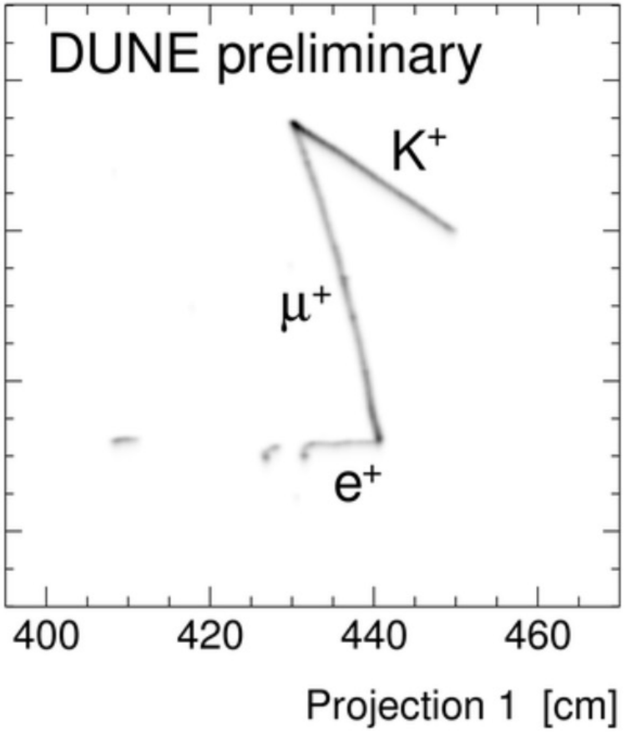
One and Two Body Current Contributions Within Many-Body Approaches

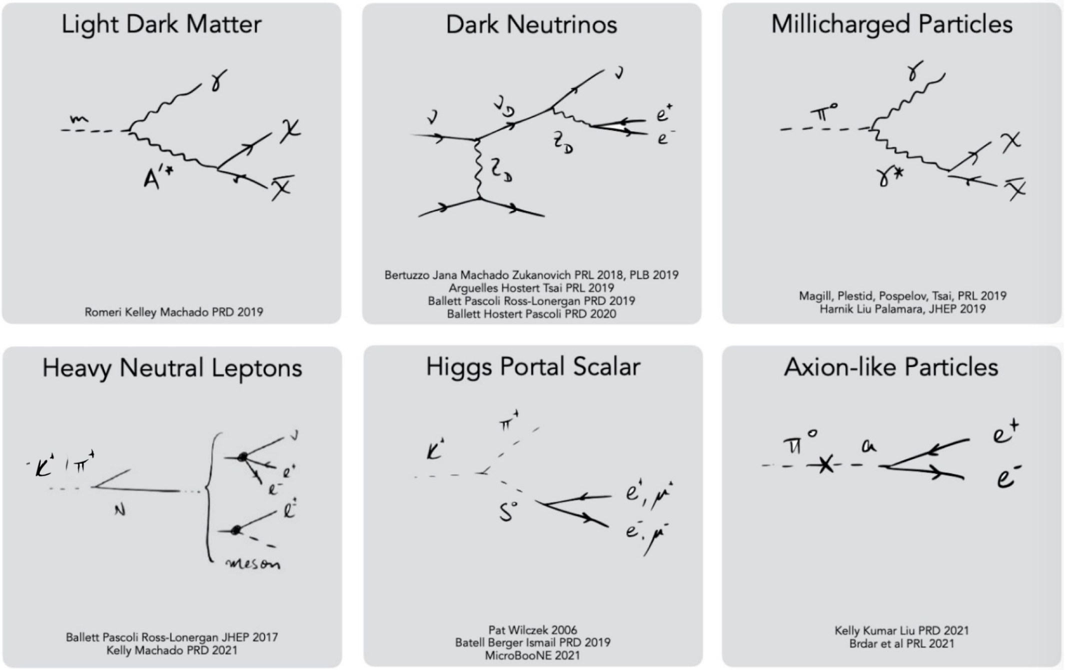
Noah Steinberg

Short-Baseline Experiment-Theory Workshop

The Good

- SBN and Long-Baseline programs provide a wealth of opportunities to look for new physics and explain long standing anomalies

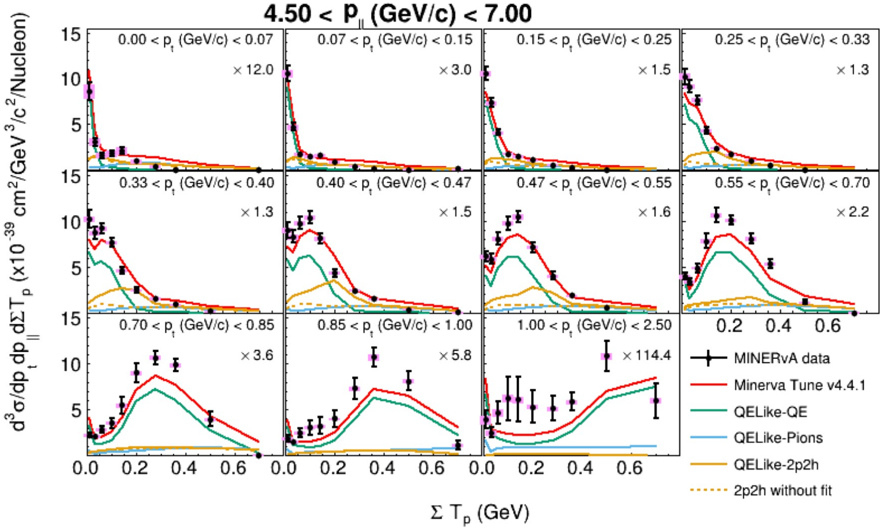




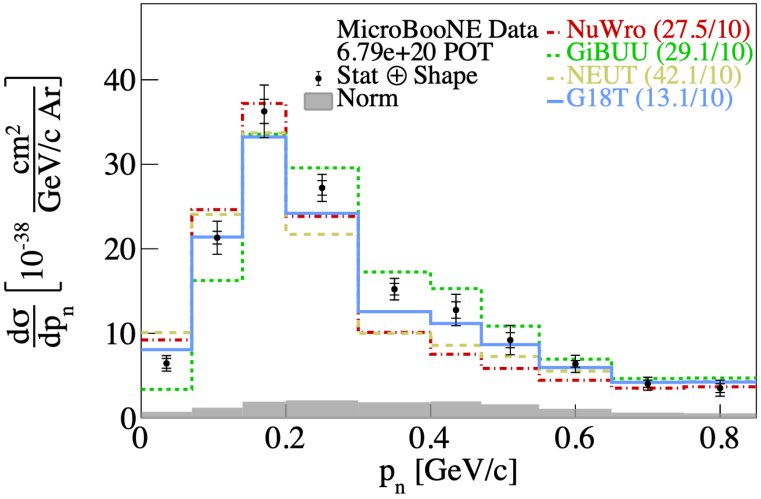
José I. Crespo-Anadón (SBND)

The Bad

- Probing these models requires an accurate understanding of the SM
  - The SM is a background for all of these processes
  - Some searches are “background free” but not all
  - In short and long baseline experiments these “backgrounds” are NOT well understood



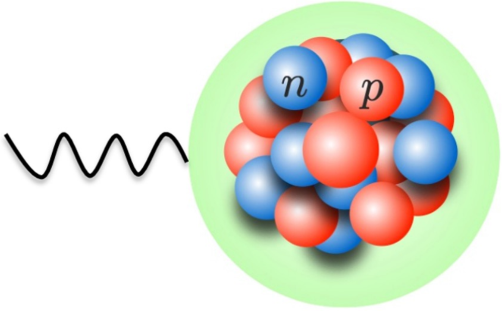
D. Ruterbories *et al.*, *Phys.Rev.Lett.* 129 (2022) 2, 021803



MicroBooNE e-Print: 2310.06082 [nucl-ex]

The One Who Knocks

Many body methods in lepton nucleus-scattering



- Nuclear dependence of cross section

$$R_{\mu\nu} = \sum_f \langle \Psi_0 | J_\mu^\dagger | \Psi_f \rangle \langle \Psi_f | J_\nu | \Psi_0 \rangle \delta(E_0 + \omega - E_f)$$

- Nuclear many body theory
  - Describe properties of nuclei as they emerge from constituent protons and neutrons

$$H = \sum_i^A \frac{\mathbf{p}_i^2}{2m_N} + \sum_{i<j}^A v_{ij} + \sum_{i<j<k}^A V_{ijk} \quad \longleftrightarrow \quad J_A^\mu(q) = \sum_i j_i^\mu(q) + \sum_{ij} j_{ij}^\mu(q)$$

- Currents must be consistent with the nuclear Hamiltonian and are linked by current conservation

Many Body Nuclear Theory

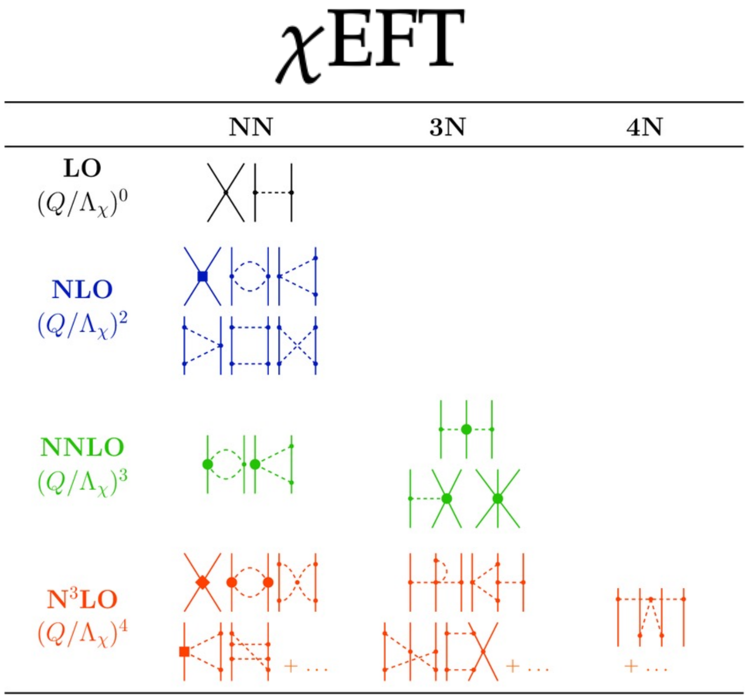
$$H = \sum_{i}^{A} \frac{\mathbf{p}_i^2}{2m_N} + \sum_{i<j}^{A} v_{ij} + \sum_{i<j<k}^{A} V_{ijk}$$

- Potentials can be provided by

Phenomenological Parameterizations

AV18 + IL7 Potentials

$$v_{ij} = \sum_{p=1}^{18} v_p(r_{ij}) O_{ij}^p$$



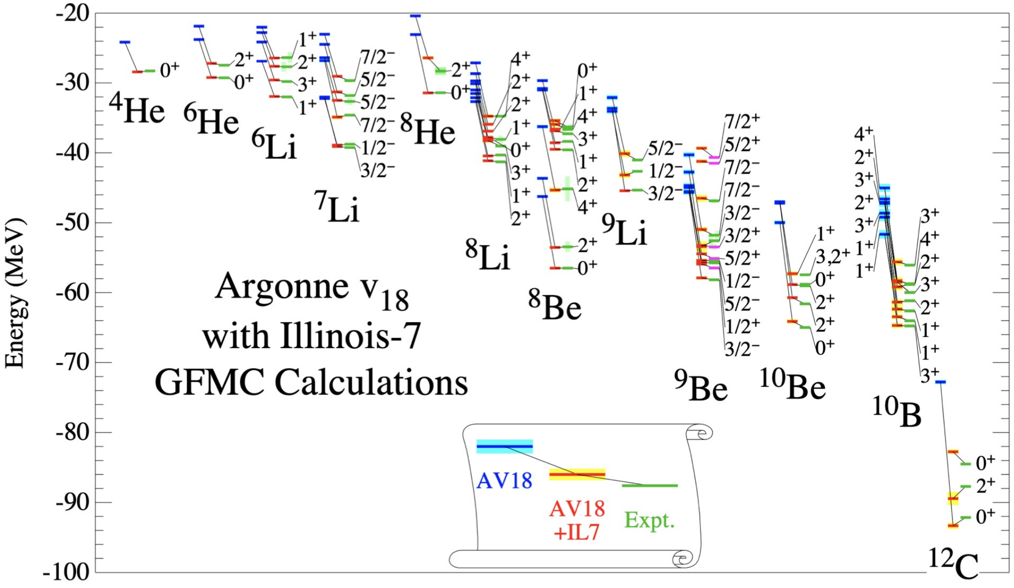
Gandolfi, S. et al *Front.in Phys.* 8 (2020) 117, *Front.Phys.* 8 (2020) 117

Many Body Nuclear Theory

- Ground state energies and wave functions computed via variational principle

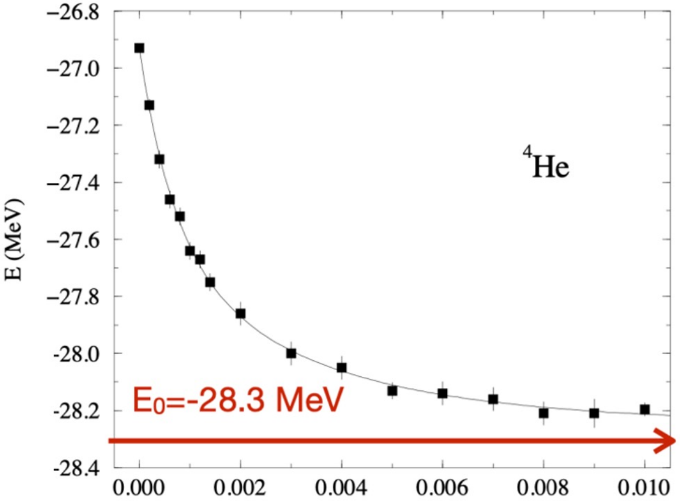
- Improved via Green’s Function Monte Carlo Techniques

$$|\Psi_T\rangle = \sum_n c_n |\Psi_n\rangle \qquad e^{-(H-E_0)\tau}|\Psi_T\rangle \rightarrow |\Psi_0\rangle$$



J. Carlson, et al. Rev.Mod.Phys. 87 (2015) 1067

B. Pudliner et al. PRC 56, 1720 (1997)



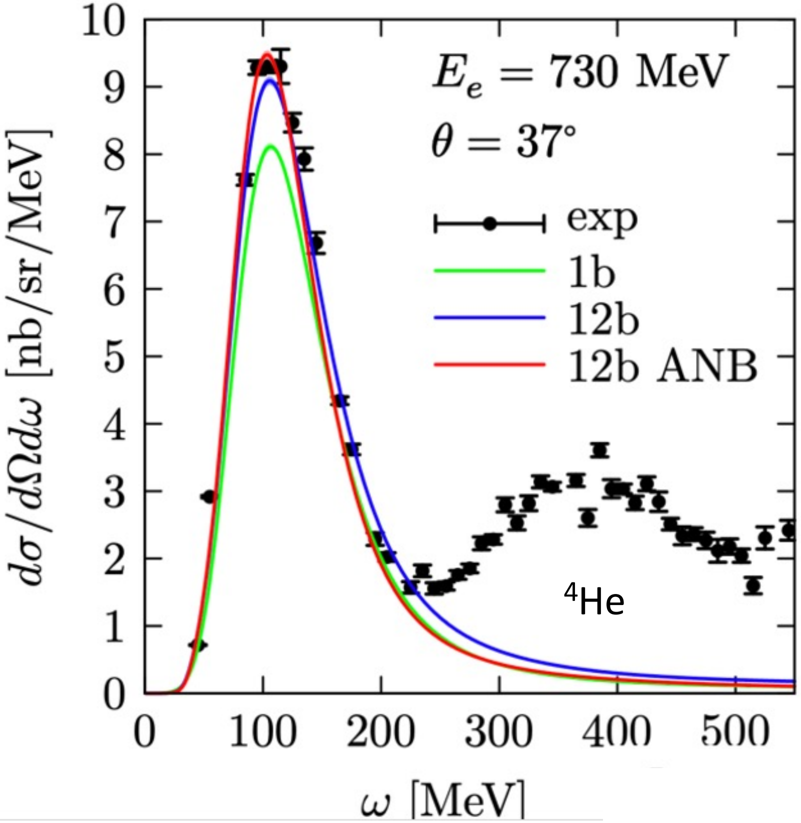
- Able to compute ground state (and excited state) energies of light nuclei with 1% precision

Many body methods in lepton nucleus-scattering

- Compute Euclidean Response, imaginary time evolve

$$E_{\alpha\beta}(\mathbf{q},\tau) = \int_{\omega_{\rm th}}^{\infty} d\omega e^{-\omega\tau} R_{\alpha\beta}(\mathbf{q},\omega)$$

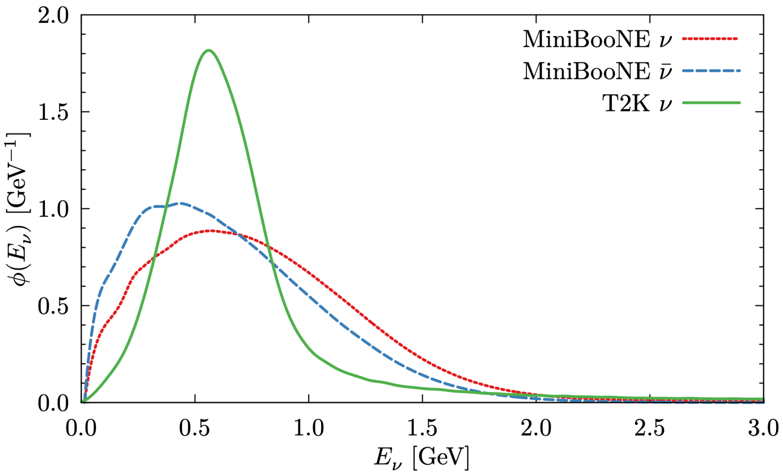
  - Inversion needed to obtain response
  - Retains initial and final state correlations
  - Virtually exact results in the quasi-elastic regime

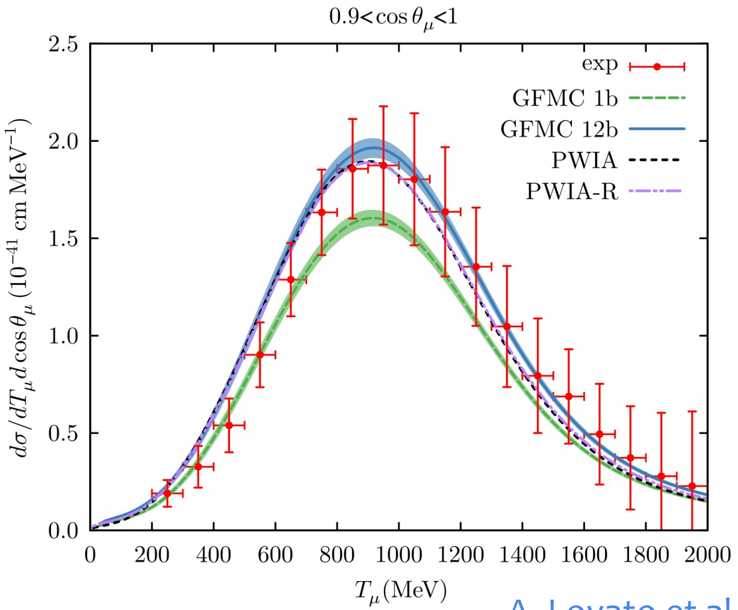


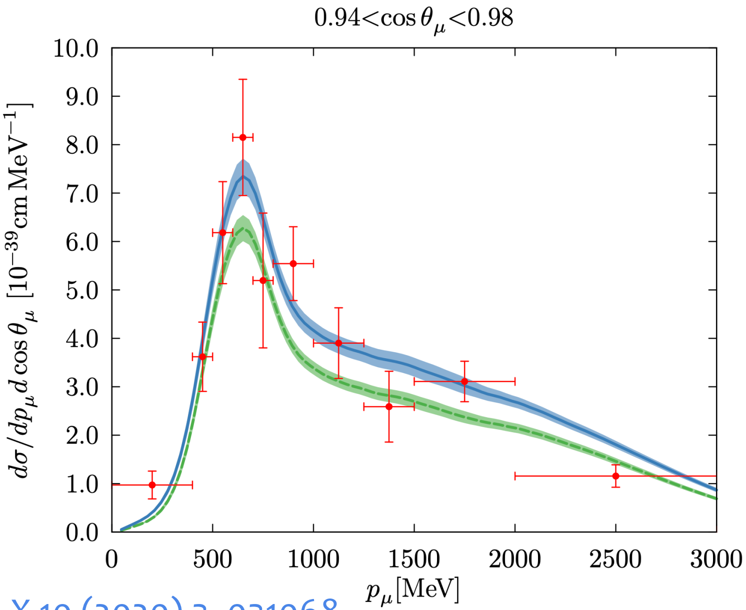
N. Rocco et al. PRC 97 (2018) no.5, 055501

GFMC Flux Folded Electroweak Cross Sections

- Comparison of GFMC EW cross sections with flux folded Neutrino Data
  - T2k & MiniBooNE
  - <E_ν> ~ 1 GeV



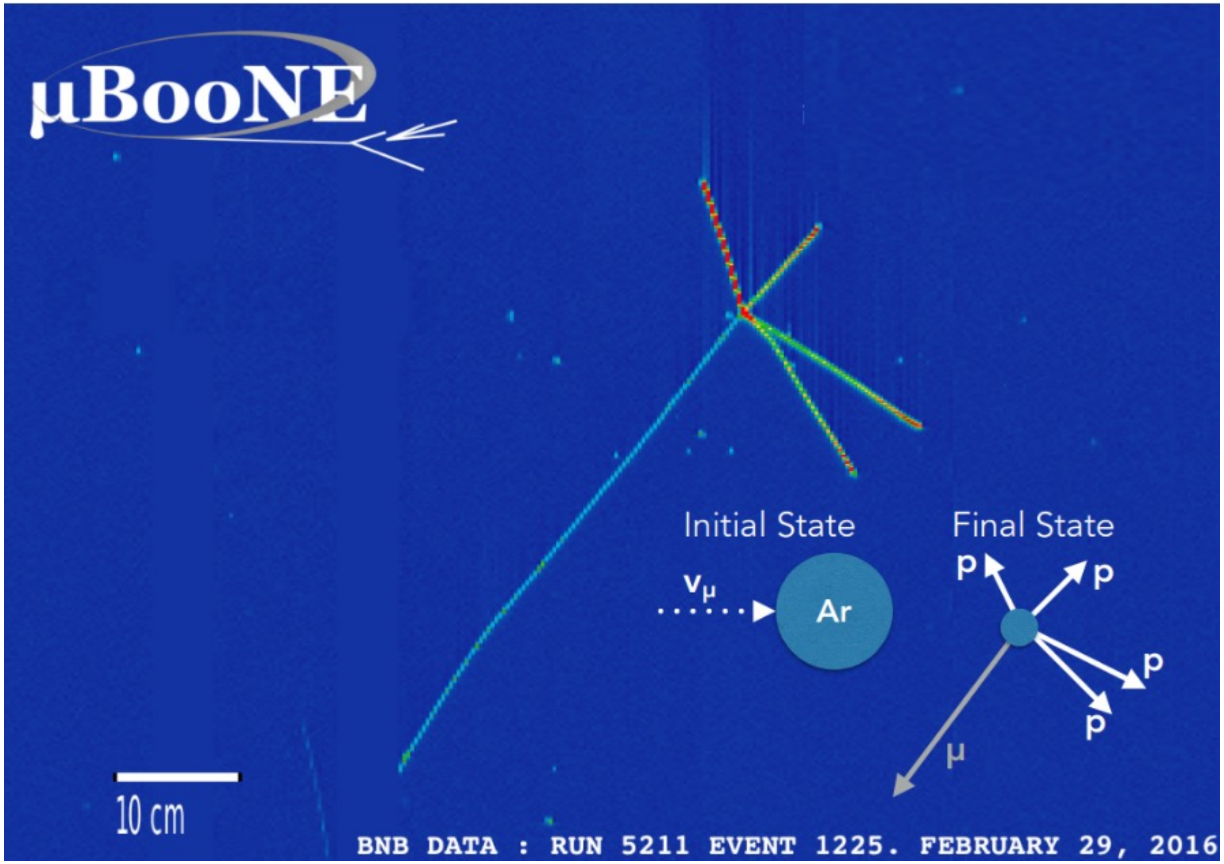




A. Lovato et al. Phys.Rev.X 10 (2020) 3, 031068

Inclusive is not enough

- SBN (and DUNE) program will reconstruct hadronic system with incredible precision
- Need exclusive predictions to study nuclear effects as well as BSM



Courtesy of Afroditi Papadopoulou

Many body methods: Spectral Function Approach

$$J^\mu = \boxed{\sum_i j_i^\mu} + \sum_{j>i} j_{ij}^\mu$$

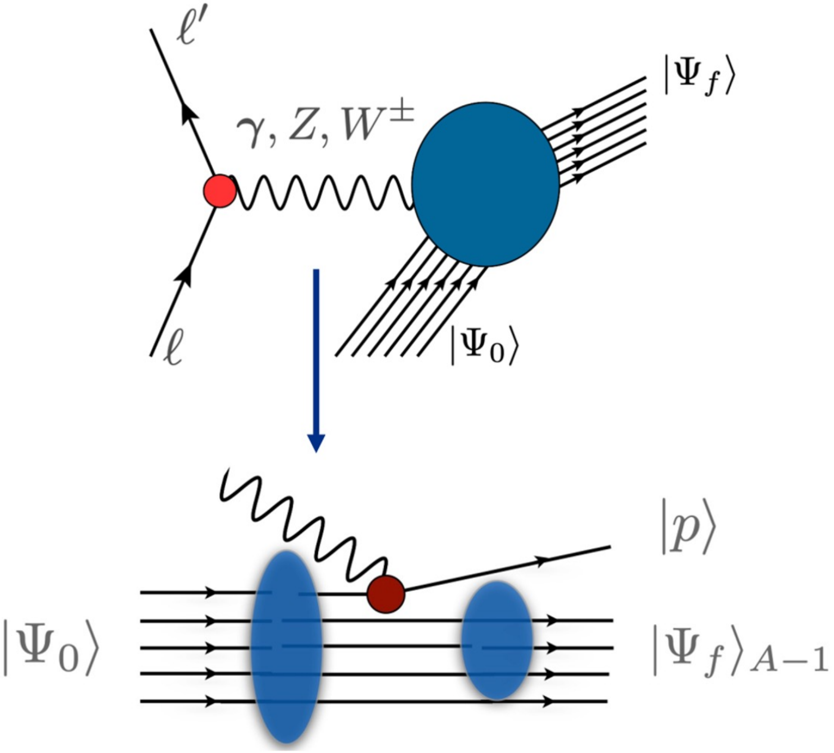
- For sufficient |**q**|, scattering factorizes
  - Incoherent sum of scattering with individual nucleons

- Single nucleon knockout

$$|f\rangle = |\mathbf{p}'\rangle \otimes |\Psi_f^{A-1}, \mathbf{p}_{A-1}\rangle$$

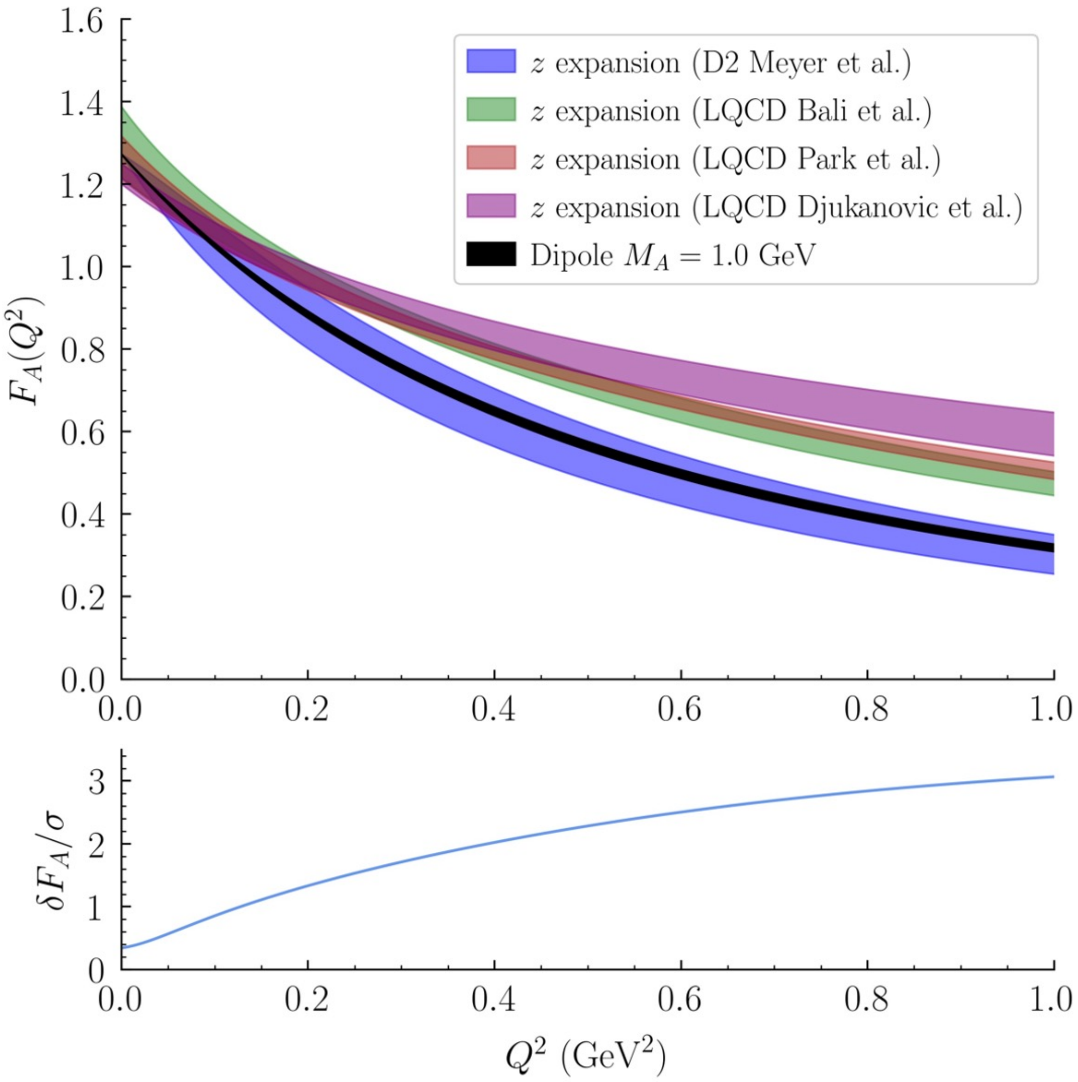
$$d\sigma = \int \boxed{(d\sigma)}_{nucleon} \boxed{P(\mathbf{p}, E)} d^3k dE$$

- Ingredients boil down to
  - Single nucleon cross sections
    - Form Factors
  - Hole Spectral function



N. Rocco et al., Phys.Rev.C 99 (2019) 2, 025502

Axial Form Factor



D. Simons, N.S. et al. 2210.02455 [hep-ph]

$$z(Q^2) = \frac{\sqrt{t_c + Q^2} - \sqrt{t_c - t_0}}{\sqrt{t_c + Q^2} + \sqrt{t_c - t_0}}$$

$$F_A(Q^2) = \sum_{k=0}^{\infty} a_k \, z(Q^2)^k \approx \sum_{k=0}^{k_{\max}} a_k \, z(Q^2)^k$$

- Dipole parameterization extracted from v-D2 underestimates uncertainty
- Meyer et al. D2 z-expansion gives similar CV but larger error
- LQCD Bali and Park extractions give much larger form factor for $Q^2 > 0.3$ GeV^2

Many body methods: Spectral Function Approach

$$J^\mu = \sum_i j_i^\mu + \boxed{\sum_{j>i} j_{ij}^\mu}$$

- Two nucleon knockout via two-body currents

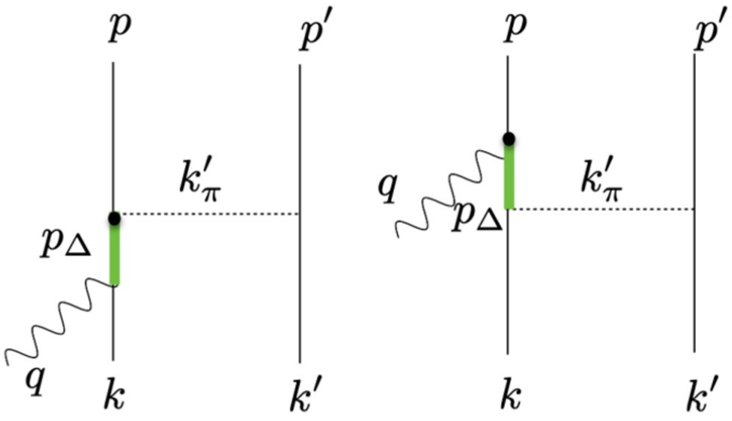
$$|\psi_f^A\rangle \rightarrow |pp'\rangle_a \otimes |\psi_f^{A-2}\rangle$$

$$\begin{aligned}
R^{\mu\nu}_{2b}(\mathbf{q},\omega) = \frac{V}{2}\int dE \frac{d^3k}{(2\pi)^3}\frac{d^3k'}{(2\pi)^3}\frac{d^3p}{(2\pi)^3}\frac{m_N^4}{e(\mathbf{k})e(\mathbf{k}')e(\mathbf{p})e(\mathbf{p}')} \\
\times \boxed{P_h \quad (\mathbf{k},\mathbf{k}',E)} \sum_{ij} \boxed{\langle k\, k'|j_{ij}^{\mu\,\dagger}|p\, p'\rangle}_a \langle p\, p'|j_{ij}^\nu|k\, k'\rangle \\
\times \delta(\omega - E + 2m_N - e(\mathbf{p}) - e(\mathbf{p}'))\,.
\end{aligned}$$

- Currents only partially constrained

$$j^\mu_{\rm CC} = (j^\mu_{\rm pif})_{\rm CC} + (j^\mu_{\rm sea})_{\rm CC} + (j^\mu_{\rm pole})_{\rm CC} + \boxed{(j^\mu_{\Delta})_{\rm CC}}$$

  - Delta current contribution dominates



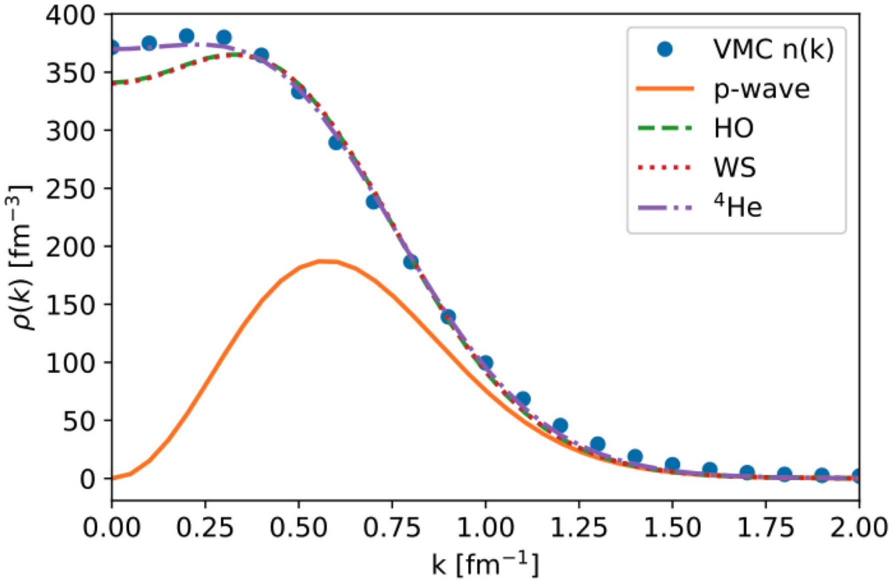
N. Rocco et al., Phys.Rev.C 99 (2019) 2, 025502

QMC Spectral Function

- One nucleon Spectral Function

$$P_{p,n}(\mathbf{k},E) = \sum_n |\langle\Psi_0^A|[|k\rangle|\Psi_n^{A-1}\rangle]|^2 \times \delta(E + E_0^A - E_n^{A-1})$$
$$= P^{MF}(\mathbf{k},E) + P^{\rm corr}(\mathbf{k},E)$$

  - Mean Field (A-1 Bound States)
  - Correlation component from continuu
  - Momentum space overlaps obtained from VMC calculations of wavefunctions



CLAS Phys.Rev.C 107 (2023) 6, L061301

- Two nucleon spectral function
  - Mean field only

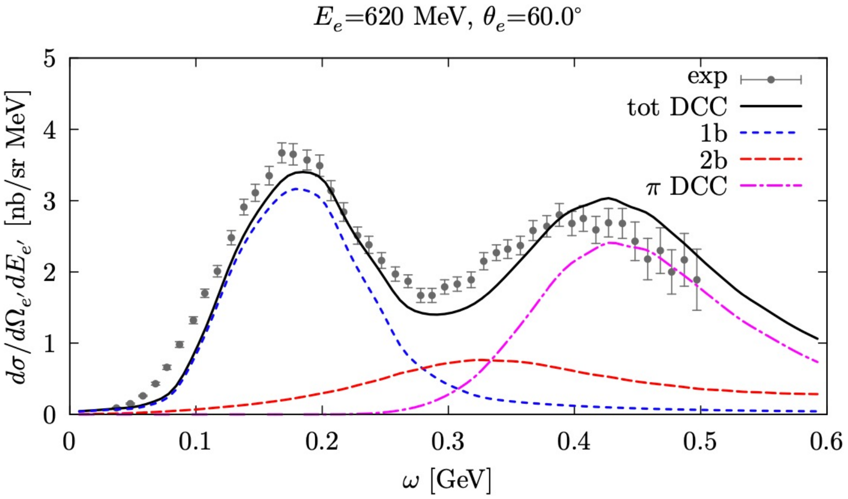
$$P^{\rm MF}_{\tau_k,\tau_k'}(\mathbf{k},\mathbf{k}',E) = n_{\tau_k,\tau_{k'}}(\mathbf{k},\mathbf{k}')$$
$$\times\, \delta\Big(E - B_0 + \bar{B}_{A-2} - \frac{\mathbf{K}^2}{2m_{A-2}}\Big)$$

Spectral Function Approach

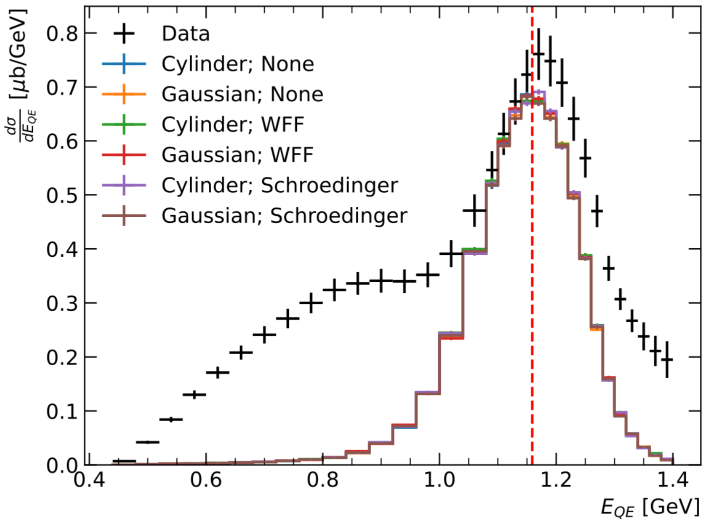
- Describes Inclusive electron scattering well
    - QE + MEC + SPP

- Exclusive QE predictions promising when compared against ν and e data

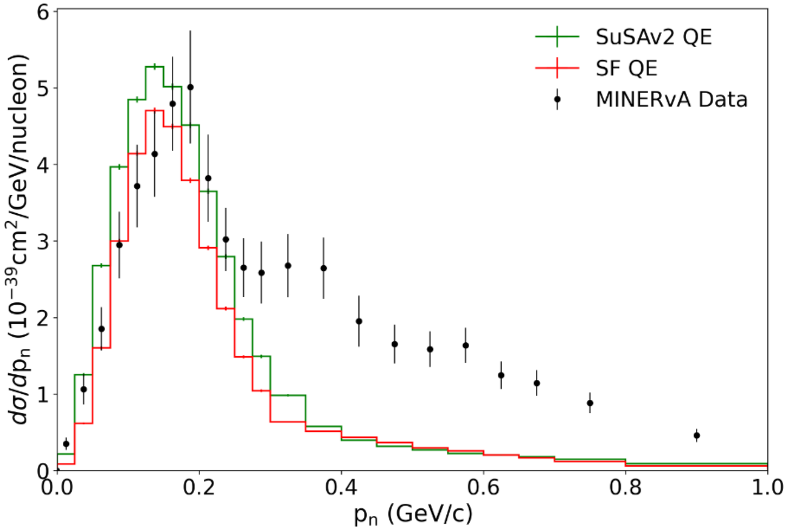
N. Rocco et al. *Phys.Rev.C* 100 (2019) 4, 045503



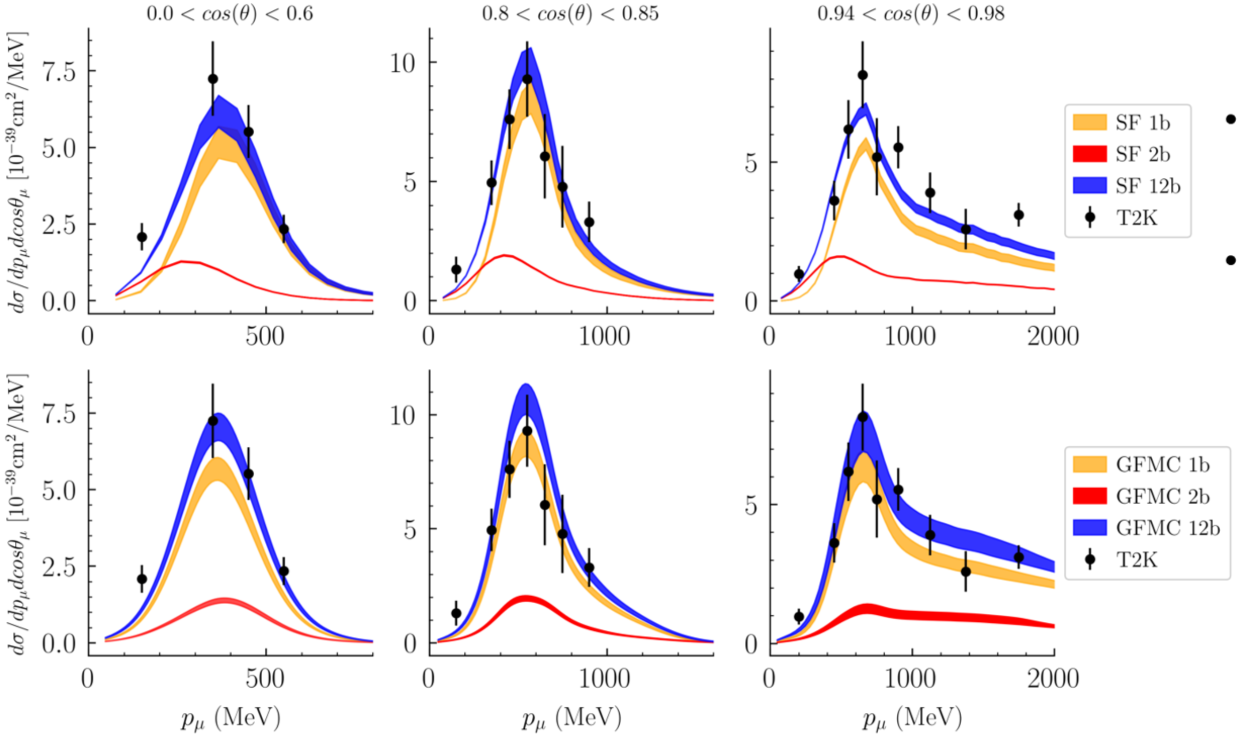
J. Isaacson et al. Phys.Rev.D 107 (2023) 3, 033007



M. Betancourt et al. *Phys.Rev.D* 108 (2023) 11, 113009



T2K – 1 and 2 Body Breakdown

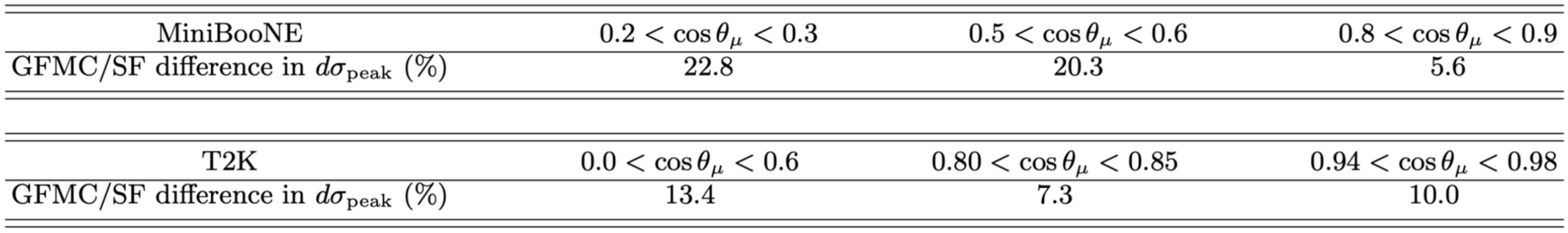


D. Simmons, N.S. et al. 2210.02455 [hep-ph]

- GMFC and SF provide excellent agreement
- T2K flux peaks at lower energies
- SF and GFMC 2 Body peaks shifted b/c of interference effects

SF vs GFMC predictions

D. Simmons, N.S. et al. 2210.02455 [hep-ph]



MiniBooNE	$0.2 < \cos\theta_\mu < 0.3$	$0.5 < \cos\theta_\mu < 0.6$	$0.8 < \cos\theta_\mu < 0.9$
GFMC/SF difference in $d\sigma_{\text{peak}}$ (%)	22.8	20.3	5.6

T2K	$0.0 < \cos\theta_\mu < 0.6$	$0.80 < \cos\theta_\mu < 0.85$	$0.94 < \cos\theta_\mu < 0.98$
GFMC/SF difference in $d\sigma_{\text{peak}}$ (%)	13.4	7.3	10.0

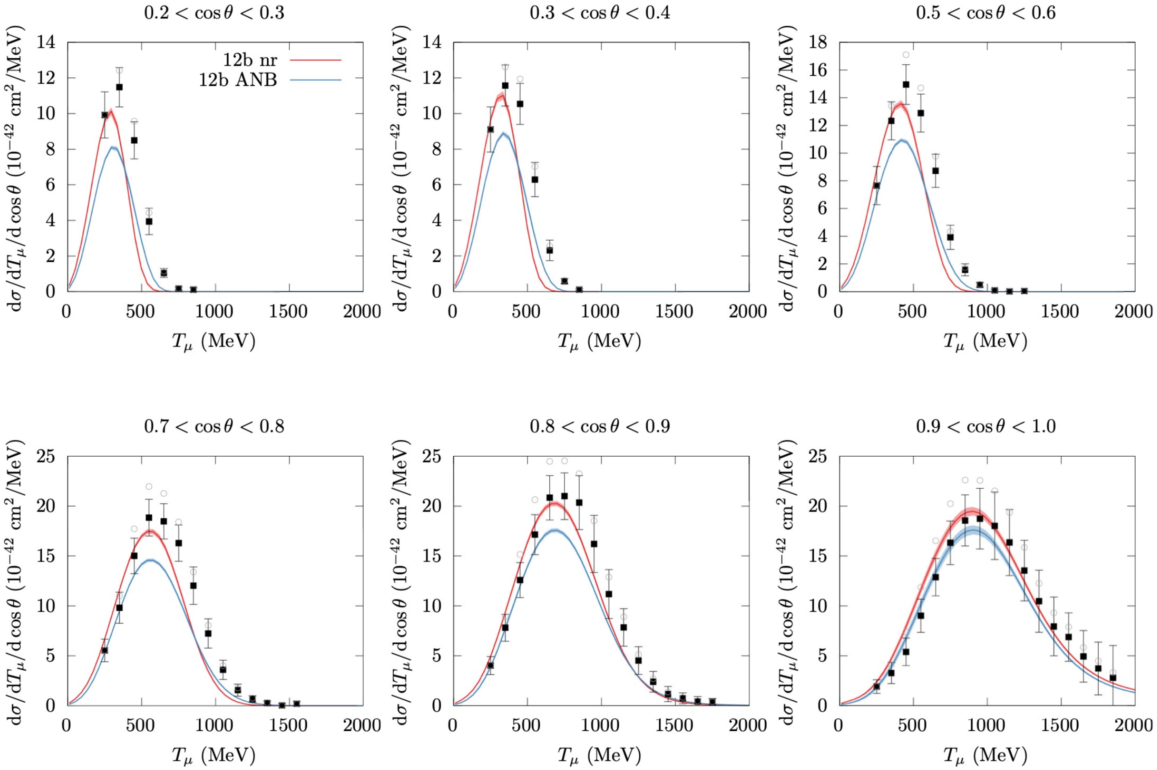
- Differences due to:
  - GFMC
    - Non-relativistic nature of GFMC
    - Static treatment of Δ propagator

  - SF
    - No FSI in factorization scheme
    - Lack of 1-2 body interference

  - First attempt at uncertainty due to factorization approach

Improvements to GFMC and SF

- Apply relativistic corrections
- Compute responses in frame that minimizes initial and final nucleon momentum
  - ANB Frame shown to reduce relativistic effects



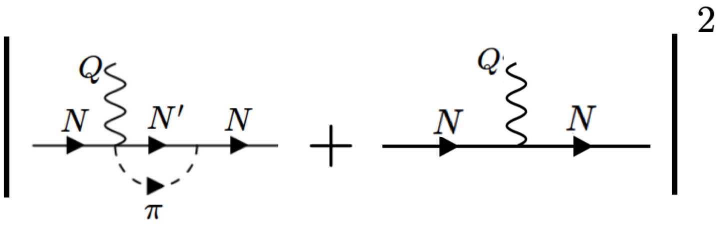
A. Nikolakopoulos et al, 2304.11772 [nucl-th]

Improvements to GFMC and SF

- SF missing interference effects between 1 and 2 body currents

$$R_{\mu\nu} \propto |\langle ph^{-1}|J_{1b}|0\rangle|^2 + |\langle pp'(hh')^{-1}|J_{2b}|0\rangle|^2$$

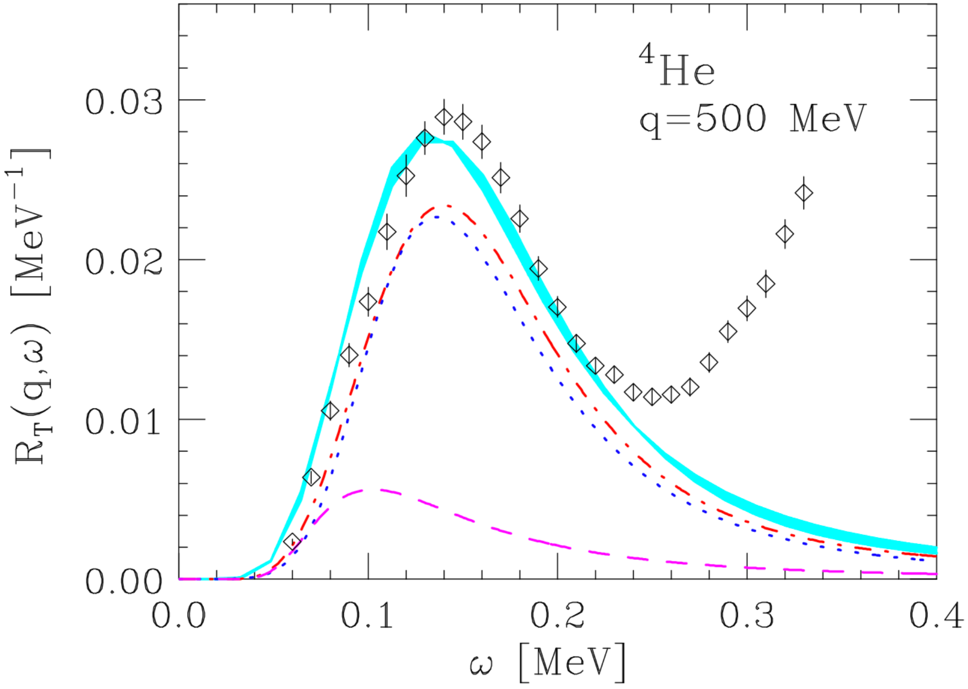
- Explicit in GFMC calculations (dominant effect of 2 body currents)



- How to compute in the SF formalism?

$$R^{\text{intf}}_{\mu\nu} \propto \langle f|J_{1b}|0\rangle\langle 0|J^{\dagger}_{2b}|f\rangle + h.c.$$

O. Benhar, N.R, A.L Phys.Rev.C 92 (2015) 2, 024602

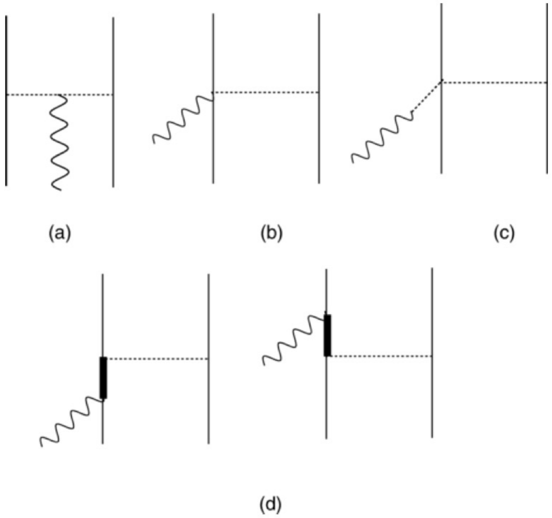


Interference in the SF

- Interference between 1 and 2 body currents leading to one nucleon knockout in inclusive lepton-nucleus scattering

$$R^{\mu\nu}_{12\mathrm{b}} = \sum_f \langle 0|j^{\mu\dagger}_{2b}|f\rangle\langle f|j^{\nu}_{1b}|0\rangle\delta(\omega + E_0 - E_f)$$

- j_{2b} excites a *1p1h* state by interacting with a second "spectator nucleon"
  - Two nucleon current operators
    - Same as appear in pure two body response
  - One body current operator contains the axial current
    - Interference therefore sensitive to axial form factor!

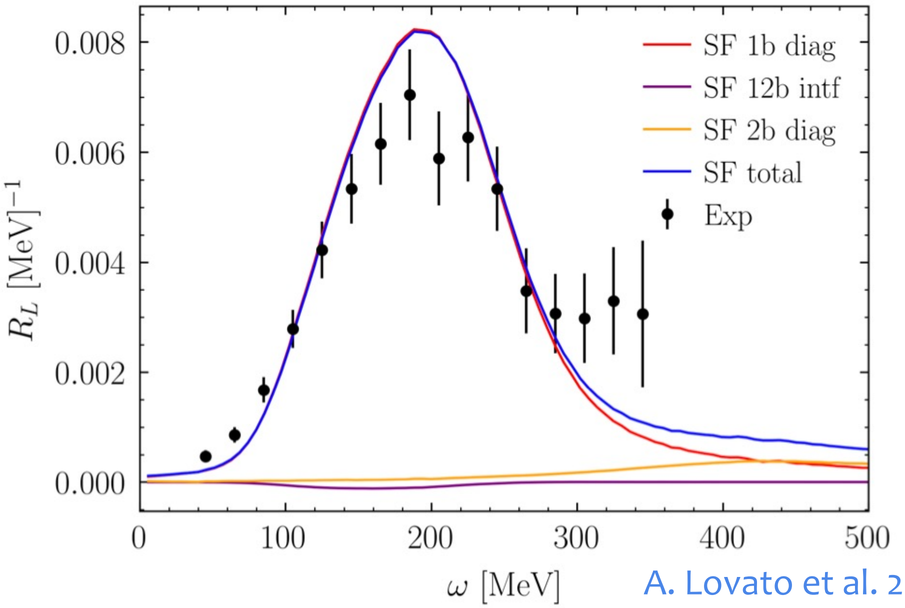


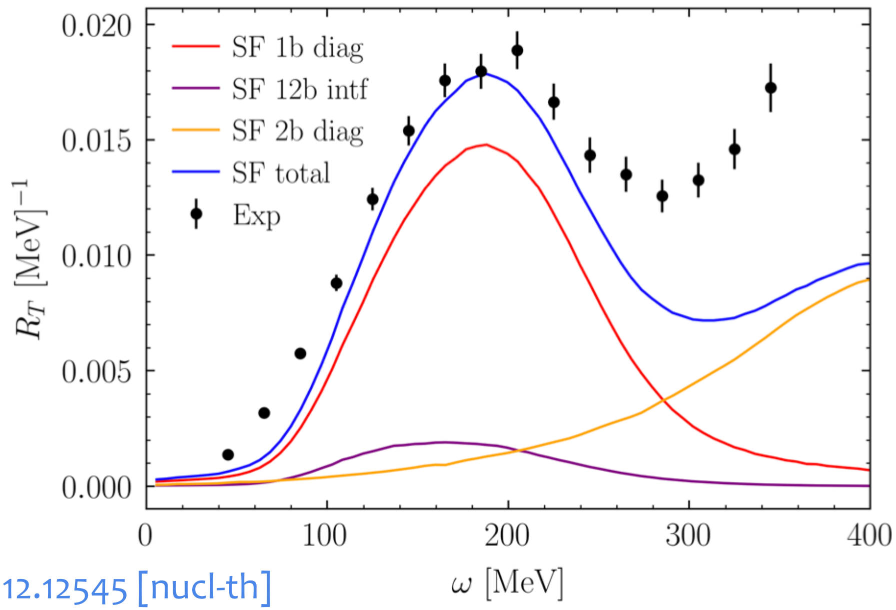
A. Lovato et al. 2312.12545 [nucl-th]

Noah Steinberg | Neutrino-Nucleus Interactions

Inclusive electron scattering - Responses

- **L** and **T** responses at **q** = 570 MeV on Carbon
- Virtually no effect on longitudinal response
- 20% total enhancement from pure two-body and interference effect in transverse response
  - Delta current is entirely transverse
- Improved agreement with data





A. Lovato et al. 2312.12545 [nucl-th]

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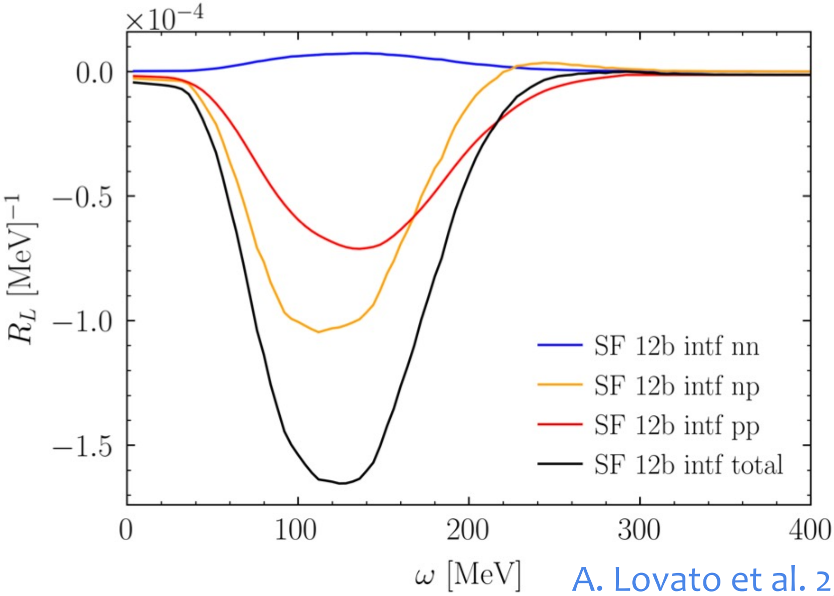
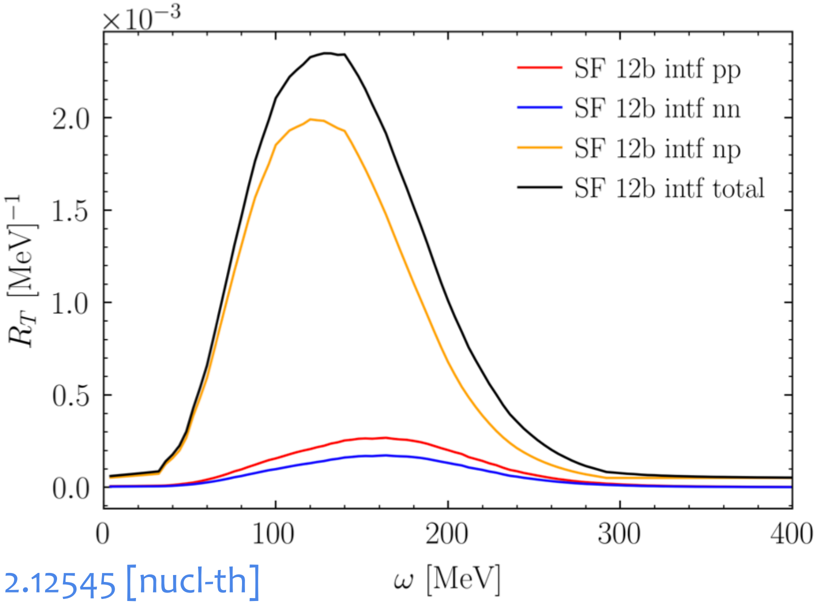
Inclusive electron scattering - Responses

- Numerical evaluation enables us to separate responses by isospin
- **L** and **T** responses for **q** = 500 MeV on Carbon
- Broken down into nn, np, and pp pairs
  - Transverse response is dominated by np pairs
  - Consistent with what is seen in SRCs in (e,e’NN) experiments on heavy nuclei
  - Sets ground work for computing isospin dependence of two-nucleon knockout





A. Lovato et al. 2312.12545 [nucl-th]

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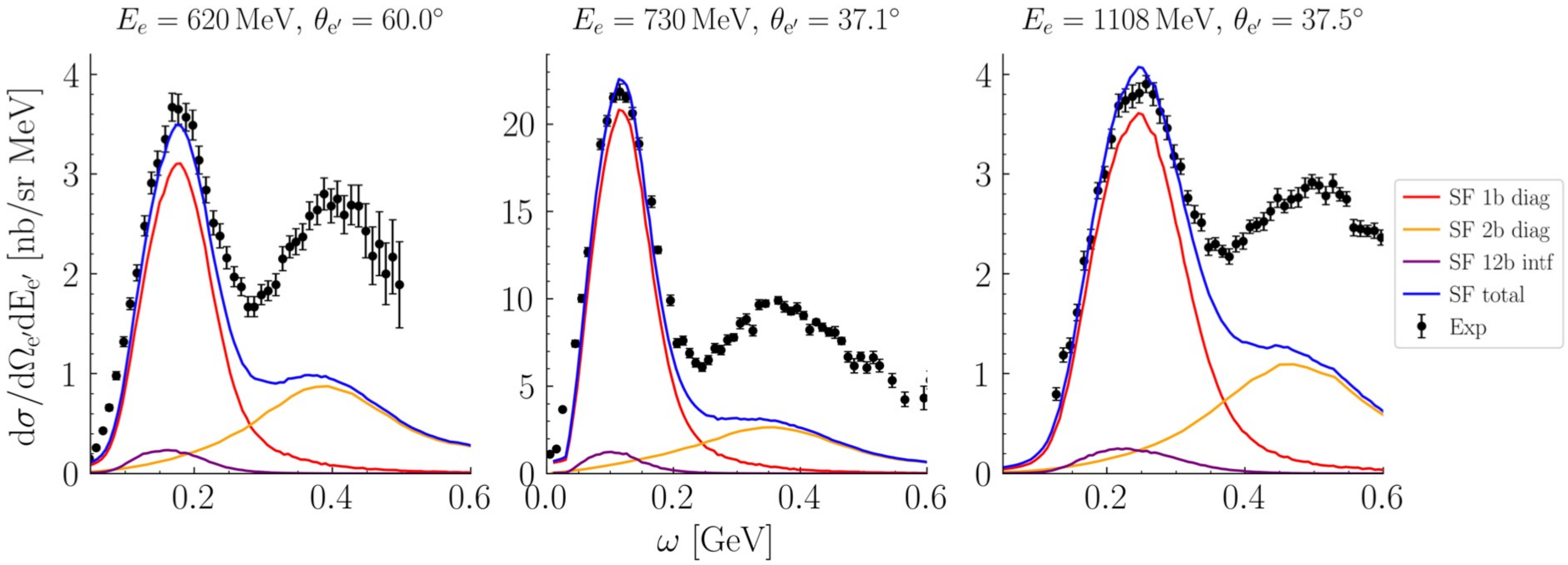
Inclusive electron scattering – Cross Sections

- Effects are more pronounced at backwards angles
  - Transverse response provides more strength in the cross section

- Cross sections agree better with data and with GFMC calculations in the QE region



A. Lovato et al. 2312.12545 [nucl-th]

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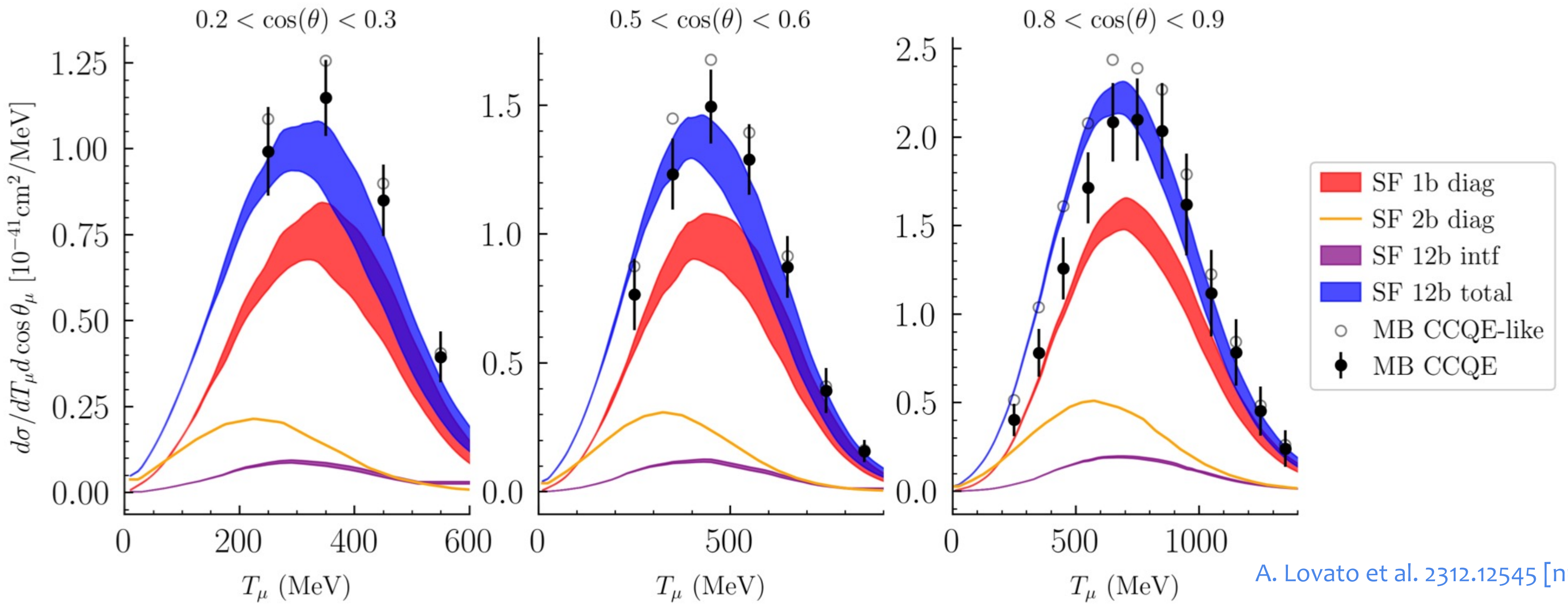
Flux Folded Neutrino scattering – Cross Sections

- Axial form factor appears in pure one-body and interference cross sections
  - Tension between deuterium extraction and Lattice QCD (MINERvA) extractions

- Bands in cross section represent an interpolation between dipole with M_A = 1 GeV and Lattice QCD z-expansion
  - Dramatic sensitivity in one-body cross section, but also small sensitivity in interference



A. Lovato et al. 2312.12545 [nucl-th]

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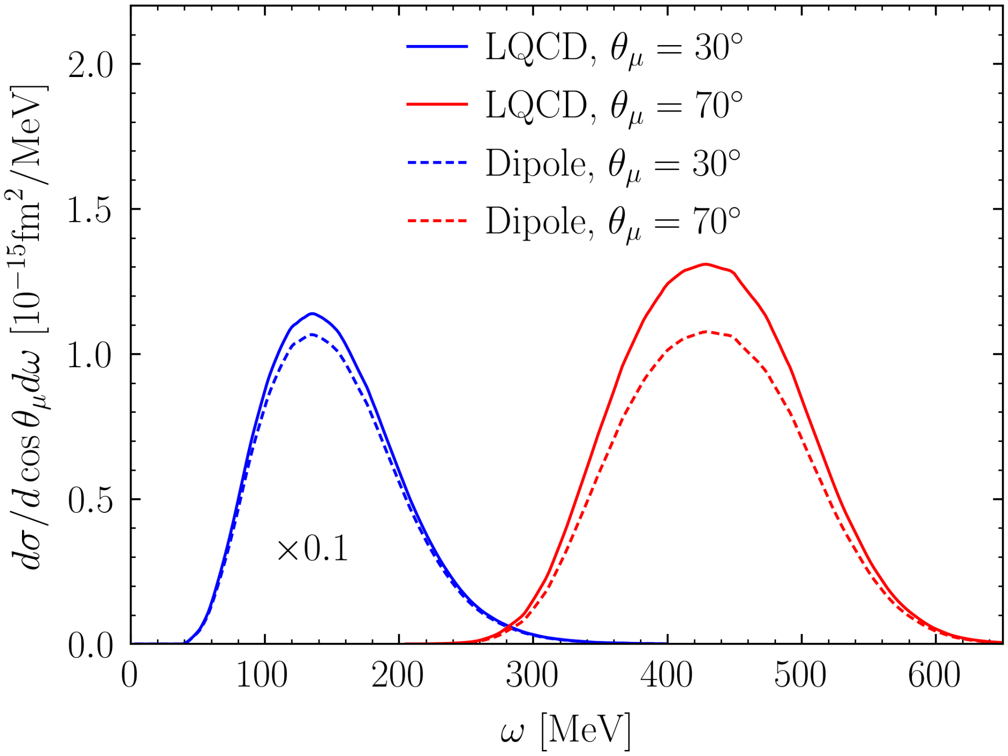
Sensitivity to FA in EW Interference Contribution

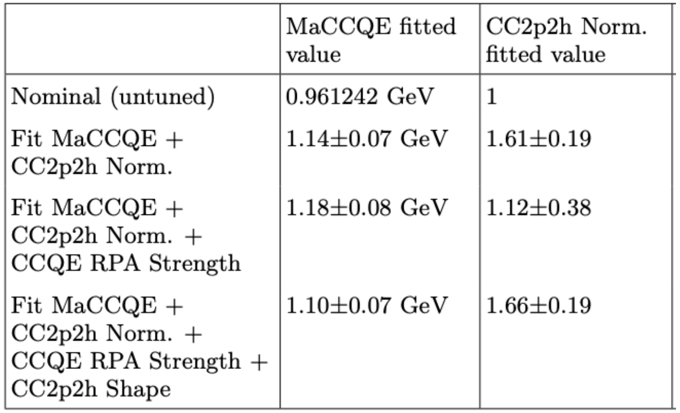
- Fixed E_ν = 1 GeV
- As in pure one-body current, the sensitivity of interference cross section to FA increases at high Q^2 (backwards angles)



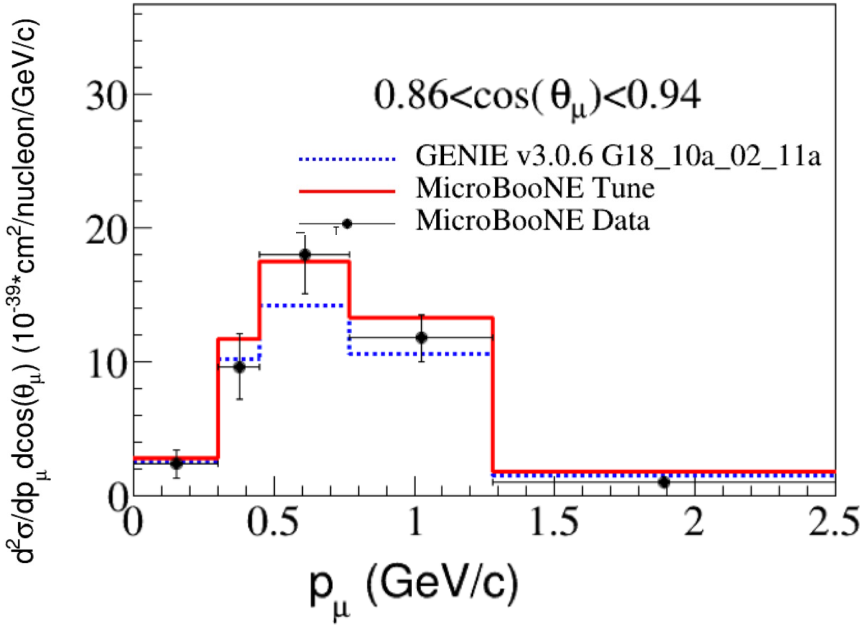
Interference effects

- Data from MiniBooNE/T2k/MicroBooNE point to normalization issues in CC0pi

- Generators tune their model parameters to fit to data
  - Ex: MicroBooNE tune to T2k CC0pi data
  - Increase M_A and 2p2h normalization

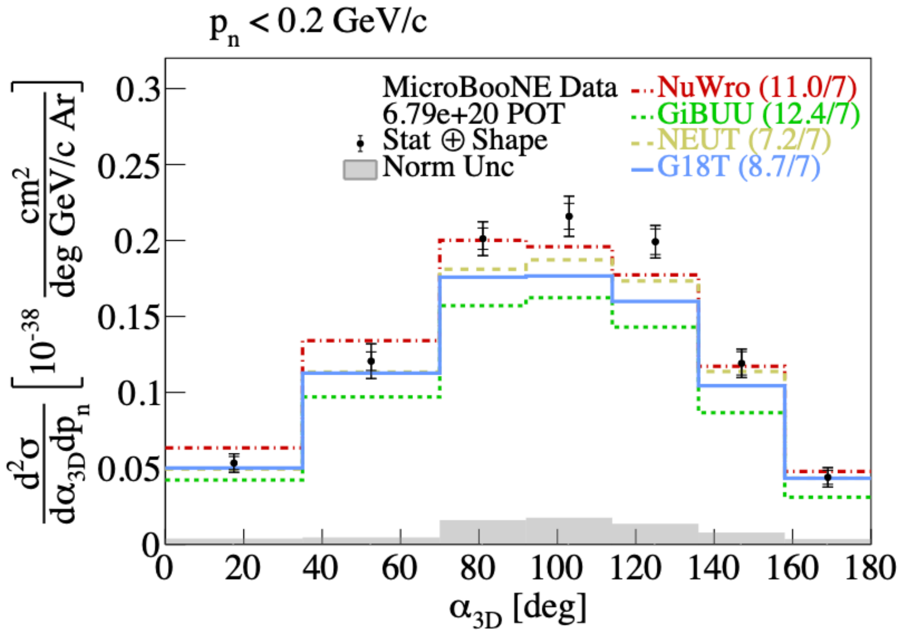


	MaCCQE fitted value	CC2p2h Norm. fitted value
Nominal (untuned)	0.961242 GeV	1
Fit MaCCQE + CC2p2h Norm.	1.14±0.07 GeV	1.61±0.19
Fit MaCCQE + CC2p2h Norm. + CCQE RPA Strength	1.18±0.08 GeV	1.12±0.38
Fit MaCCQE + CC2p2h Norm. + CCQE RPA Strength + CC2p2h Shape	1.10±0.07 GeV	1.66±0.19



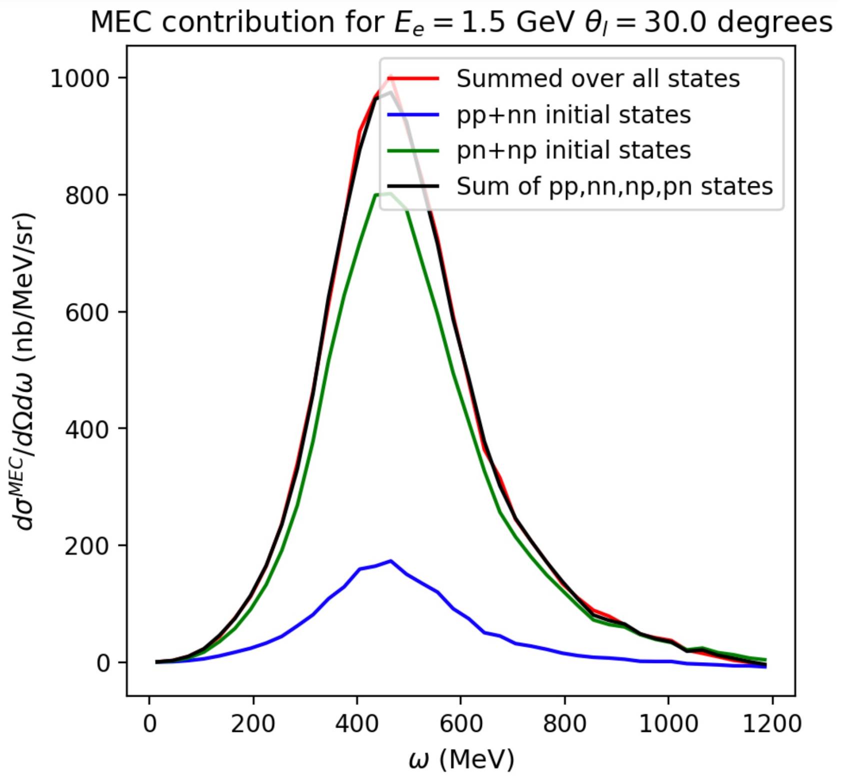
Interference effects

- Problem persists in very pure exclusive QE-like data
- Inclusion of these interference effects could alter these tunes
- Warning that tuning with incomplete models gives you the wrong physics



Future work and Relevance to the SBN program

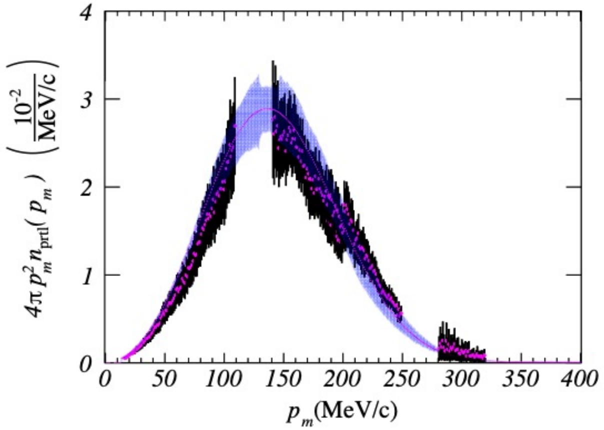
- Exclusive calculations of interference and pure two-body current contributions
  - Exclusive predictions for pure one-body currents already show big differences
  - Exclusive interference contribution relatively straightforward since isospin dependence already computed
  - Pure two-body exclusive predictions on the way



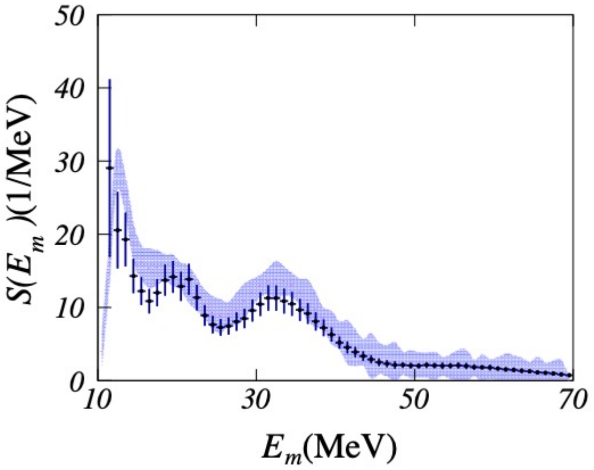
Future work and Relevance to the SBN program

- Extension of these predictions to asymmetric nuclei like Ar
  - Certain cancellations in symmetric nuclei are not present– more diagrams
  - Use of Ar and Ti Spectral functions extracted from Jlab

L. Jiang et al., Phys.Rev.D 105 (2022) 11, 112002



(b) $30 < E_m < 54$ MeV



- Explore A dependence and dependence on number of np pairs
  - (^{12}C intf/ ^{12}C QE) > (^{40}Ar intf/ ^{40}Ar QE)?

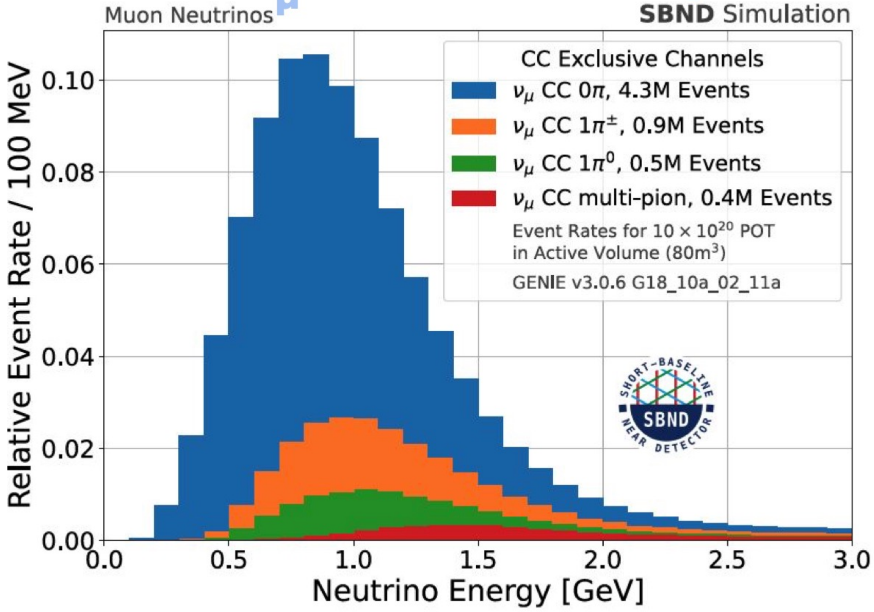
Future work and Relevance to the SBN program

- SBND's massive statistics will enable precision studies and allow us to constrain our interaction models in an unprecedented way

- Test applications of NMBT in EW interactions in the GeV region with high precision

- Constrain unknown axial form factors
  - Important for 2p2h and resonance
  - Currently known only to ~15%

ν_μ ~2million / year



Conclusion

- Unlocking full potential of SBN program requires precise SM predictions and understanding of neutrino-nucleus cross sections

- Many body nuclear theory is the optimal starting point for building neutrino-nucleus scattering models
  - Systematic control
  - Comparison with virtually exact results

- Important to incorporate all relevant reaction mechanisms within a consistent picture so that we are not tuning incorrect physics!

- Consistent implementation of all of this is coming

- Stay Tuned

Thank you!