Axial-vector Form Factor from Lattice QCD

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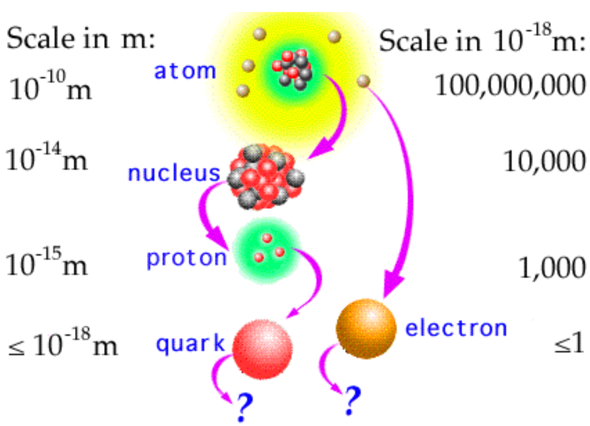

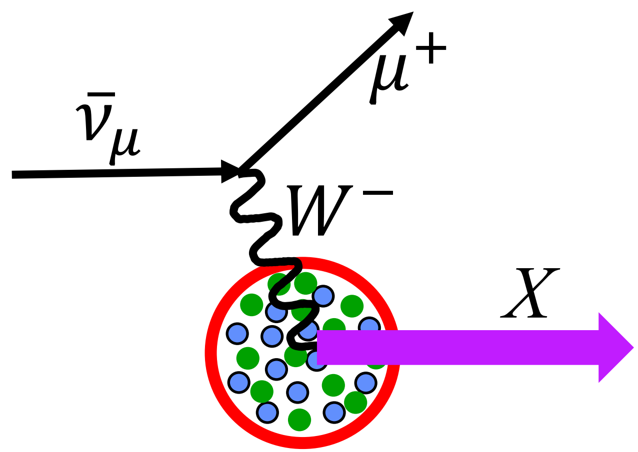

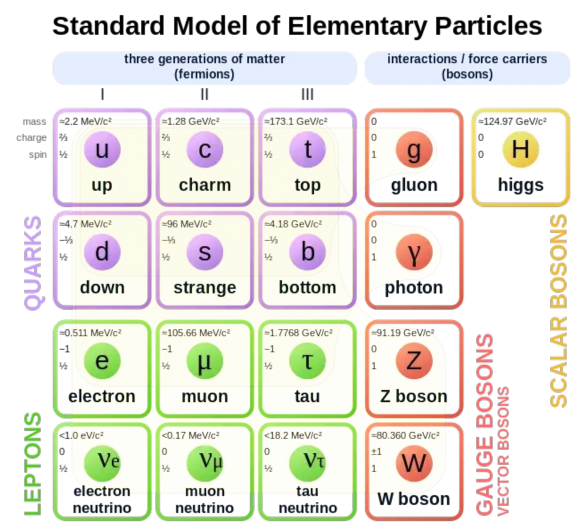

LA-UR: 23-29580

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Thanks for computer resources

OLCF (INCITE HEP133), ERCAP@NERSC (HEP, NP), USQCD@JLAB, LANL IC

- USQCD Community white paper:
  Lattice QCD and Neutrino-Nucleus Scattering, *Eur.Phys.J.A* 55 (2019) 11, 196
- Snowmass 2021 White Paper
  Theoretical tools for neutrino scattering: interplay between lattice QCD, EFTs, nuclear physics, phenomenology, and neutrino event generators. e-Print: 2203.09030 [hep-ph]
- Rajan Gupta, Review at Lattice 2023: arXiv:2401.16614

Publications on Form Factors

AFF: R. Gupta et al, (PNDME) PRD 96, 114503 (2017)

VFF: Y-C Jang, et al, (PNDME) PRD 101, 014507 (2020)

AFF: Y-C Jang et al, (PNDME) PRL 124, 072002 (2020)

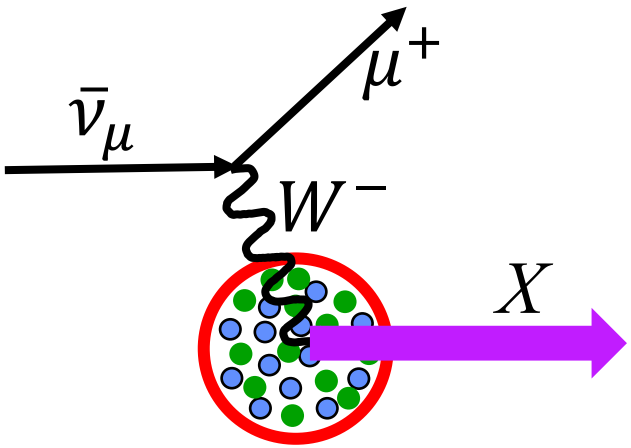
Both: S. Park, et al, (NME) PRD 105, 054505 (2022)

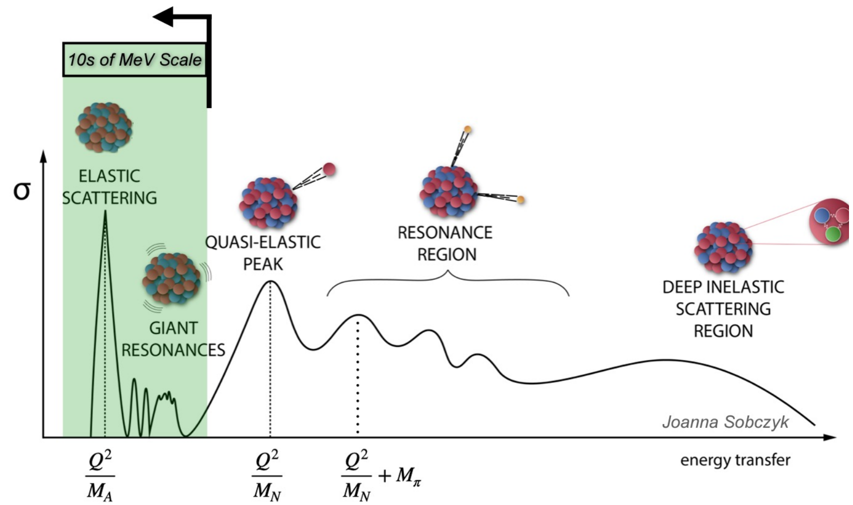
AFF: Y-C Jang, et al, (PNDME) PRD 109, 014503 (2024)

AFF: Tomalak, Gupta, Bhattacharya PRD 108, 074514 (2023)

Review at Lattice 2023: arXiv:2401.16614

Neutrino-nucleus scattering experiments





- Incoming neutrino energy and flux not known precisely
- Dynamics of struck Argon nucleus is too complex to simulate directly and connect to final states seen in the detectors

Need to know event-by-event the
- Neutrino energy
- Neutrino-nucleus cross-section

To resolve the
- Mass hierarchy between $(\nu_e, \nu_\mu, \nu_\tau)$
- Mixing angles $\theta_{12}, \theta_{23}, \theta_{13}$
- Size of CP violation angle δ_{CP}

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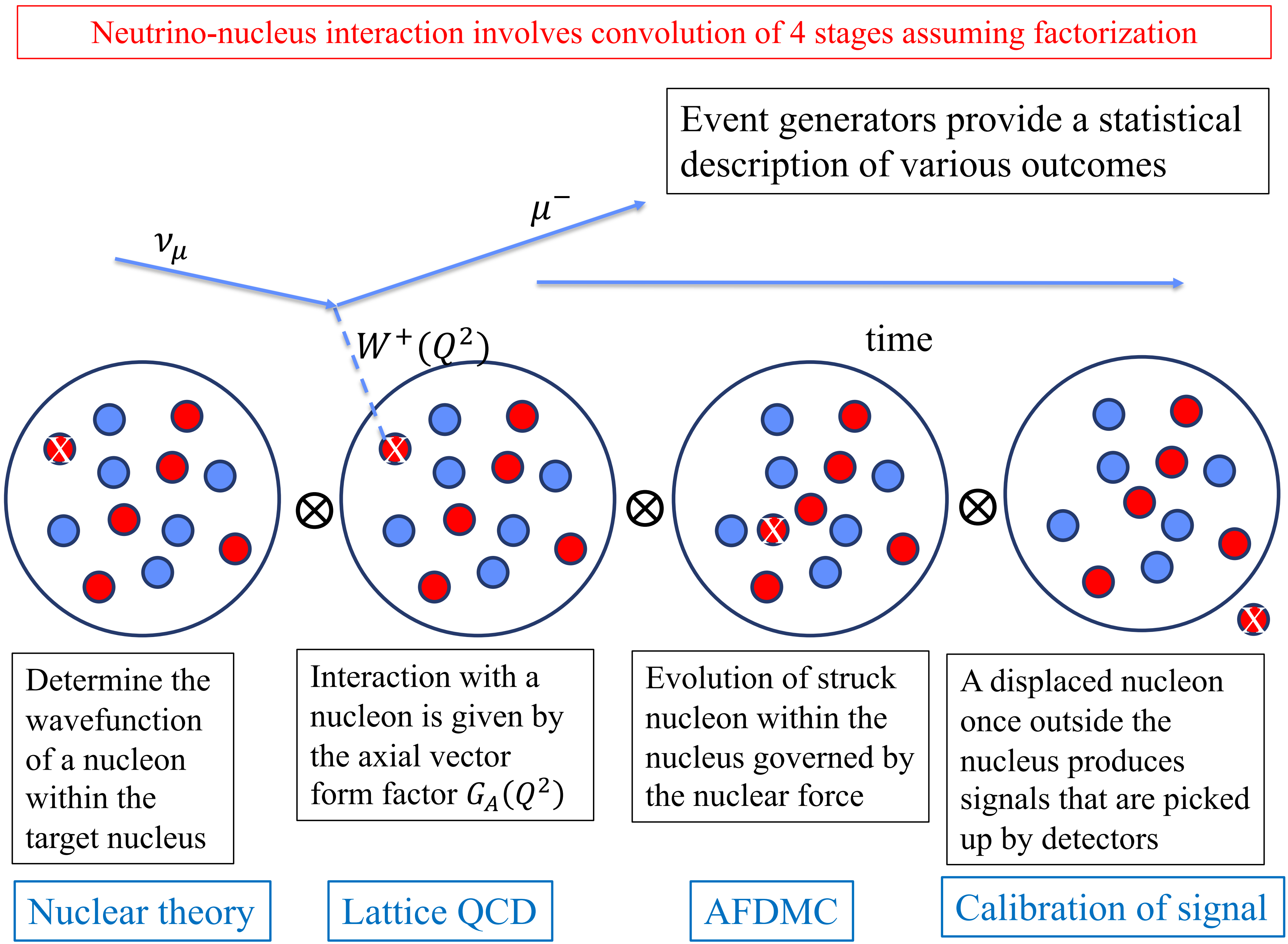
Theory → Event Generators

Factorization of the process

1. Wavefunction of the initial state of the "struck" nucleon within the nucleus
2. Axial vector FF of the nucleon
3. Intra nucleus evolution of the struck nucleon using nuclear many body theory
4. Evolution of final state particles to the detectors

Complete implementation of these within Monte Carlo event generators with uncertainty quantification at each step needed for determining neutrino oscillation parameters



Neutrino-nucleus interaction involves convolution of 4 stages assuming factorization
Event generators provide a statistical description of various outcomes
ν_μ
μ^-
$W^+(Q^2)$
time
⊗
⊗
⊗
Determine the wavefunction of a nucleon within the target nucleus
Interaction with a nucleon is given by the axial vector form factor $G_A(Q^2)$
Evolution of struck nucleon within the nucleus governed by the nuclear force
A displaced nucleon once outside the nucleus produces signals that are picked up by detectors
Nuclear theory
Lattice QCD
AFDMC
Calibration of signal

The ν-n differential cross-section:

$$\frac{d\sigma}{dQ^2}\begin{pmatrix}\nu_l + n \to l^- + p \\ \bar{\nu}_l + p \to l^+ + n\end{pmatrix}$$

$$= \frac{M^2 {G_F}^2 cos^2\theta_c}{8\pi {E_\nu}^2}\left\{A(Q^2) \pm B(Q^2)\frac{(s-u)}{M^2} + C(Q^2)\frac{(s-u)^2}{M^4}\right\},$$

$$\begin{aligned} A(Q^2) &= \frac{(m^2+Q^2)}{M^2}\left[(1+\tau)F_A^2 - (1-\tau)F_1^2 + \tau(1-\tau)F_2^2 + 4\tau F_1 F_2 \right. \\ &\quad \left. -\frac{m^2}{4M^2}\left((F_1+F_2)^2 + (F_A+2F_P)^2 - 4\left(1+\frac{Q^2}{4M^2}\right)F_P^2\right)\right], \\ B(Q^2) &= \frac{Q^2}{M^2}F_A(F_1+F_2), \\ C(Q^2) &= \frac{1}{4}(F_A^2 + F_1^2 + \tau F_2^2). \end{aligned}$$

F_A = axial form factor
$G_E = F_1 - \tau F_2$ Electric
$G_M = F_1 + F_2$ Magnetic
$\tau = Q^2/4M^2$
$M = M_p$ = 939 MeV
m=mass of the lepton

$\langle N A_\mu N \rangle \to$ linear combination of F_A , $\tilde{F}_P$

$\langle N V_\mu N \rangle \to G_E$, G_M

Lattice QCD Inputs for DUNE

Ideal: Matrix elements (form factors) for $\nu - {}^{40}Ar$ scattering

$$\langle X \,|\, A_\mu(q) \,|\, {}^{40}Ar \rangle$$

$$\langle X \,|\, V_\mu(q) \,|\, {}^{40}Ar \rangle$$

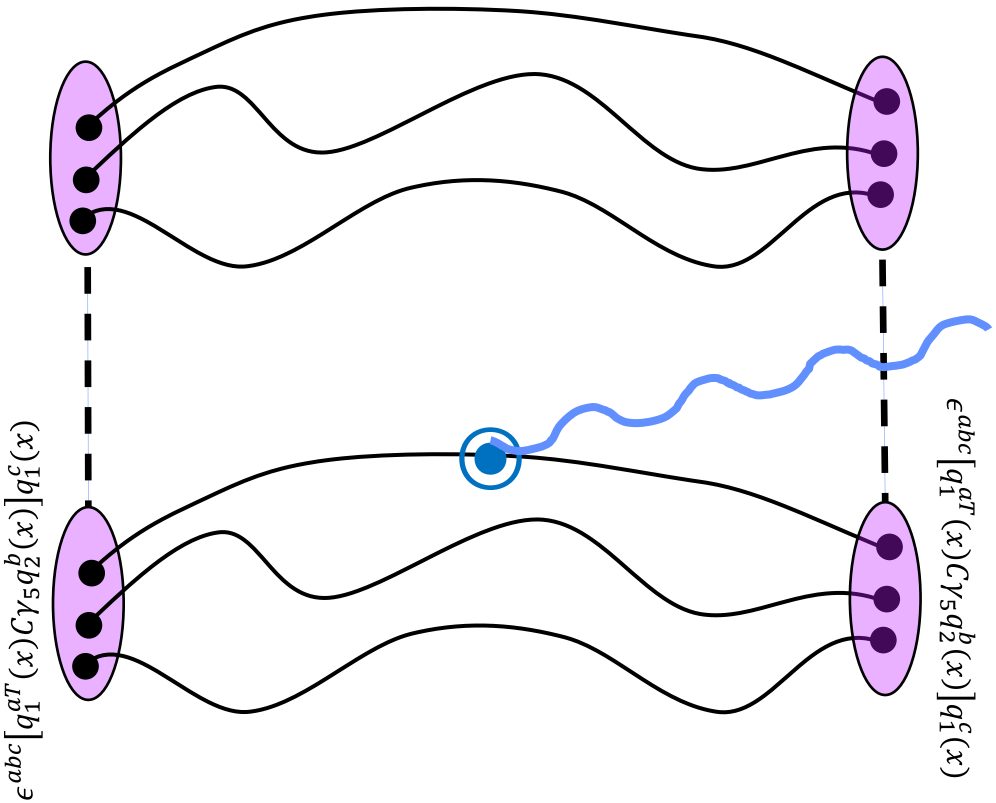
Start with nucleons and different energy regions: factorization

$\langle p \,|\, J^w_\mu(q) \,|\, n \rangle$	Quasi-elastic
$\langle n\pi \,|\, J^w_\mu(q) \,|\, n \rangle, \langle \Delta \,|\, J^w_\mu(q) \,|\, n \rangle$	Resonant
$\langle n\, p \,|\, J^{w+}_\mu(q) \,|\, n\, n \rangle$	2-nucleon
$\langle \mathrm{X} \,|\, J^w_\mu(q) \,|\, n \rangle$	DIS

Build these into the nuclear many body Hamiltonian

Why simulating ^{40}Ar is challenging

Wick contraction of $\Gamma_N^2 = \left\langle \Omega \left| \sum_x \overline{Ar}(x,t)\ A_\mu\ Ar(0,0) \right| \Omega \right\rangle$



Ar = 18p + 22n
=58u + 62d quarks

1) Number of all possible contractions of u and d quarks and insertion of A_μ is still "impossible" to program and simulate

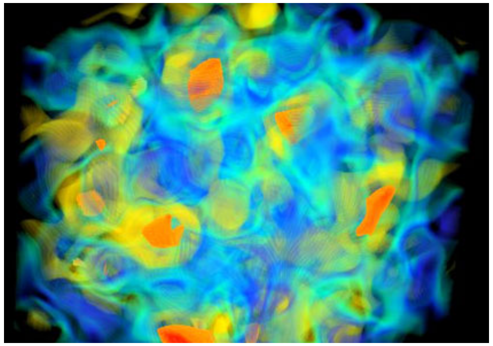
2) The signal will fall off with a high power of $e^{-(M_N - 1.5M_\pi)t}$

LQCD is QCD (a Quantum Field Theory) discretized on a lattice.

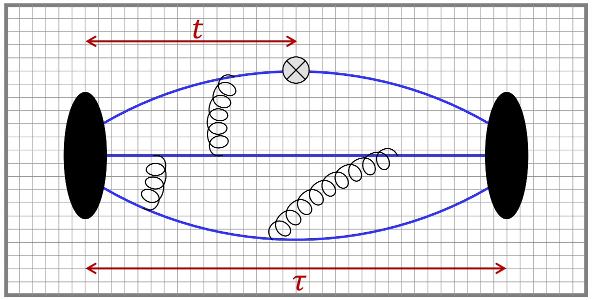
Wick rotation turns QFT into a stochastic computational problem.

Simulations of LQCD provide

- The quantum vacuum of QCD
  - ➢ ensembles of gauge configurations

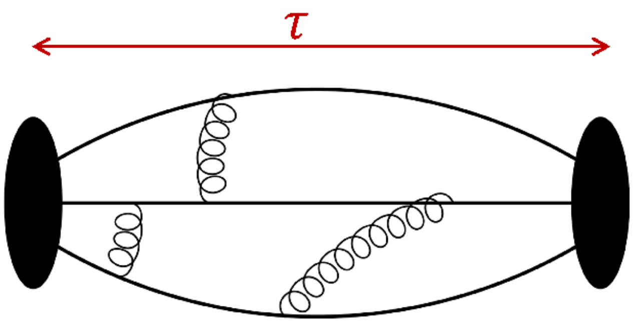


- N-point correlation functions
  - ➢ Hadrons and their interactions are built up using external probes on this vacuum
- Matrix elements $\langle N(p_f) | \boldsymbol{\mathcal{O}}(Q^2) | N(p_i) \rangle$ calculated between fully quantum hadronic states (wavefunctions)



Lattice QCD gives us Γ^n

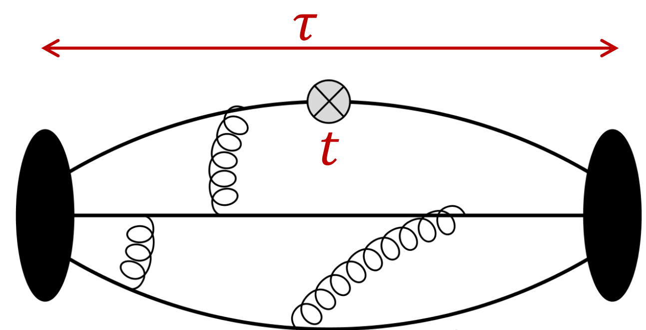
2-point function



$$\langle \Omega \mid \widehat{N_\tau}^\dagger \ \widehat{N}_0 \mid \Omega \rangle$$

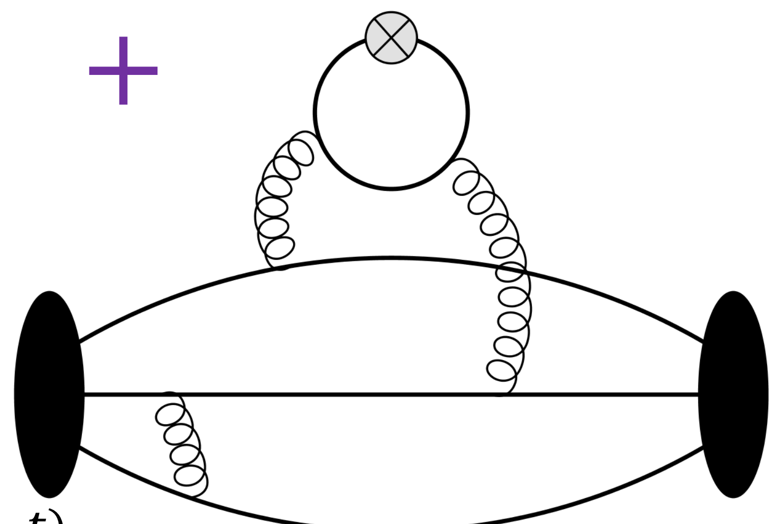
$$\Gamma^{2pt}(\tau) = \sum_i |A_i|^2 e^{-E_i \tau}$$

3-point functions



Connected

+

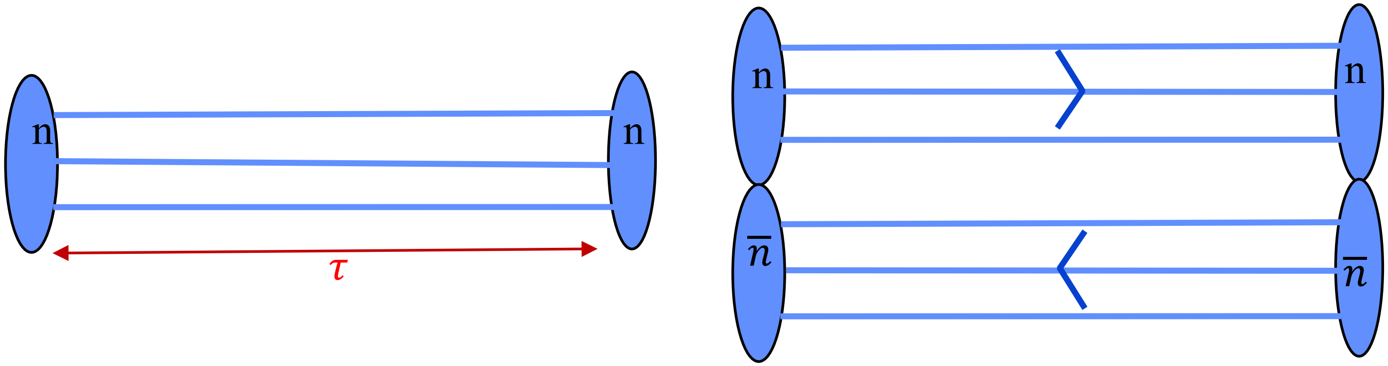


Disconnected

$$\langle \Omega \mid \widehat{N_\tau}^\dagger \, O(t) \, \widehat{N}_0 \mid \Omega \rangle$$

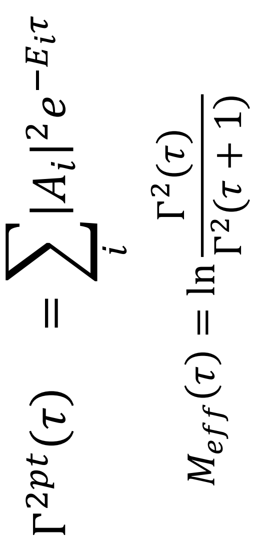
$$\Gamma_O^{3pt}(t,\tau) = \sum_{i,j} A_i^* A_j \langle i | O | j \rangle e^{-E_i t - E_j(\tau - t)}$$

Signal-to-noise falls as $e^{-(M_N - 1.5 M_\pi)\tau}$ in nucleon n-point functions



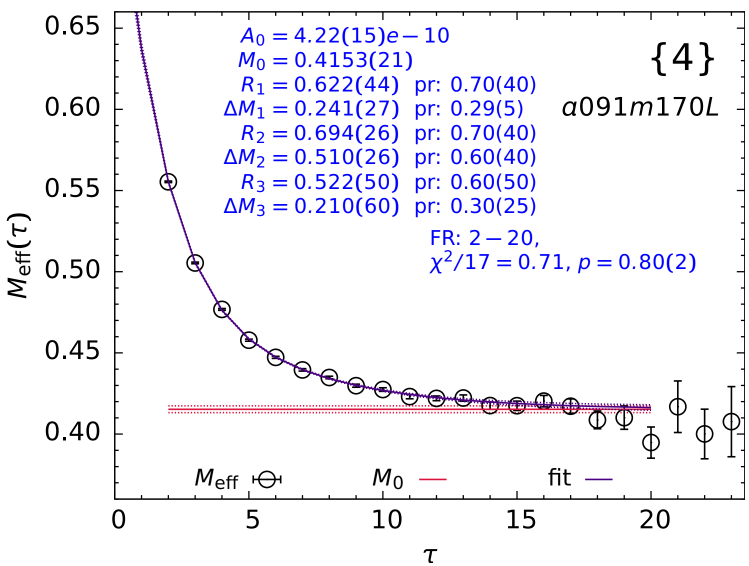
Signal: $\Gamma^2 = e^{-E_N \tau}$

Variance: $e^{-3E_\pi \tau}$



$$\Gamma^{2pt}(\tau) = \sum_i |A_i|^2 e^{-E_i \tau}$$

$$M_{eff}(\tau) = \ln \frac{\Gamma^2(\tau)}{\Gamma^2(\tau+1)}$$

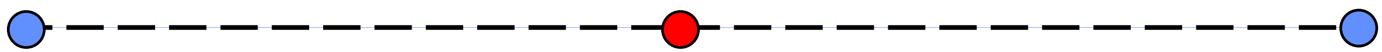


➢ To resolve a *small* mass gap *($M_1 - M_0$) requires* large *t*

Spectral decomposition of Γ^3

Three-point function for matrix elements of axial current $\mathcal{A}_\mu$

$$\langle \Omega | \mathbb{N}\, \mathcal{A}_\mu(\mathrm{t}) \overline{\mathbb{N}}(0) | \Omega \rangle$$



Insert $T = e^{-H\Delta t} \sum_i |n_i\rangle\langle n_i|$ at each Δt with $T\,|n_i\rangle \equiv e^{-H\Delta t}|n_i\rangle = e^{-E_i\Delta t}|n_i\rangle$

$$\langle \Omega | \mathbb{N}(\tau) \cdots e^{-\mathrm{H}\Delta t} \sum_j |n_j\rangle\langle n_j|\, \mathcal{A}_\mu e^{-\mathrm{H}\Delta t} \sum_i |n_i\rangle\langle n_i| \cdots \overline{\mathbb{N}}(0)|\Omega\rangle$$

$$\sum_{i,j} \underbrace{\langle \Omega | \mathbb{N} | n_j \rangle}_{A_j^*} e^{-\mathrm{E_j}(\tau - t)} \underbrace{\langle n_j | A_\mu | n_i \rangle}_{\text{Matrix Elements}} e^{-\mathrm{E_i} t} \underbrace{\langle n_i | \overline{\mathbb{N}} | \Omega \rangle}_{A_i}$$

$E_0, E_1, \ldots$ energies of the ground & excited states

$\mathrm{A}_0, \mathrm{A}_1, \ldots$ corresponding amplitudes

Extracting Nucleon Charges

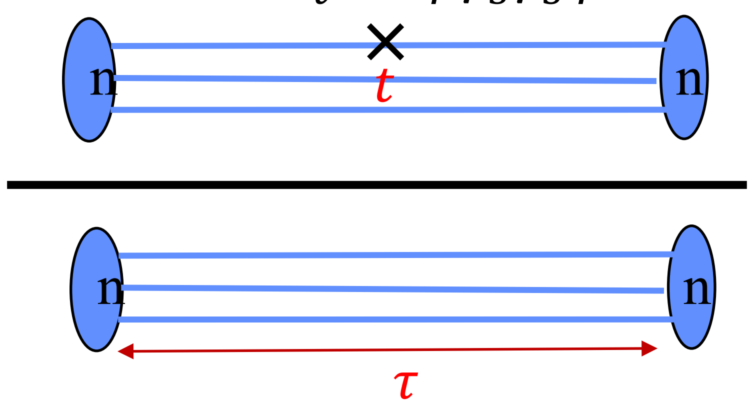
$$\Gamma^2 = \sum_i A_i^* A_i \, e^{-E_i \tau} \qquad \Gamma^3 = \sum_{i,j} A_i^* A_j \langle N_i | O | N_j \rangle e^{-E_i t} \, e^{-E_j (\tau - t)}$$

If only the ground state contributes ($\tau \to \infty$)

$$\frac{\Gamma^3}{\Gamma^2} = \frac{\langle \Omega | \overline{\mathbb{N}} \, A_\mu \mathbb{N} | \Omega \rangle}{\langle \Omega | \mathbb{N} \, \overline{\mathbb{N}} | \Omega \rangle} \to \langle N(p_i) | A_\mu \, (Q^2 = 0) | N(p_i) \rangle \to \boldsymbol{g_A}$$

$$O_t = \bar{\psi} \gamma_3 \gamma_5 \psi$$

$\dfrac{\Gamma^3}{\Gamma^2} =$



$\xrightarrow[\tau \to \infty]{} \boldsymbol{g_A}$

- Otherwise, fit Γ^3 using its spectral decomposition.
- Requires knowing the spectrum (energies E_i) & amplitude A_0 (from and Γ^2???)

Calculations of nucleon 2,3-point functions using LQCD are mature

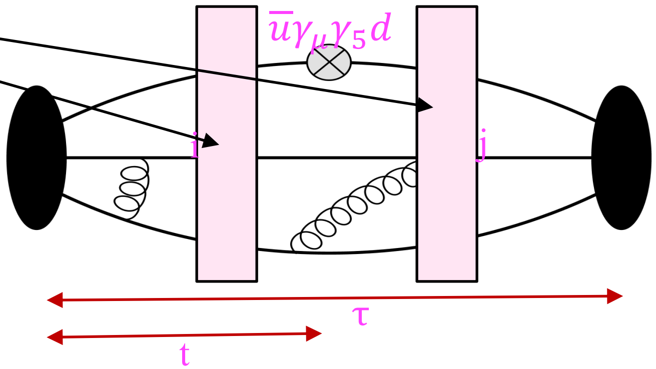
Spectrum (energies E_i & amplitudes A_i) and ME are extracted from fits to the spectral decomposition of 2- and 3-point functions

$$\Gamma^{2pt}(\tau) = \sum_i |A_i|^2 e^{-E_i\tau}$$

$$\Gamma_O^{3pt}(t,\tau) = \sum_{i,j} A_i^* A_j \langle i|O|j\rangle e^{-E_i t - E_j(\tau - t)}$$

Extract $\langle 0|\, O \,|\, 0 \rangle$

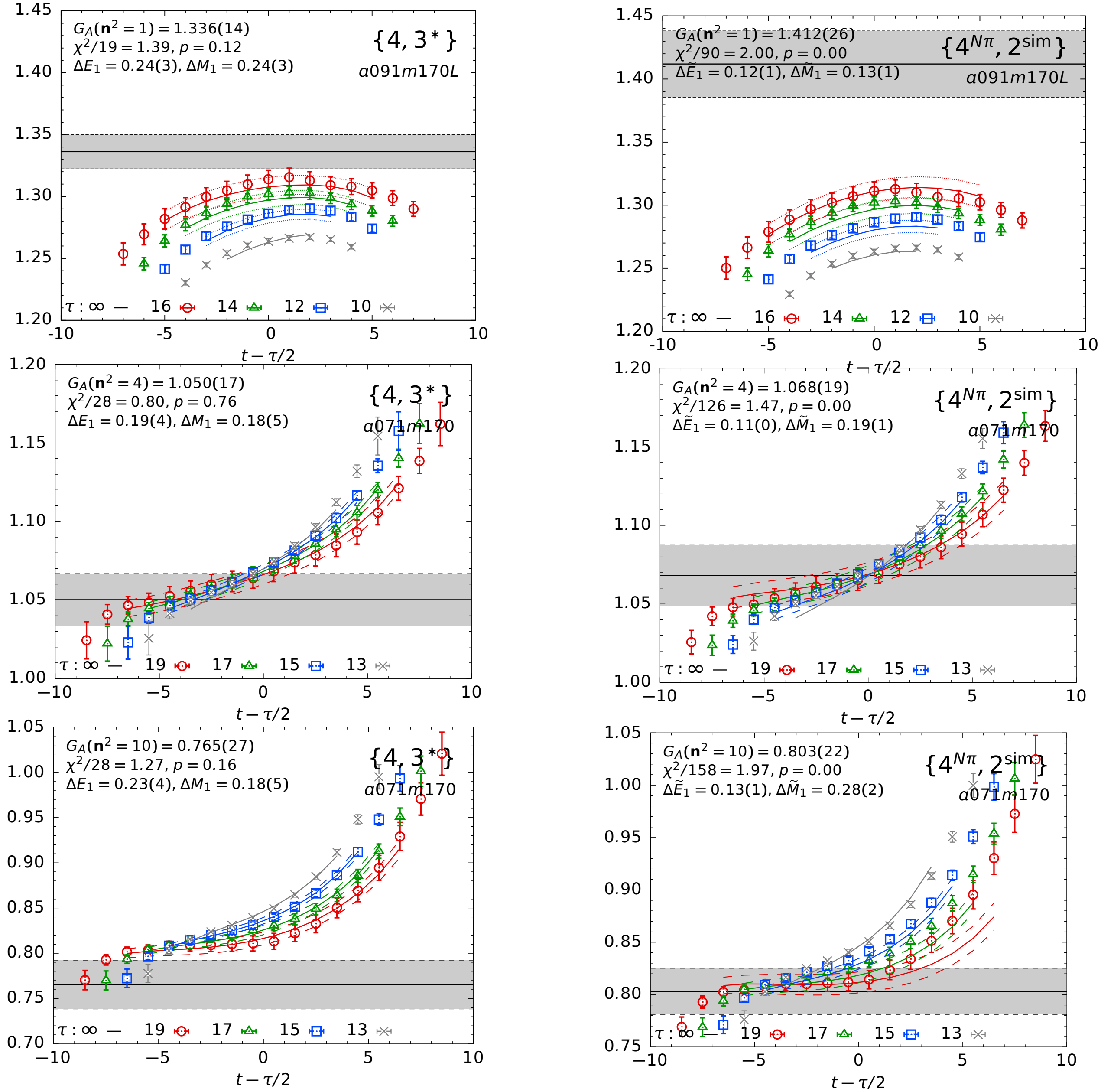
All states $|i\rangle$ with the same quantum numbers as N contribute unless A_i (or A_j) is zero



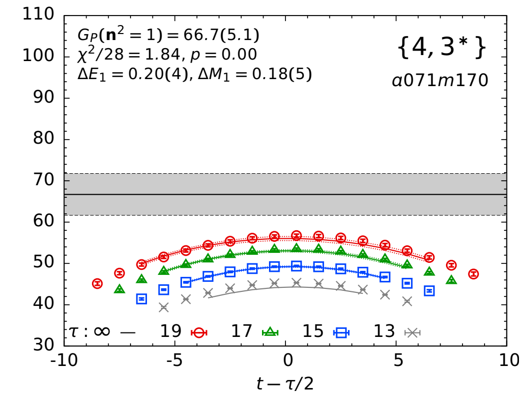
Radial excited States:
N(1440), N(1710)
Towers of multihadrons states
$N(\vec{k})\pi(-\vec{k})$ > 1200 MeV
$N(0)\pi(\vec{k})\pi(-\vec{k})$ > 1200 MeV

removing ESC from multihadron states remains a challenge

What do data look like?

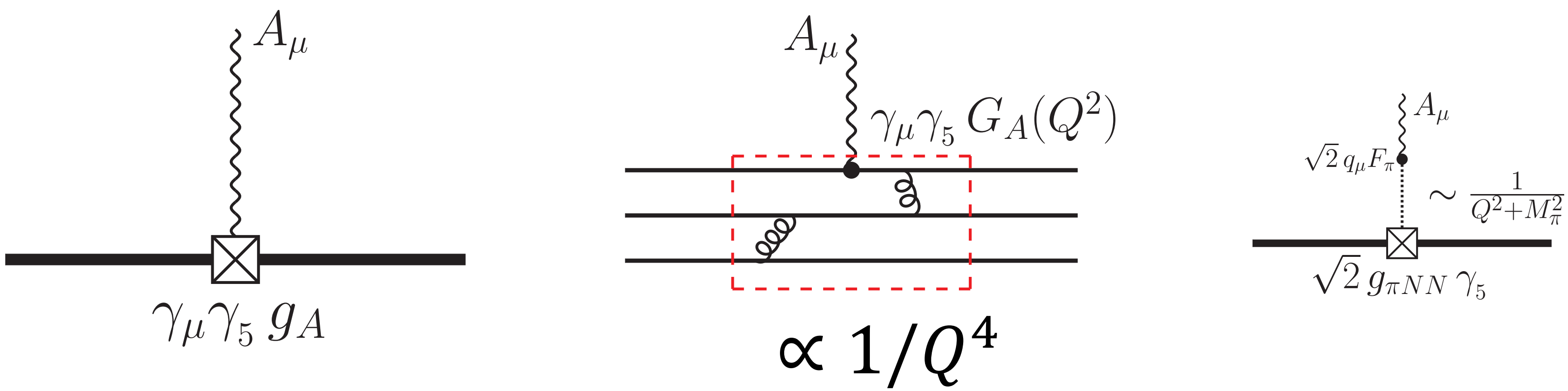


Systematics in lattice calculations: Summary



- Statistics
  - Signal falls as $e^{-(M_N-1.5M_\pi)\tau}$
- Excited state contributions (ESC)
  - Towers of $N\pi$ /$N\pi\pi$ multihadron states starting at ~1200 MeV
  - Which ($N\pi$ /$N\pi\pi$, radial, …) states contribute?
  - Fits to the spectral decomposition of Γ^n (truncated at 3 states)
- Chiral-Continuum-Finite-Volume (CCFV) extrapolation
  - $\sigma_{\pi N}(\mathrm{a},\mathrm{M}_\pi,\mathrm{M}_\pi\mathrm{L}) = \sigma_{\pi N}(0,\mathrm{M}_\pi = 135\mathrm{MeV},\infty) + \cdots$

$\Gamma^n \to ME \to$ Axial-vector Form Factors, $G_A, \widetilde{G}_P, G_P$



On each [iso-symmetric] ensemble characterized by $\{a, M_\pi, M_\pi L\}$

$$\langle N(p_f)|A^\mu(q)|N(p_i)\rangle = \bar{u}(p_f)\left[\gamma^\mu G_A(q^2) + q_\mu \frac{\tilde{G}_P(q^2)}{2M}\right]\gamma_5 u(p_i)$$

$$\langle N(p_f)|P(q)|N(p_i)\rangle = \bar{u}(p_f) G_P(q^2)\gamma_5\, u(p_i)$$

PCAC [$\partial_\mu A_\mu = 2mP$] relates G_A, $\tilde{G}_P$, G_P

Constraints once FF are extracted from ground state matrix elements

1) PCAC ($\partial_u A_u = 2\hat{m}\mathrm{P}$) requires

$$2\hat{m}G_P(Q^2) = 2M_N G_A(Q^2) - \frac{Q^2}{2M_N}\tilde{G}_P(Q^2)$$

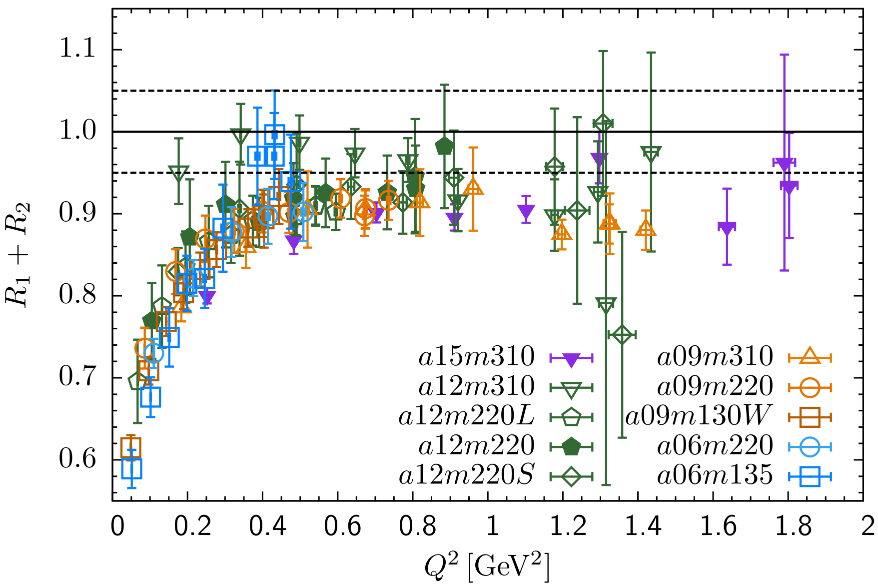
2) In any [nucleon] ground state

$$\partial_4 A_4 = \left(E_q - M_0\right)A_4$$

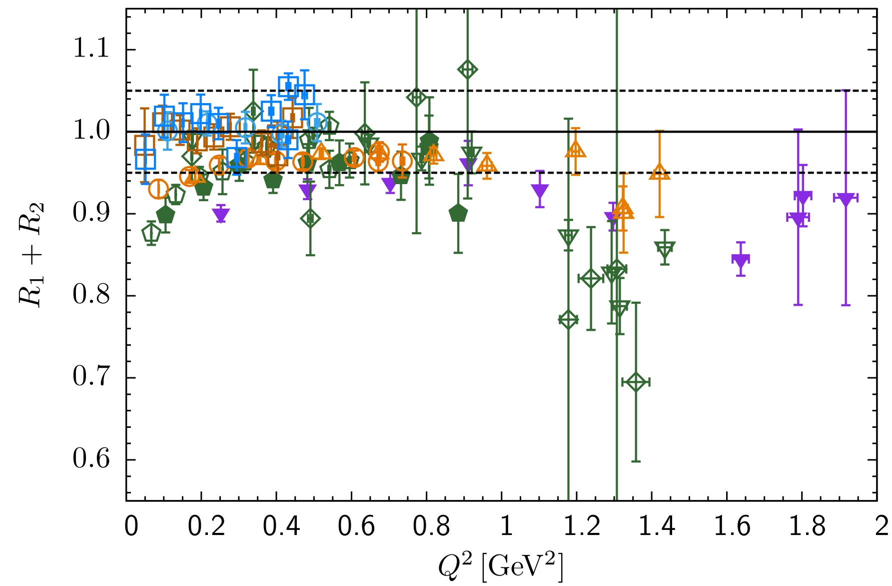
3) G_A, $\tilde{G}_P$ extracted from $\langle N(p_f)|A_i(q)|N(p_i)\rangle$

must be consistent with $\langle N(p_f)|A_4(q)|N(p_i)\rangle$

Satisfying PCAC relation



Standard Analysis
Pre 2019



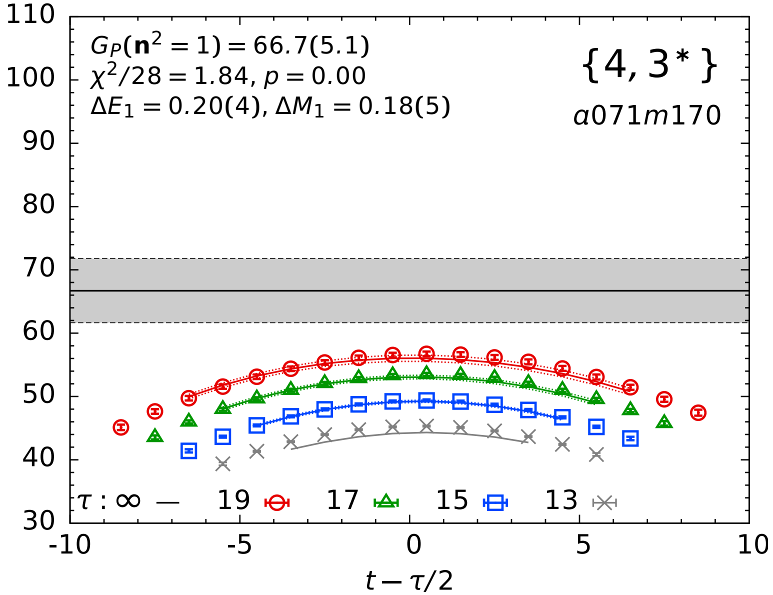
With $N\pi$
Post 2019

Oliver Bär: Phys. Rev. D 99, 054506 (2019), Phys. Rev. D 100, 054507 (2019)

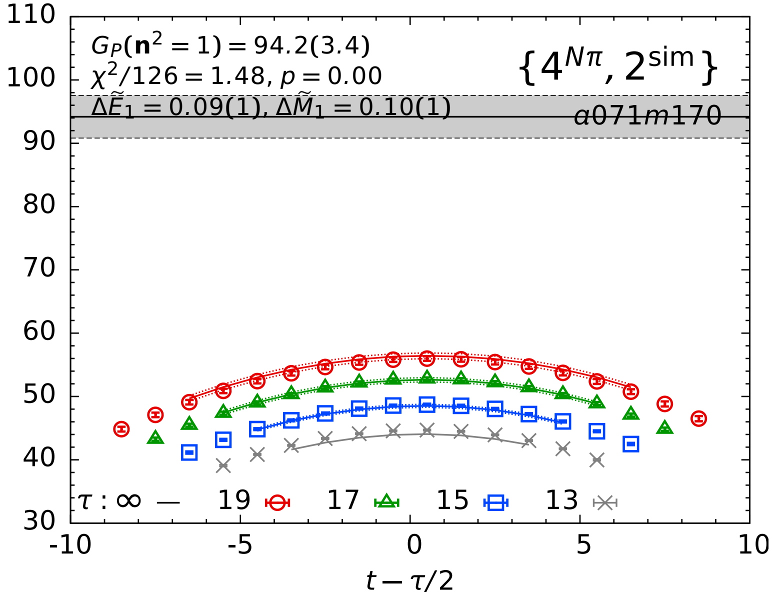
How large is the "$N\pi$" effect?

Output of a simultaneous fit to $\langle A_i \rangle, \langle A_4 \rangle, \langle P \rangle$ (called $\{4^{N\pi}, 2^{sim}\}$ fit) increases the form factors by:

- $G_A \sim 5\ \%$
- $\tilde{G}_P \sim 45\ \%$
- $G_P \sim 45\ \%$

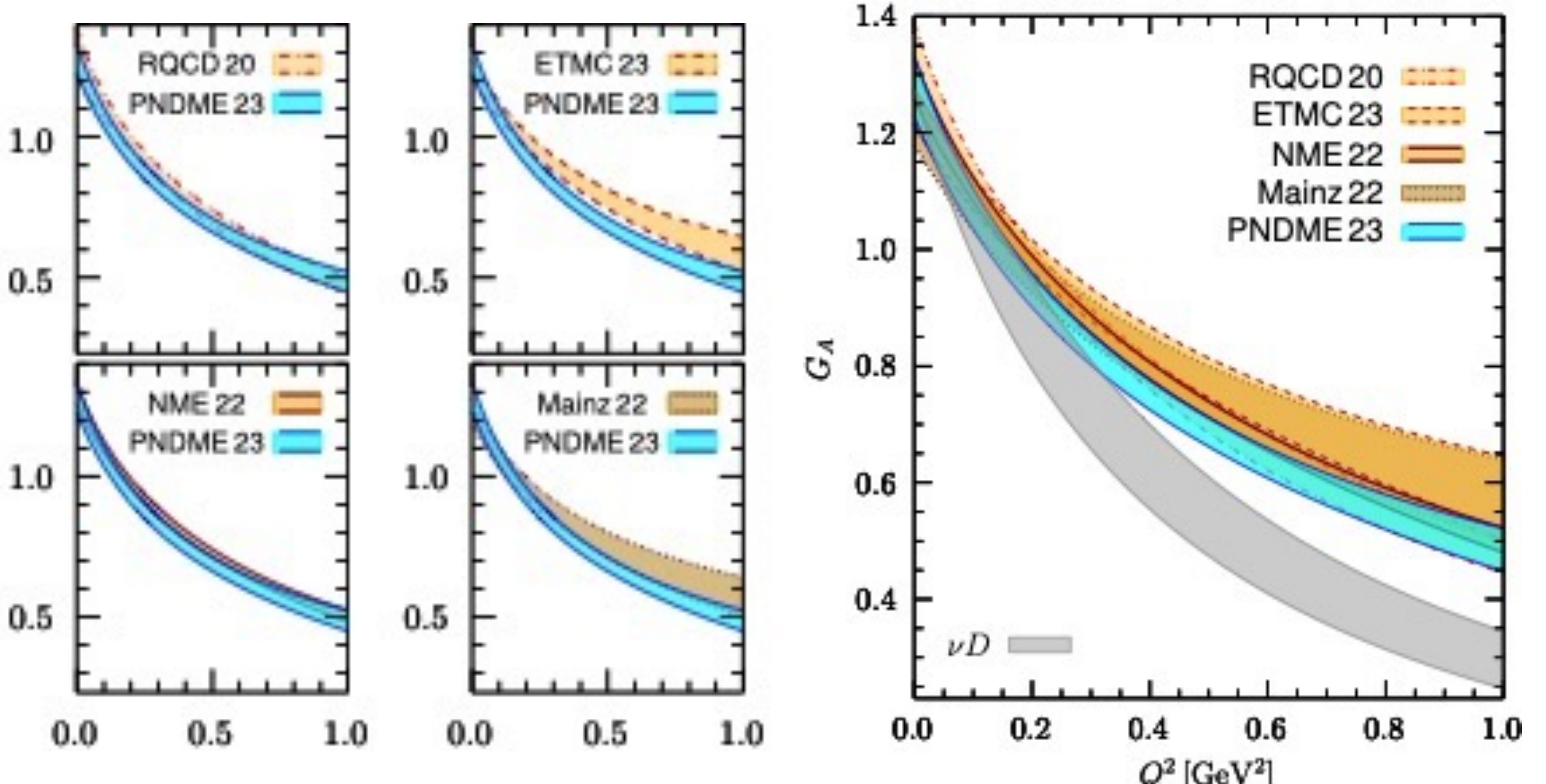


Standard 3-state fit to $\langle P \rangle$



Simultaneous 2-state fit to $\langle A_i \rangle, \langle A_4 \rangle, \langle P \rangle$ correlators

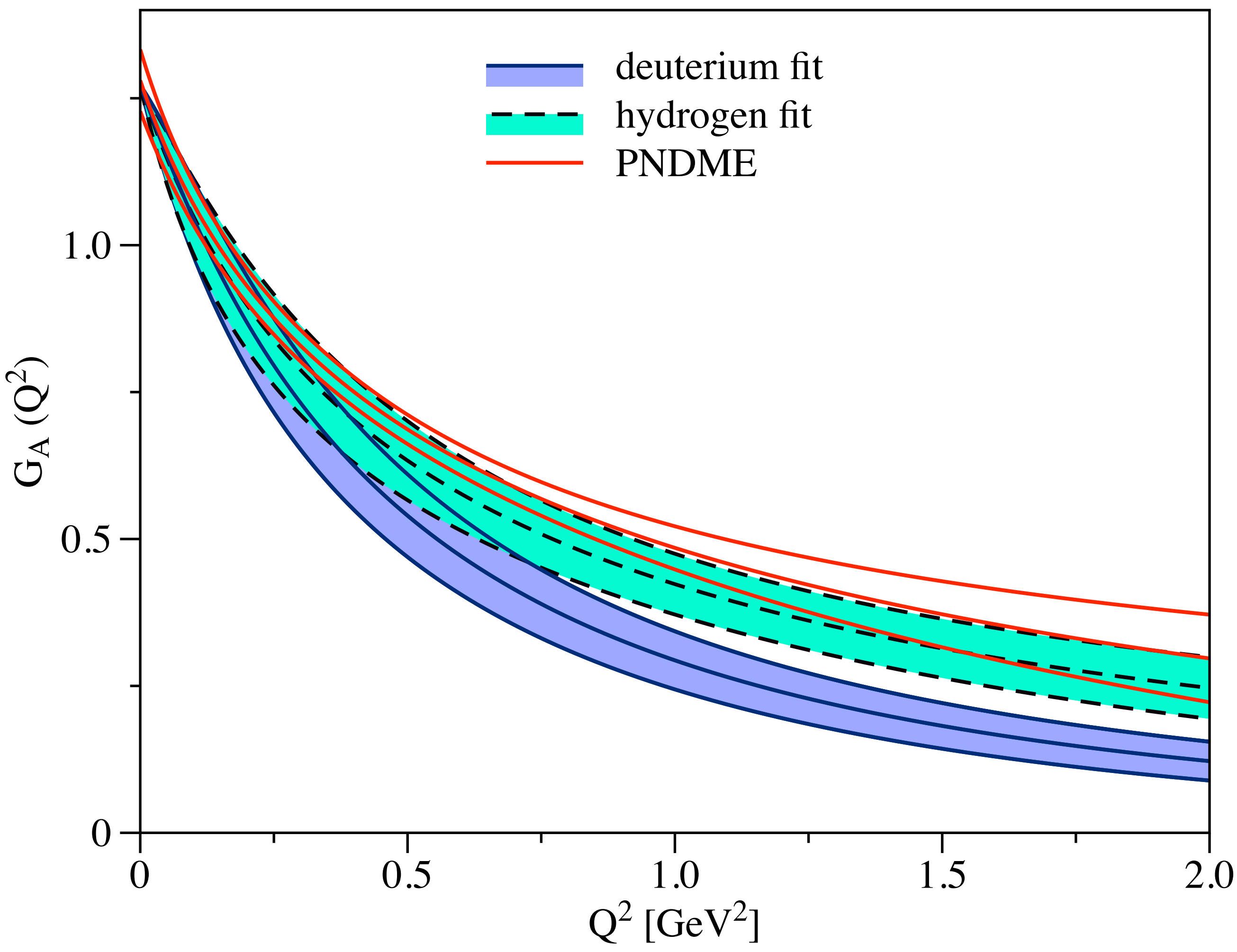
Comparing axial form factor from LQCD



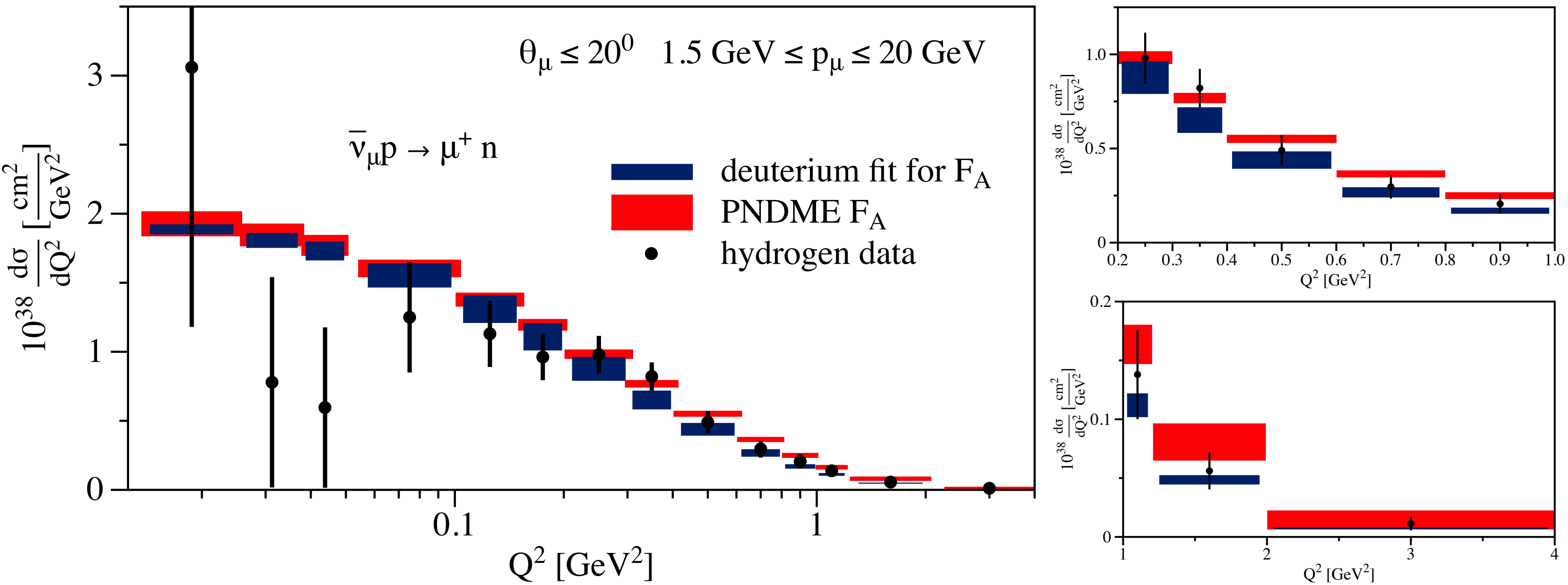
$g_A = 1.281(53)$
$\langle r_A^2 \rangle = 0.498(56)\text{fm}^2$

A consensus is emerging

Axial vector form factor



Comparing prediction of x-section using AFF from $\nu - D$ and PNDME with MINERvA data



$Q^2 < 0.2$ GeV^2 $0.2 < Q^2 < 1$ $Q^2 > 1$ GeV^2

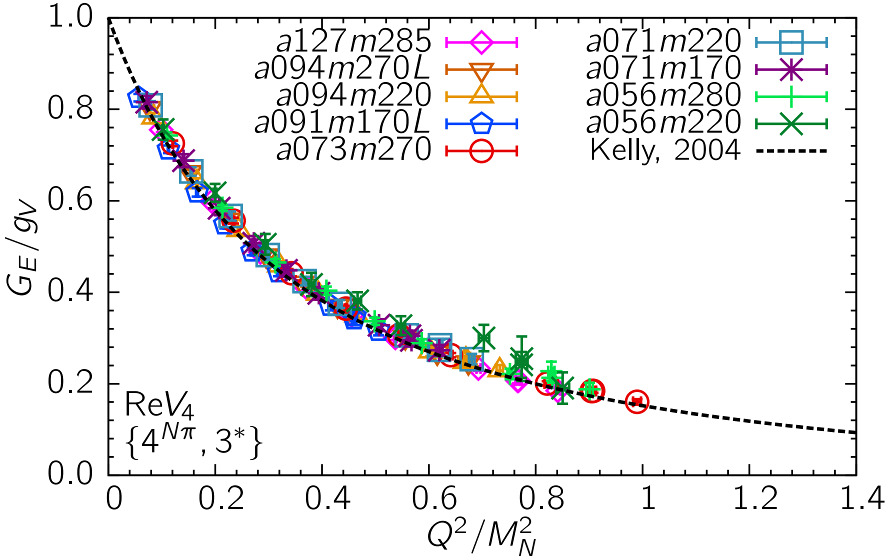
T. Cai, et al., (MINERvA) Nature **volume 614**, pages 48–53 (2023); Phys. Rev. Lett. 130, 161801 (2023)

Oleksandr Tomalak, Rajan Gupta, Tanmoy Bhattacharya, PRD 108 (2023) 074514

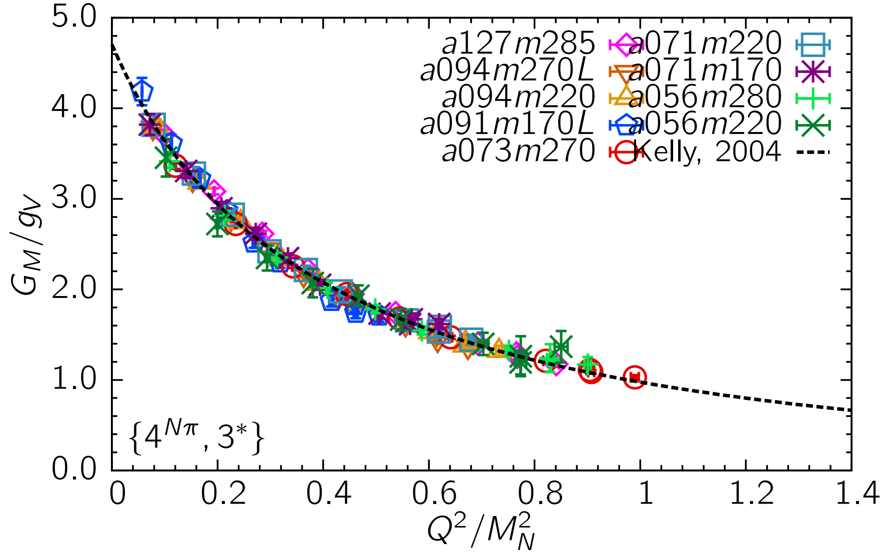
Mapping the AFF

- $0 < Q^2 < 0.2$ GeV^2
  - This region will get populated by simulations with $M_\pi \approx 135$ MeV , a → 0, $M_\pi L > 4$
  - MINERνA data has large errors
  - Characterized by g_A and $\langle r_A^2 \rangle$
  - $G_A(Q^2)$ parameterized by a z-expansion with a few terms
- $0.2 < Q^2 < 1$ GeV^2
  - Lattice data mostly from $M_\pi > 200$ MeV simulations
  - Competitive with MINERνA data. Cross check of each other
- $Q^2 > 1$ GeV^2
  - Lattice needs new ideas
  - MINERνA and future experiments

Electric & Magnetic FF



Electric



Magnetic

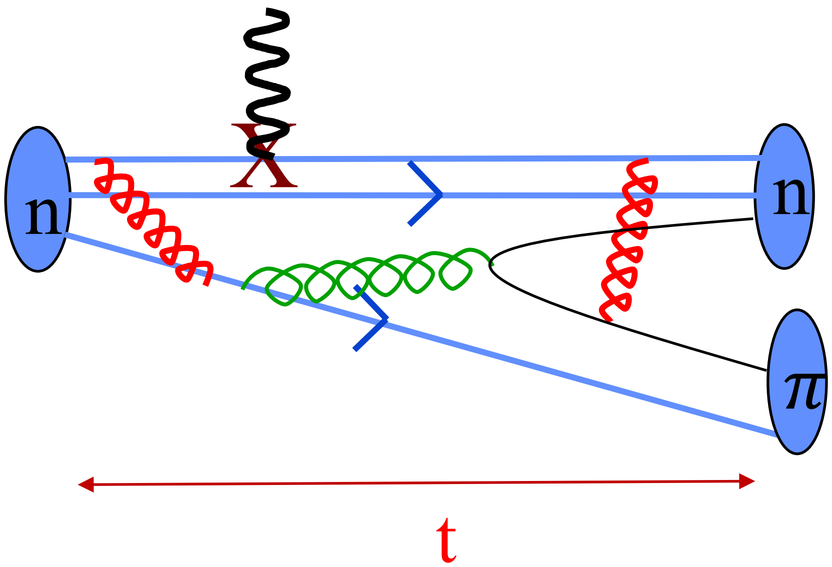
- The extraction of electric and magnetic form factors is insensitive to the details of the excited states
- Vector meson dominance → $N\pi\pi$ state should contribute (some evidence)
- The form factors do not show significant dependence on the lattice spacing or the quark mass
- Good agreement with the Kelly curve. Validates the lattice methodology
- Improve precision and get data over larger range of parameter values

Looking ahead

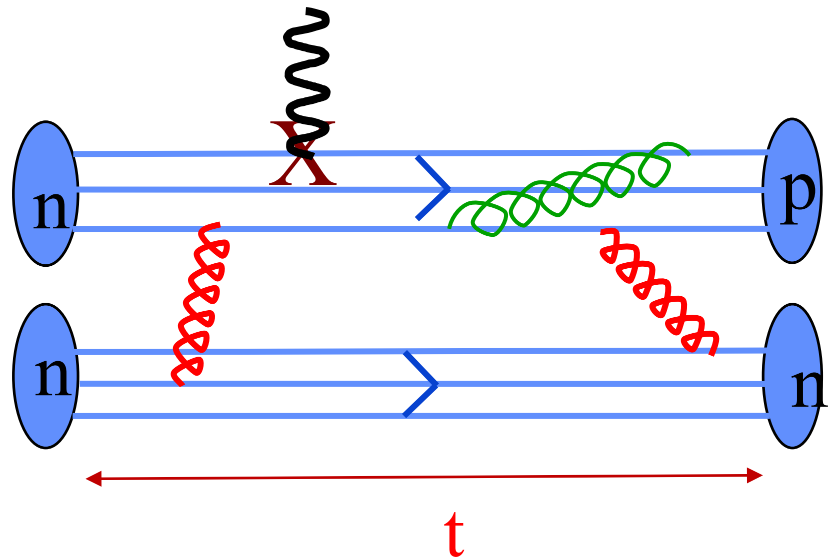
Variational with Multi-hadron states

NPB205 [FS5] (1982) 188

$$\hat{N} \rightarrow \hat{N} + c\,\widehat{N\pi}$$



$$\langle\, n\,\pi^{+} \,|\, J_{\mu}^{+}(q) \,|\, n \,\rangle$$



$$\langle\, n\, p \,|\, J_{\mu}^{+}(q) \,|\, n\, n \,\rangle$$

See

- Barca et al, 2211.12278 , 2110.11908
- NPLQCD Collaboration, *Phys.Rev.Lett.* 120 (2018) 15, 152002
- Nuclear matrix elements from lattice QCD for electroweak and beyond-Standard-Model processes, 2008.11160 [hep-lat]

Q^2_{max} under improvements in lattice calculations

$M_\pi \to 135$; $a \to 0$; $L \to \infty$

- $Q^2 = p^2 - (E(p) - M)^2$
- $p = \frac{2\pi}{La}\boldsymbol{n} = \frac{2\pi}{La}(n_1, n_2, n_3)$
- Fixed $\beta = 6/g^2$ (fixed a)
  - $M_\pi \to 135$ MeV keeping $M_\pi L$ fixed $\Rightarrow Q^2$ decreases
- Fixed M_π , take $a \to 0$ keeping L in fermi fixed
  - La fixed $\Rightarrow Q^2$ stays constant
- Fixed M_π and a: take $L \to \infty$
  - p decreases $\Rightarrow Q^2$ decreases

Q^2_{max} in lattice data will decrease but DUNE requires larger Q^2_{max}

Summary

- Challenges in lattice calculations of nucleon matrix elements:
  - Signal to noise degrades as $e^{-(M_N-1.5M_\pi)t}$
  - removing multi-hadron excited states to get ground state ME
  - including multi-hadron in initial and/or final state for transition ME
- Continue to develop a robust analysis strategy for identifying and removing dominant excited states in various nucleon matrix elements
- Improve chiral and continuum extrapolation. Simulate at more $\{a, M_\pi\}$
- Current $0.04 < Q^2 < 1$ GeV^2. Extend to larger Q^2 for DUNE
- Transition matrix elements
- Goal: Perform a comprehensive analysis of scattering data with input of lattice results for g_A, $G_E(Q^2)$, $G_M(Q^2)$, $G_A(Q^2)$, $\tilde{G}_P(Q^2)$

Improvements in algorithms and computing power are needed to reach few percent precision