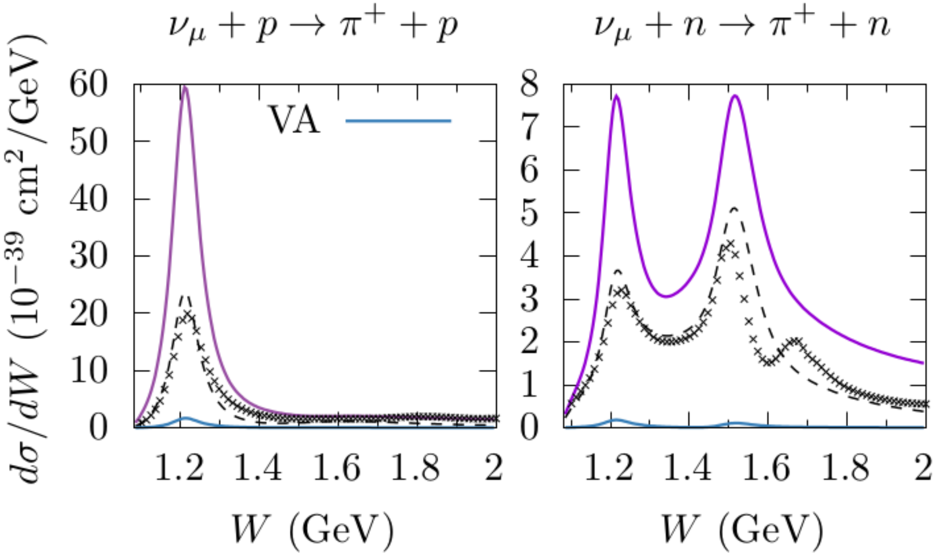

[H. Damien, UGent (2020)]

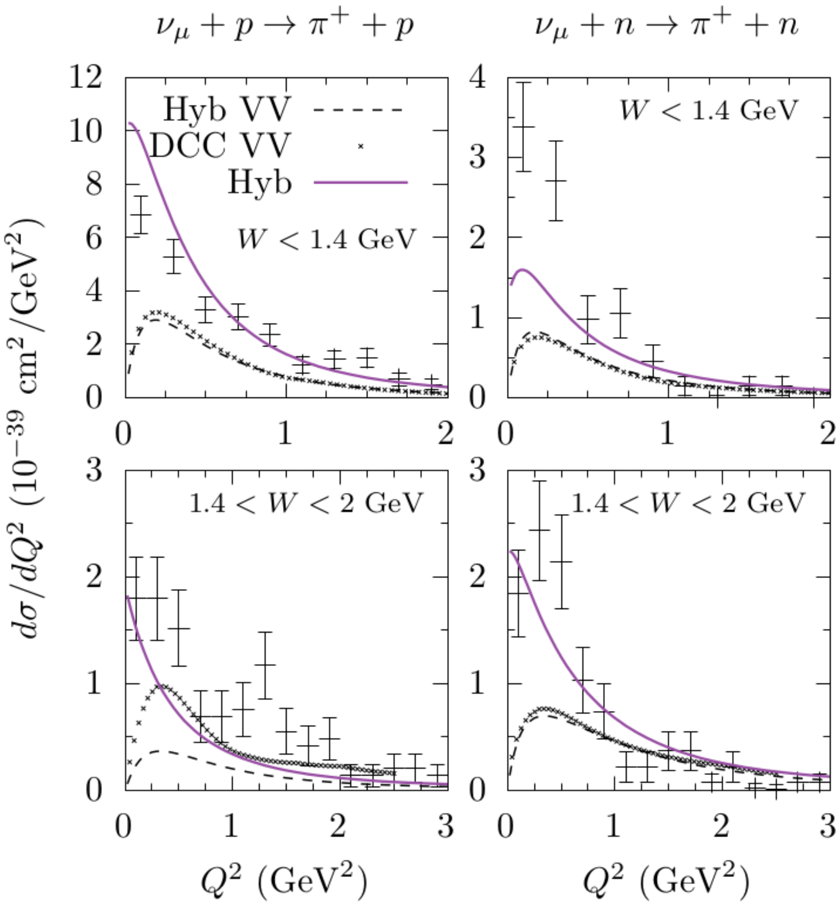
4 < W < 8 GeV

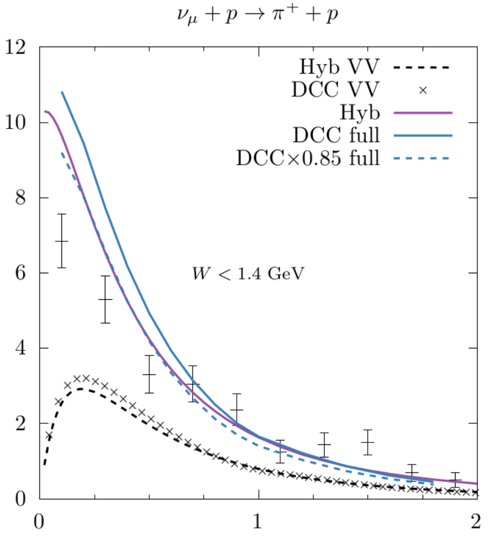

[H. Damien, UGent (2020)]

8 GeV < W

# BEBC flux-folded = VV + AA





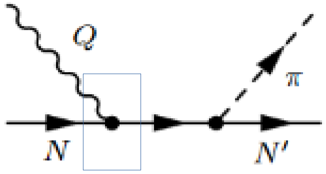


At high energy VA terms are negligible

→ Can isolate AA contribution

# Background contributions: nucleon form-factors

Hernandez, Nieves, Valverde
[Phys.Rev.D76, 033005 (2007)]



$$\mathcal{L}^{\sigma}_{\rm int} = \frac{g_A}{f_\pi}\bar{\Psi}\gamma^\mu\gamma_5\frac{\vec{\tau}}{2}(\partial_\mu\vec{\phi})\Psi \qquad \vec{V}^\mu = \bar{\Psi}\gamma^\mu\frac{\vec{\tau}}{2}\Psi \qquad \vec{A}^\mu = f_\pi\partial^\mu\vec{\phi} + g_A\bar{\Psi}\gamma^\mu\gamma_5\frac{\vec{\tau}}{2}\Psi$$

$$\overline{u}\left[\Gamma^\mu_V - \Gamma^\mu_A\right]u = \overline{u}\left[\gamma^\mu - g_A\left(\gamma^\mu + q^\mu\frac{\not{q}}{m_\pi - q^2}\right)\gamma^5\right]u$$

$$\downarrow$$

$$\overline{u}\left[\tilde{\Gamma}^\mu_V(q^2) - \tilde{\Gamma}^\mu_A(q^2)\right]u$$

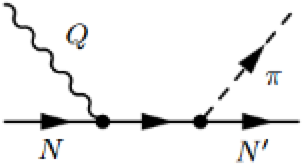
$$\tilde{\Gamma}^\mu_V(q^2) = F_1(q^2)\gamma^\mu + i\frac{F_2(q^2)}{2M_N}\sigma^{\mu\nu}q_\nu$$

$$\tilde{\Gamma}^\mu_A(q^2) : \; g_A \rightarrow G_A(q^2) = \frac{g_A}{(1 - q^2/M_A^2)^2}$$

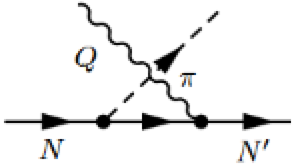
# Background contributions: vector current

Hernandez, Nieves, Valverde
[Phys.Rev.D76, 033005 (2007)]

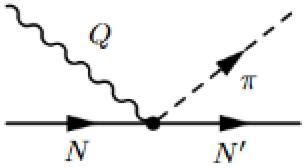
$$J_V^\mu = \overline{u}\left[\mathcal{O}^\mu_{NP,V} + \mathcal{O}^\mu_{CNP,V} + \mathcal{O}^\mu_{CT,V} + \mathcal{O}^\mu_{PIF,V}\right] u$$



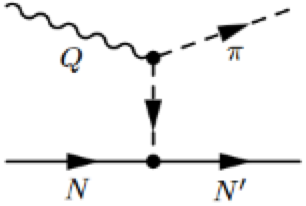
Nucleon pole (NP)



Crossed nucleon pole (CNP)



Contact-term (CT)



Pion-in-flight (PIF)

For $F_1 = 1$ $Q_\mu J_V^\mu = 0 \ (\mathrm{CVC})$

Introduction of $F_1(Q^2)$ in NP and CNP breaks conservation for $Q^2 > 0$

$$\mathcal{O}^\mu_{CT,V} = \frac{g_A}{\sqrt{2} f_\pi} F_{CT}(q^2)\gamma^\mu\gamma^5 \rightarrow \boxed{F_{CT}(q^2) = F_1(q^2)}$$

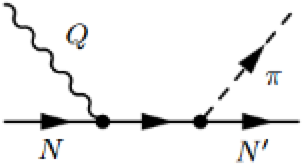
$$\mathcal{O}^\mu_{PF,V} = \frac{g_A}{\sqrt{2} f_\pi} F_{PF}(q^2)\frac{(2k_\pi - q)^\mu}{t^2 - m_\pi^2} 2M\gamma^5 \rightarrow \boxed{F_{PF}(q^2) = F_1(q^2)}$$

Introduce of $F_1(Q^2)$ in CT and PF to recover CVC

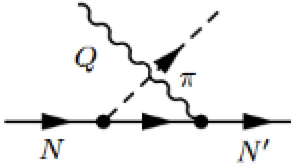
# Background contributions: axial current

Hernandez, Nieves, Valverde
[Phys.Rev.D76, 033005 (2007)]

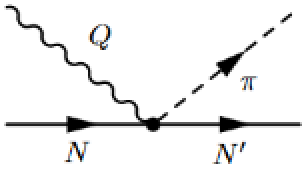
$$J^\mu_A = \overline{u}\left[\mathcal{O}^\mu_{NP,A} + \mathcal{O}^\mu_{CNP,A} + \mathcal{O}^\mu_{CT,A} + \mathcal{O}^\mu_{PP}\right] u$$



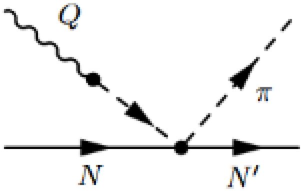
Nucleon pole (NP)



Crossed nucleon pole (CNP)



Contact-term (CT)



Pion-pole (PP)

$$\mathcal{O}^\mu_{CT,A} = \frac{1}{\sqrt{2}f_\pi}\gamma^\mu \to \frac{F_\rho(t)}{\sqrt{2}f_\pi}\gamma^\mu$$

Rho-meson propagator to regularize CT,A
“rho dominance of the ππNN coupling”

→ Need to include it in the PP term

$$\mathcal{O}^\mu_{PP} = \frac{1}{\sqrt{2}f_\pi}\frac{q^\mu}{q^2 - m_\pi^2}\not{q} \to \frac{F_\rho(t)}{\sqrt{2}f_\pi}\frac{q^\mu}{q^2 - m_\pi^2}\not{q}$$

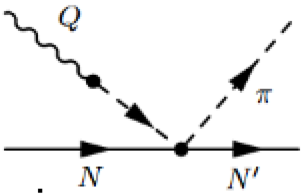
$$\kappa_\rho = 0$$

$$\mathcal{O}^\mu_{PP} + \mathcal{O}^\mu_{CT,A} =$$

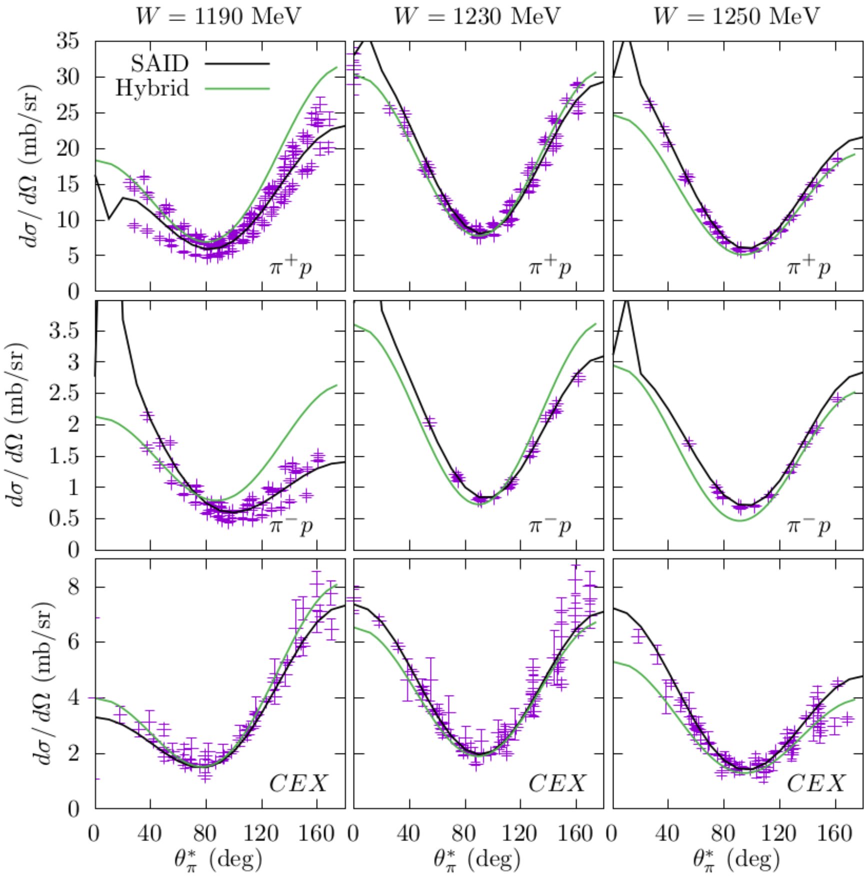
$$\mathcal{O}^\mu_\rho = \frac{g_{\rho NN}\, g_{W\rho\pi}}{t - m_\rho^2}\cancelto{1}{F_A}(Q^2)\{g^{\mu\alpha} + \frac{Q^\mu Q^\alpha}{Q^2 + m_\pi^2}\}\left(\gamma_\alpha + i\frac{\cancel{k_\rho}}{2M_N}\sigma_{\alpha\nu}K^\nu_\rho\right)$$

# Pion-nucleon scattering as limiting case

$$\vec{A}^{\mu} = f_\pi \partial^\mu \vec{\phi} + g_A \bar{\Psi}\gamma^\mu\gamma_5 \frac{\vec{\tau}}{2}\Psi + \frac{1}{2f_\pi}\bar{\Psi}\gamma^\mu(\vec{\phi}\times\vec{\tau})\Psi$$



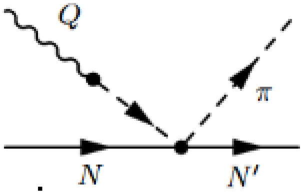
Axial-current has a pion-pole → the same dynamics for πN scattering



- Can gauge model with πN angular distributions

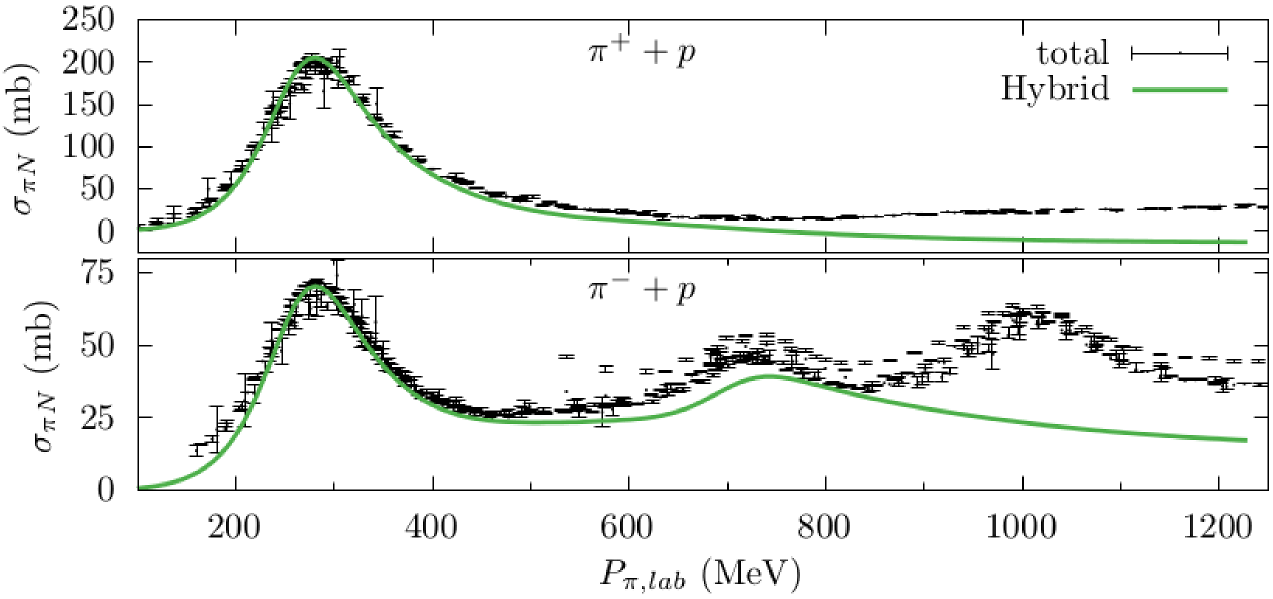
# Pion-nucleon scattering

$$\vec{A}^{\mu} = f_{\pi}\partial^{\mu}\vec{\phi} + g_A\bar{\Psi}\gamma^{\mu}\gamma_5\frac{\vec{\tau}}{2}\Psi + \frac{1}{2f_{\pi}}\bar{\Psi}\gamma^{\mu}(\vec{\phi}\times\vec{\tau})\Psi$$



Axial-current has a pion-pole → the same dynamics for πN scattering

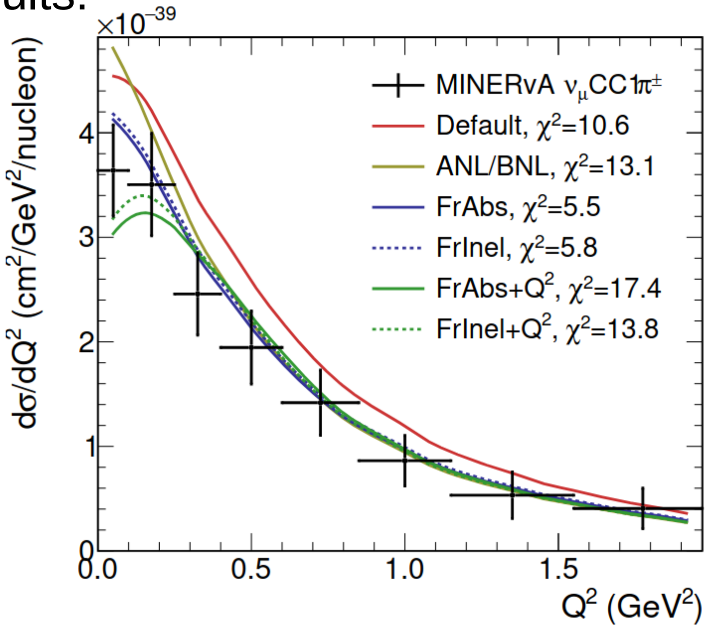
- Can gauge model with πN angular distributions
- Total CS sensitive to Im(A)



# Tensions in the resonance region ?

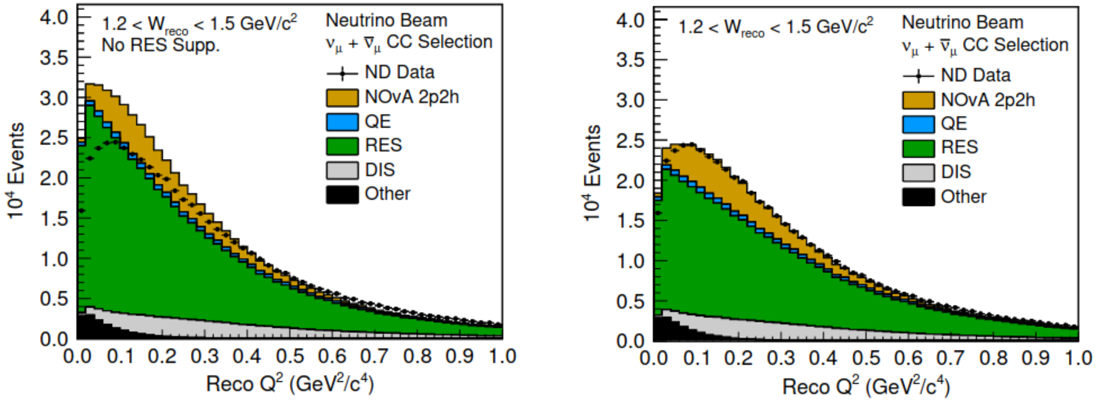
To resolve tension between deuteron / carbon results:

> “An additional *ad hoc* correction for
> the low-Q2 region,
> where collective nuclear effects are expected
> to be large”



[MINERvA PRD100, 072005 (2019)]

Similar correction introduced by NOvA

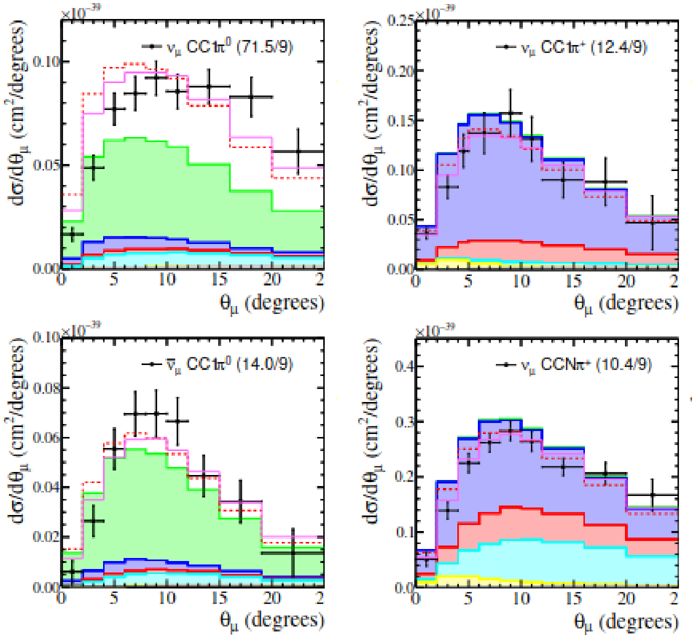


[NOvA Eur. Phys. J. C 80, 1119 (2020)]

# Tensions in the resonance region ?

In [PRD 100, 072005 (2019)] a simultaneous fit of

1. ANL/BNL bubble-chamber data for pion production on deuteron
2. MINERvA pion production data on carbon



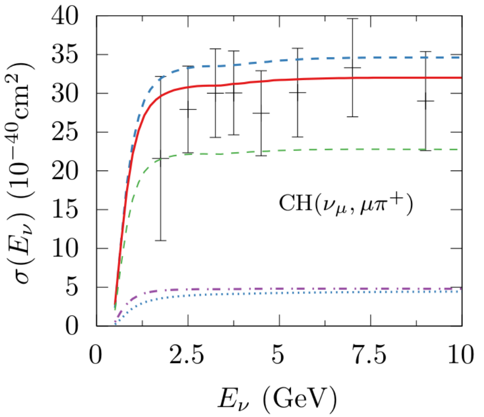
## Conclusion ?

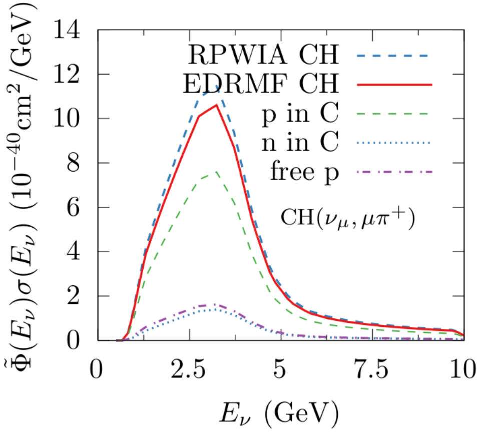
> “the Monte Carlo models which are currently widely used in the field are unable to explain multiple data sets, even when they are from a single Experiment.”

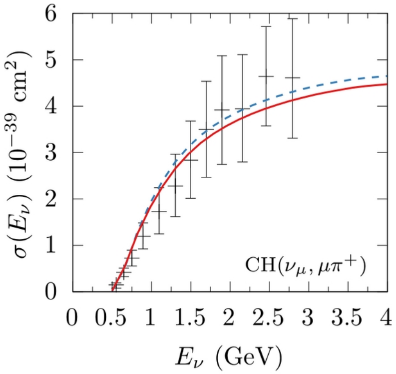
# Neutrinoproduction of charged pions on CH

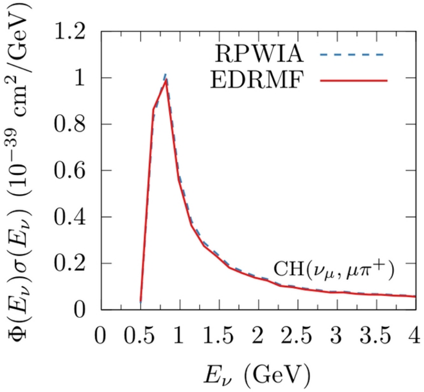
MINERνA experiment $1\pi^+$
$W_{(exp)} < 1.4$ GeV

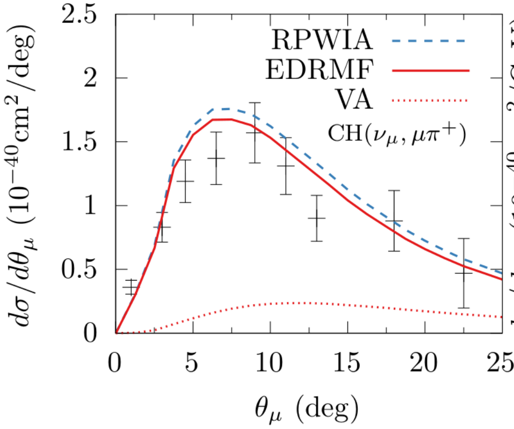
T2K experiment $1\pi^+$

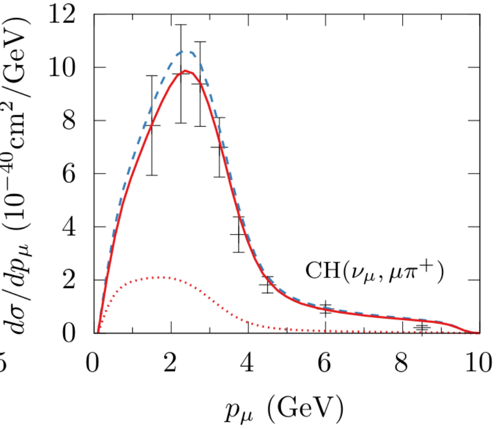


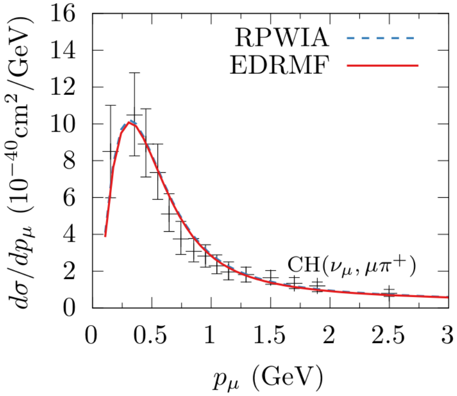






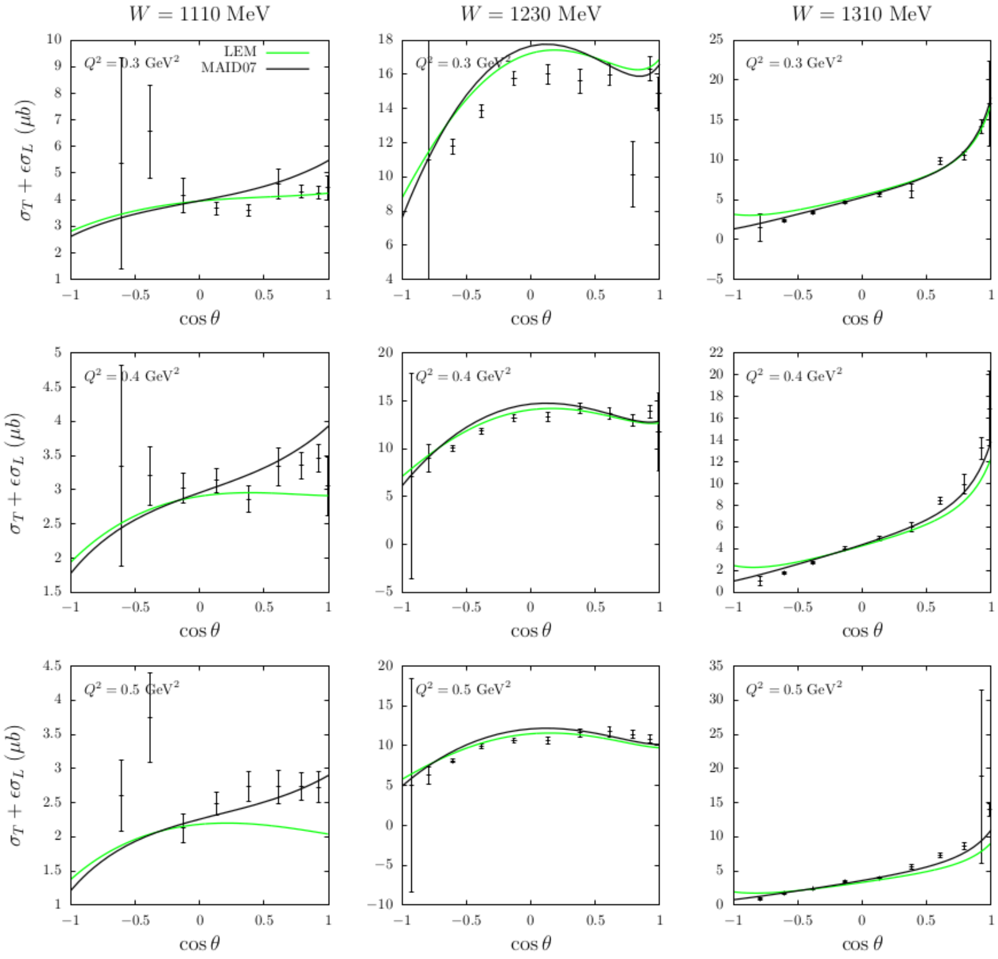






# Electron-induced SPP: angular-dependence structure functions

$$\frac{d\sigma_e}{d\Omega^*} = \boxed{\sigma_T+\epsilon\sigma_L} + \sqrt{2\epsilon(1+\epsilon)}\sigma_{LT}\cos\left(\phi^*\right) + \epsilon\sigma TT\cos\left(2\phi^*\right) + h\sqrt{2\epsilon\left(1-\epsilon\right)}\sigma_{LT'}\sin\phi^*$$



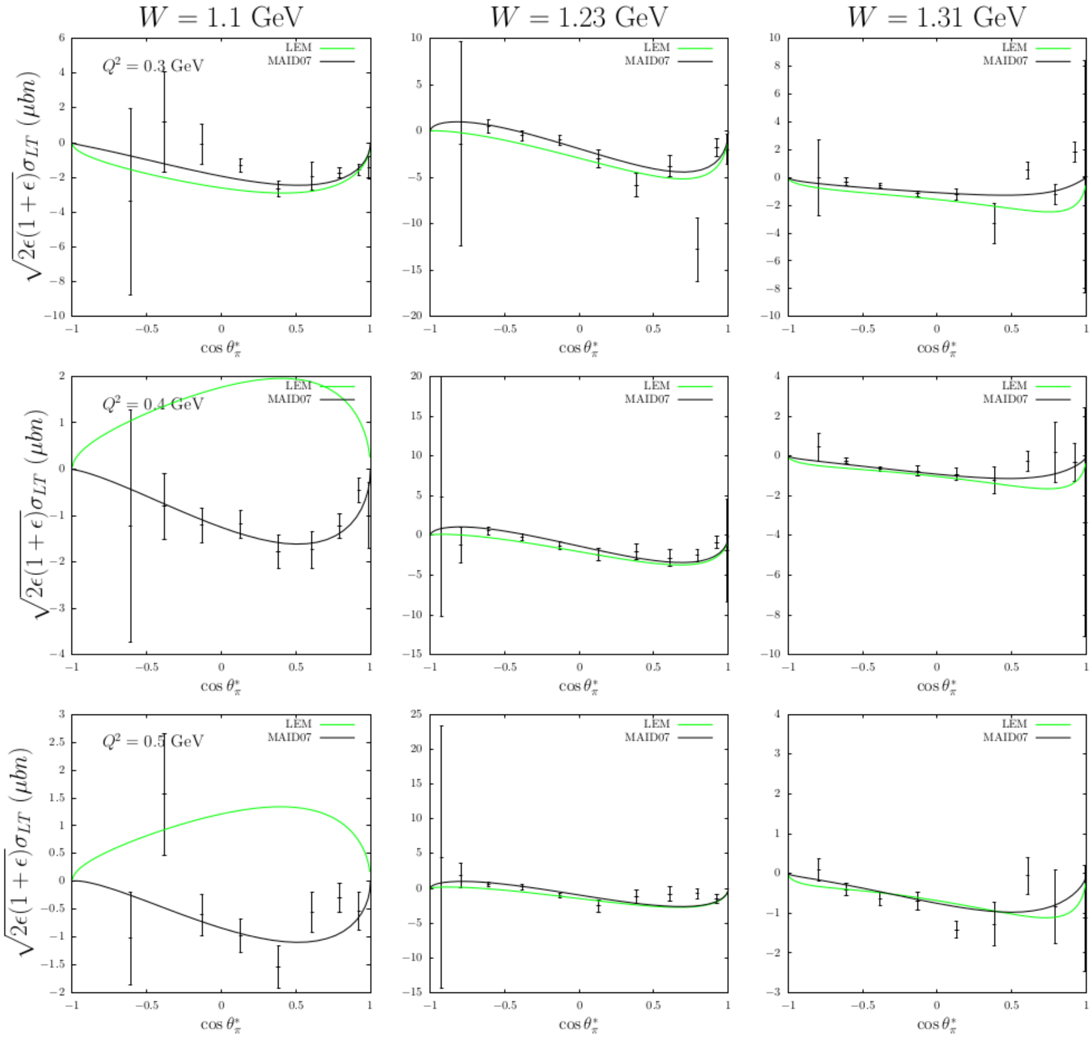
– LEM from
[R.G.J et al. PRD 95, 113007 (2017)]
(based on
[HNV PRD76, 033005, 2007])

– MAID07

CLAS data

# Electron-induced SPP: angular-dependence structure functions

$$\frac{d\sigma_e}{d\Omega^*} = \sigma_T+\epsilon\sigma_L+\sqrt{2\epsilon(1+\epsilon)}\boxed{\sigma_{LT}}\cos(\phi^*)+\epsilon\sigma TT\cos(2\phi^*)+h\sqrt{2\epsilon\,(1-\epsilon)}\sigma_{LT'}\sin\phi^*$$



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