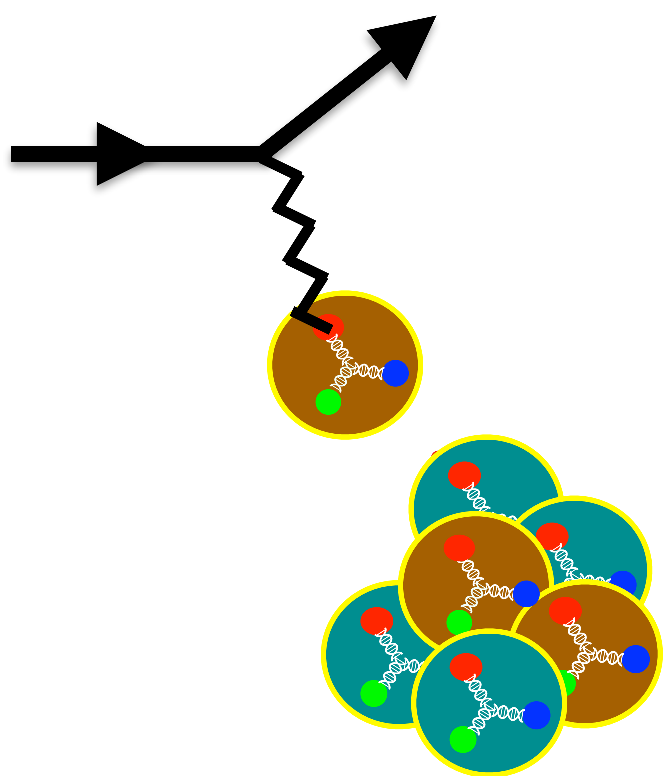
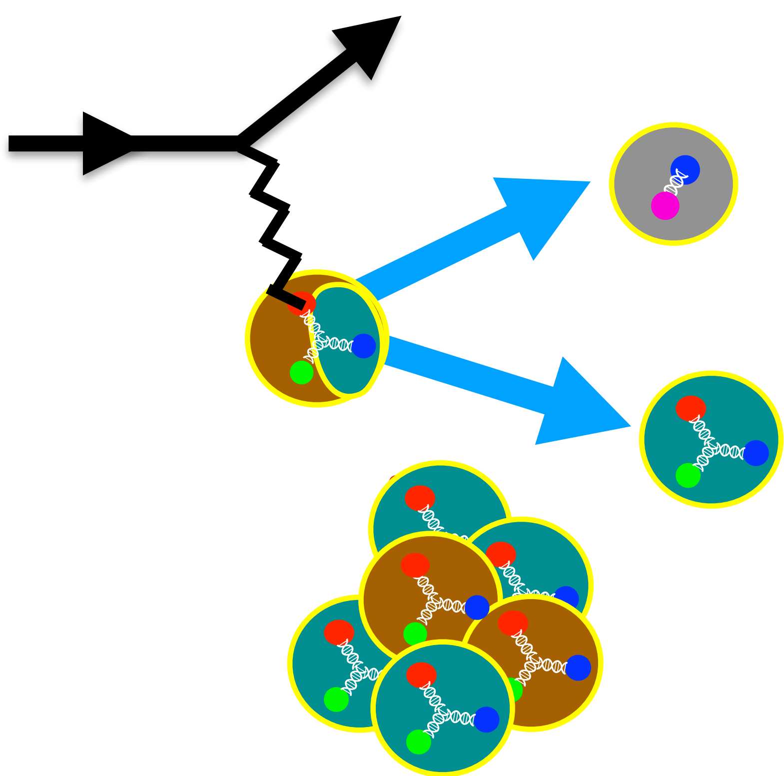
Theory uncertainties in neutrino-nucleus interactions





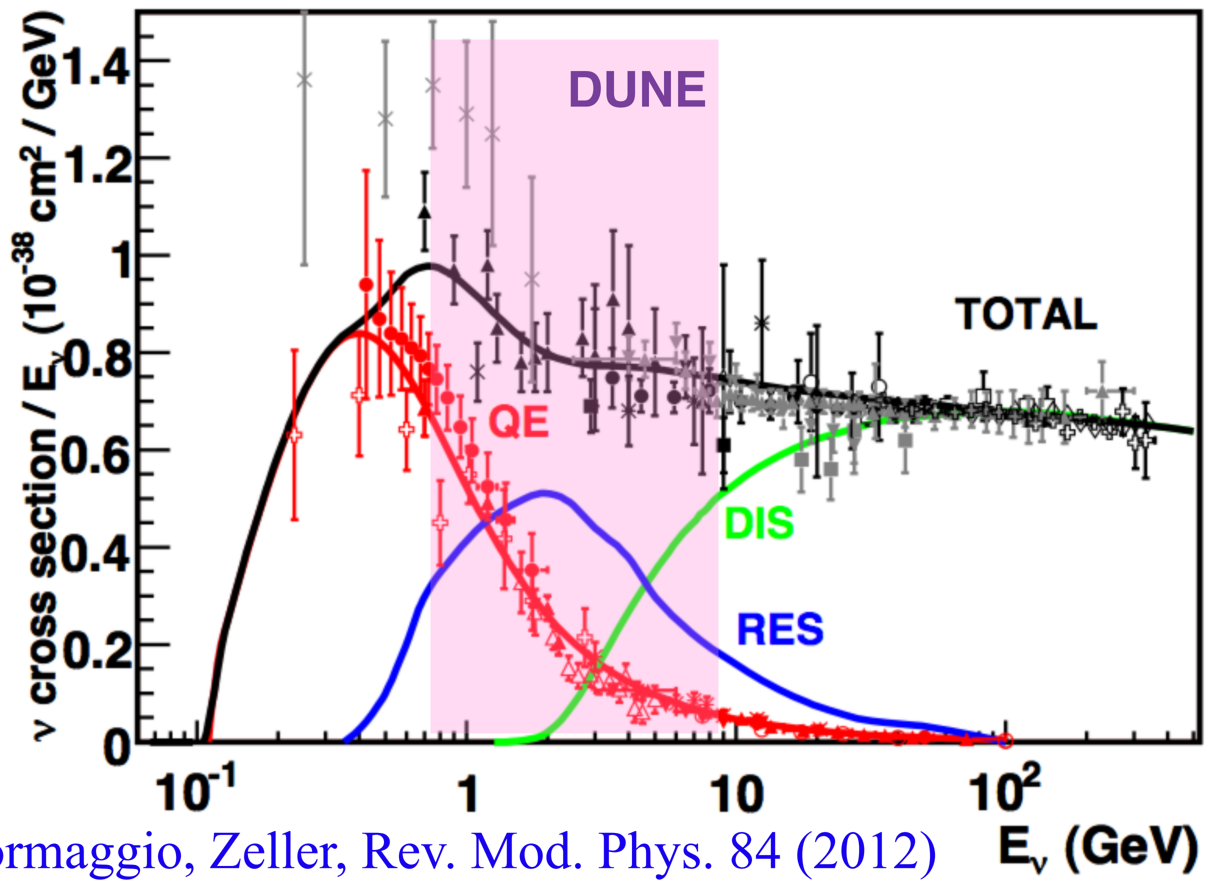
Michael Wagman

2nd Short-Baseline Experiment-Theory Workshop

Fermilab

April 3, 2024

Neutrino-nucleus scattering



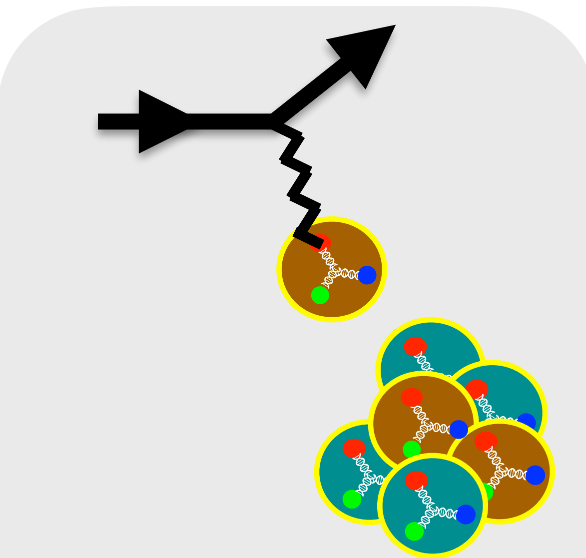
Formaggio, Zeller, Rev. Mod. Phys. 84 (2012)

Accelerator neutrino fluxes cover a wide range of energies where different processes dominate cross-section:

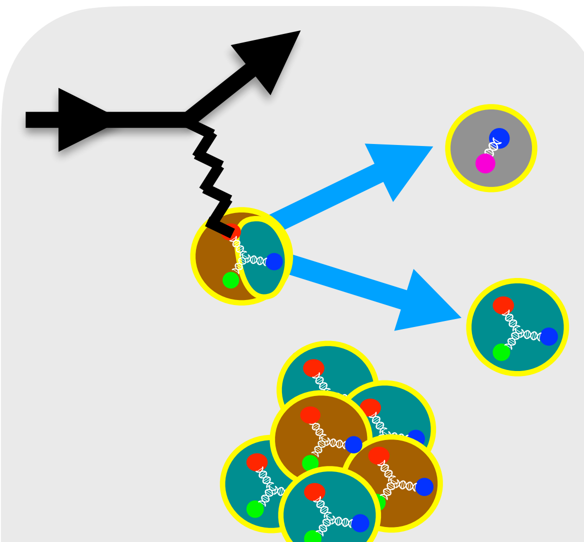
- Quasi-elastic nucleon scattering
- Resonance production
- Deep inelastic scattering

Theory input required to decompose cross section into such processes and therefore predict its energy dependence

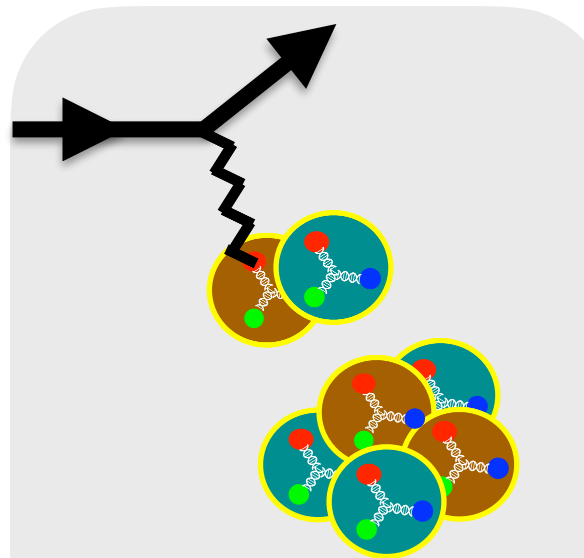
Effective theories for different energies require different inputs



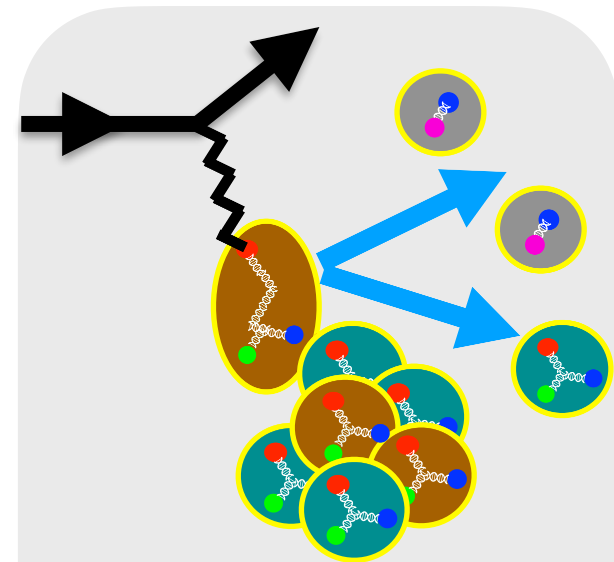
Nucleon form factors



Resonance production



Two-body currents

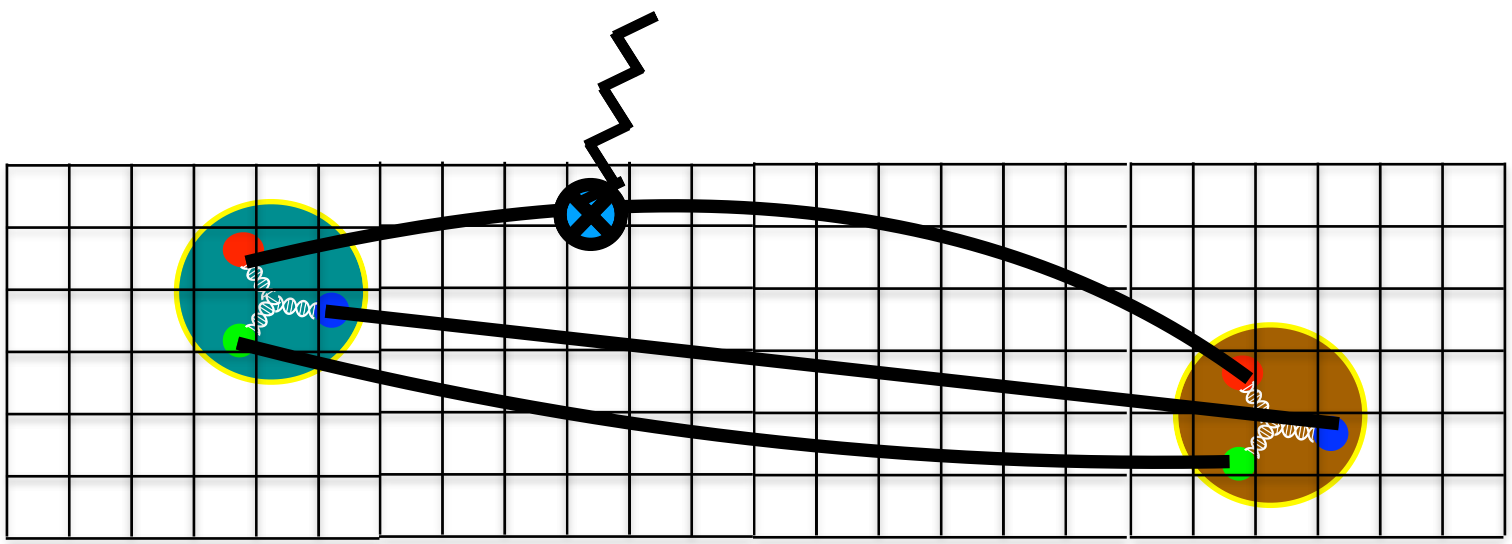


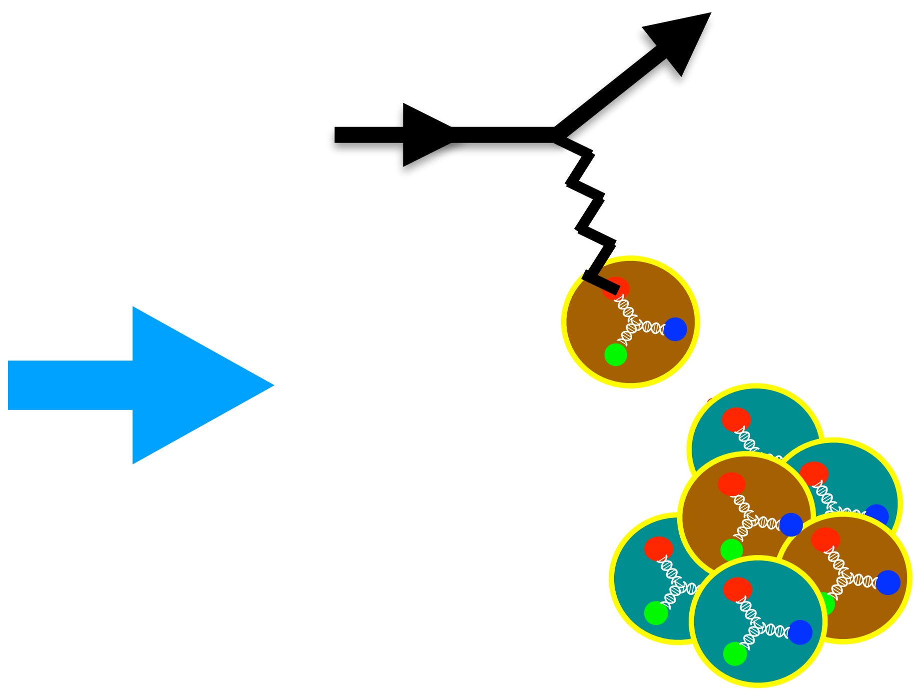
Quark and gluon PDFs

Lattice QCD and neutrino-nucleus

Lattice QCD provides reliable methods for numerically computing properties of QCD including nucleon form factors encoding responses to electroweak currents

Neutrino-nucleon scattering amplitudes can be computed straightforwardly once nucleon electroweak form factors known





Connecting **nucleon** form factors to neutrino-**nucleus** scattering is more complicated

- Lattice QCD can constrain inputs to nuclear EFTs and models
- Constraints from lattice QCD and experiment are often complementary

Quarks and gluons on a lattice

Lattice QCD uses a path integral version of quantum mechanics

— Quark propagators provide explicit solutions to the quark field path integral

— Gluon field path integrals are performed numerically using Monte Carlo:
random field values are drawn from a probability distribution similar to the integrand

Compromises:

Lattice spacing	Finite-volume	Imaginary time
$a \to 0$	$L \to \infty$	$t \to it$

Imaginary time turns complex quantum probability amplitudes

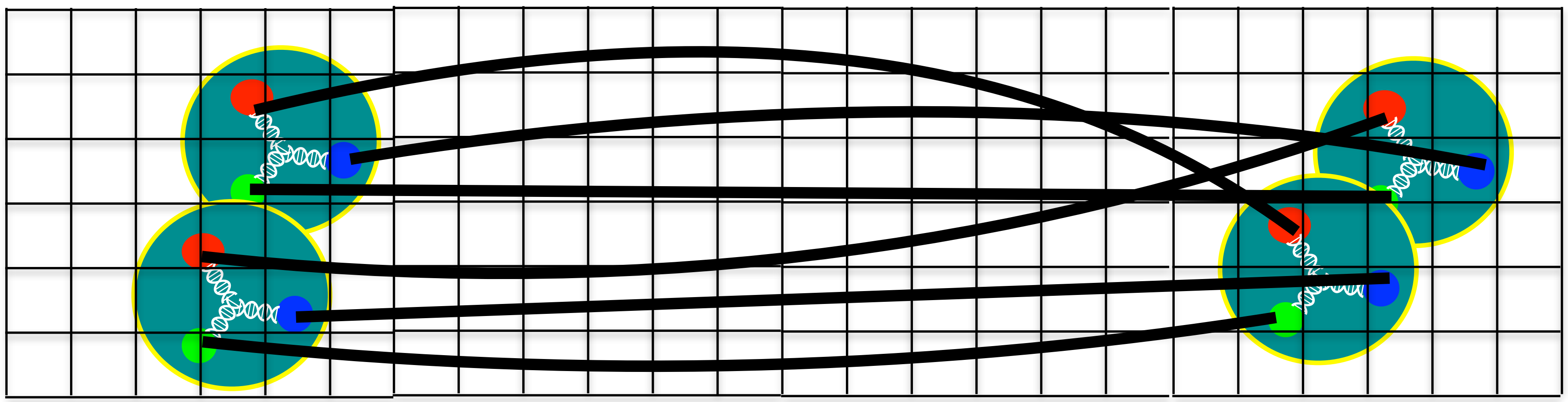
$$\text{probability amplitude} \sim e^{iS}$$

into positive-definite functions that can be interpreted as probabilities for random numbers in a Monte Carlo simulation

$$\text{probability amplitude} \sim e^{-S}$$

Observables in LQCD

LQCD energy spectrum determined from 2-point correlation functions



In imaginary time, correlation functions can be written as sums of exponentials

$$C_A(t) = \langle 0|A(t)A^\dagger(0)|0\rangle = \sum_n \langle 0|A(0)e^{-Ht}|n\rangle \langle n|A^\dagger(0)|0\rangle + \dots$$

$$= \sum_n |Z_n|^2 e^{-E_n t}$$

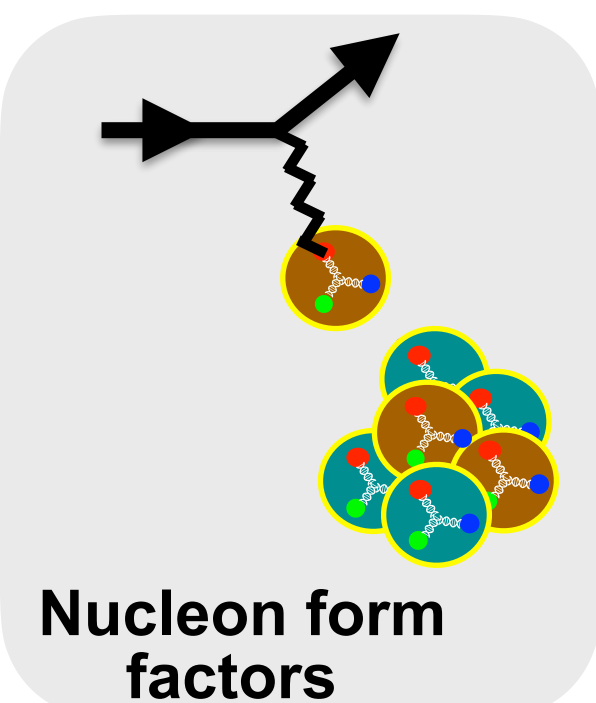
Imaginary time evolution $e^{-iHt_{\mathrm{real}}} = e^{-H(it_{\mathrm{real}})}$

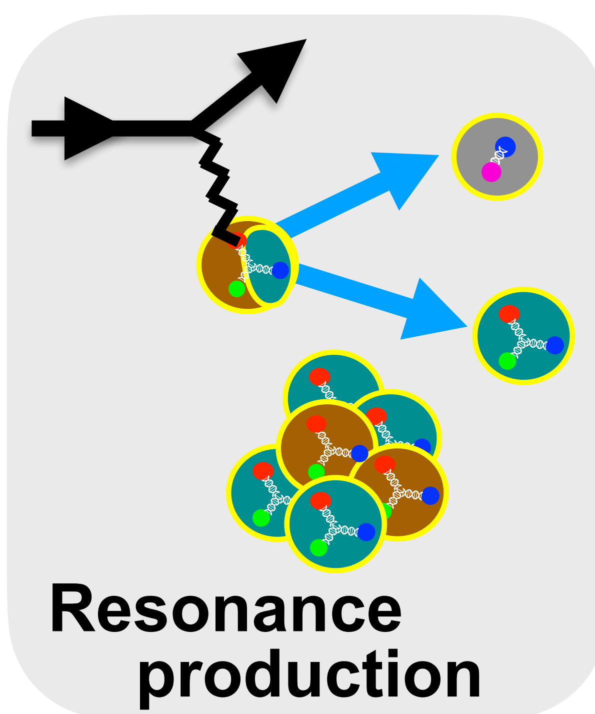
Ground state dominates for large t $\quad C_A(t) \propto e^{-E_0 t} + \dots$

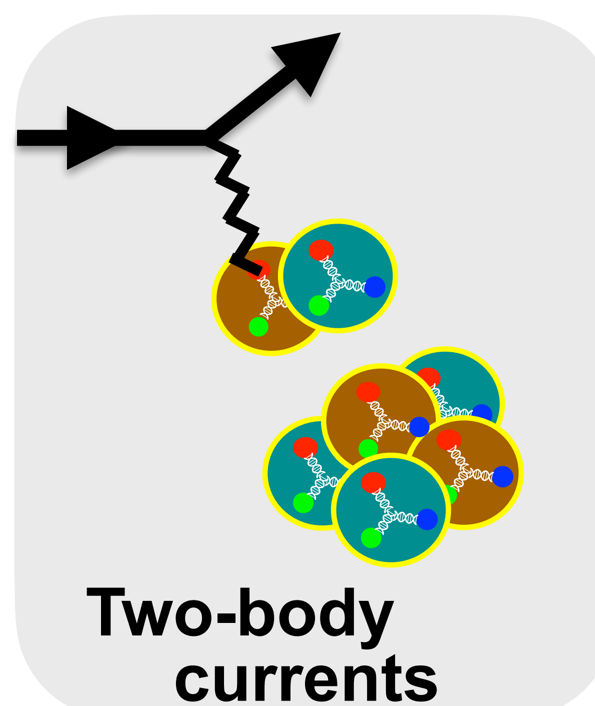
Reaction mechanisms

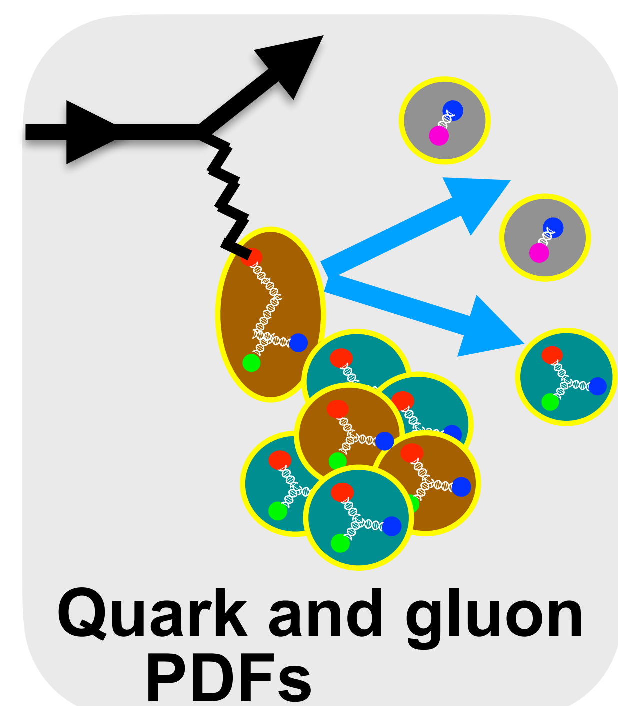
Process	Neutrino Energy Range	Example Final State
Coherent Elastic Scattering	$\lesssim 50$ MeV	$\nu + A$
Inelastic Scattering	$\lesssim 100$ MeV	$e + {}^{\mathrm{A}}(Z+1)^{*}(\rightarrow {}^{\mathrm{A}}(Z+1) + n\gamma)$
Quasi-Elastic Scattering	100 MeV–1 GeV	$l + p + X$
Two-Nucleon Emission	1 GeV	$l + 2N + X$
Resonance Production	1–3 GeV	$l + \Delta(\rightarrow N + \pi) + X$
Shallow Inelastic Scattering	3–5 GeV	$l + n\pi + X$
Deep Inelastic Scattering	$\gtrsim 5$ GeV	$l + n\pi + X$

Alvarez-Ruso, MW et al, arXiv:2203.09030



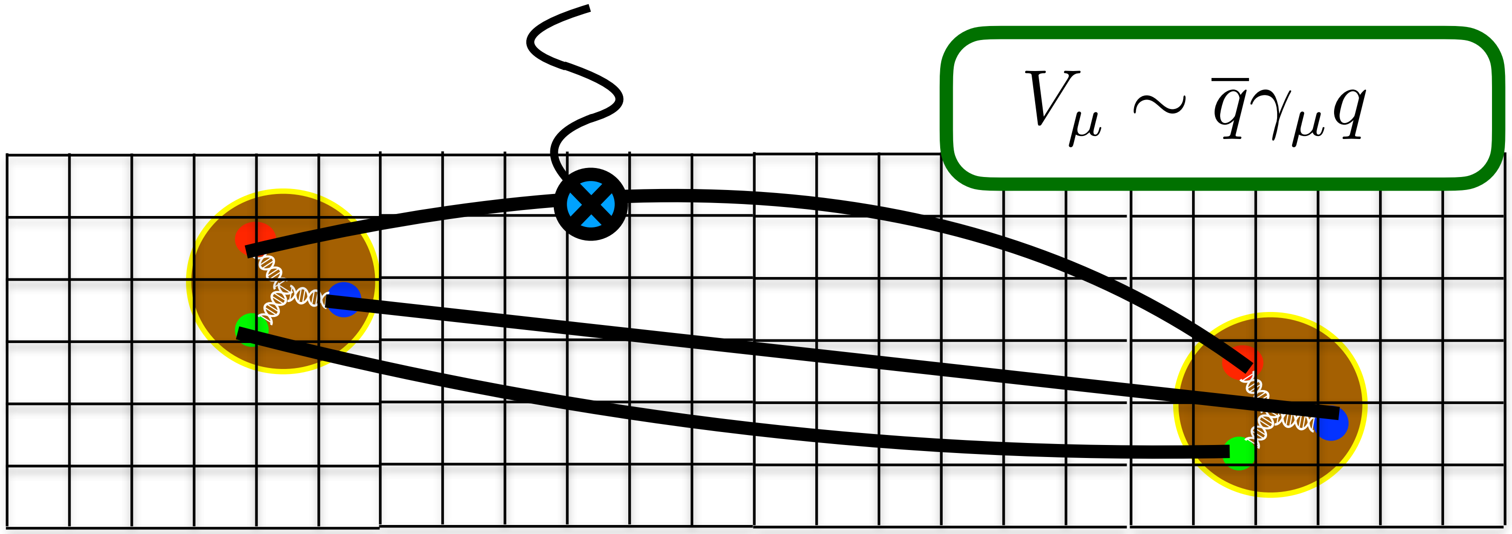






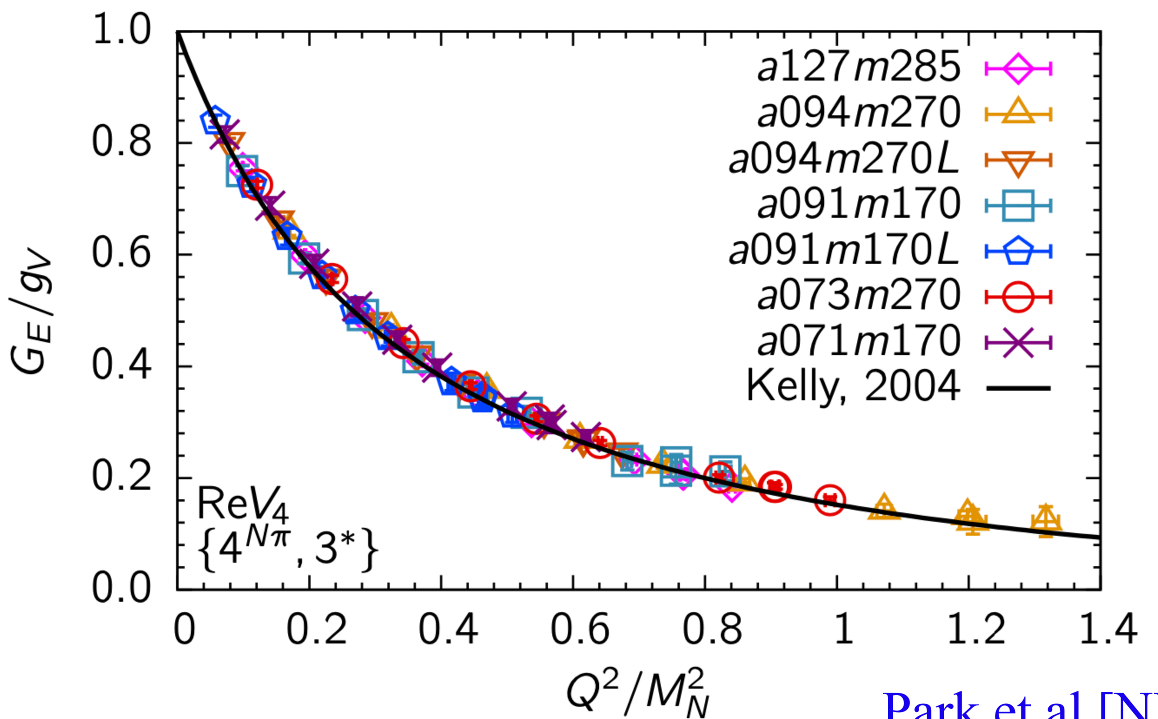
LQCD and nucleon form factors

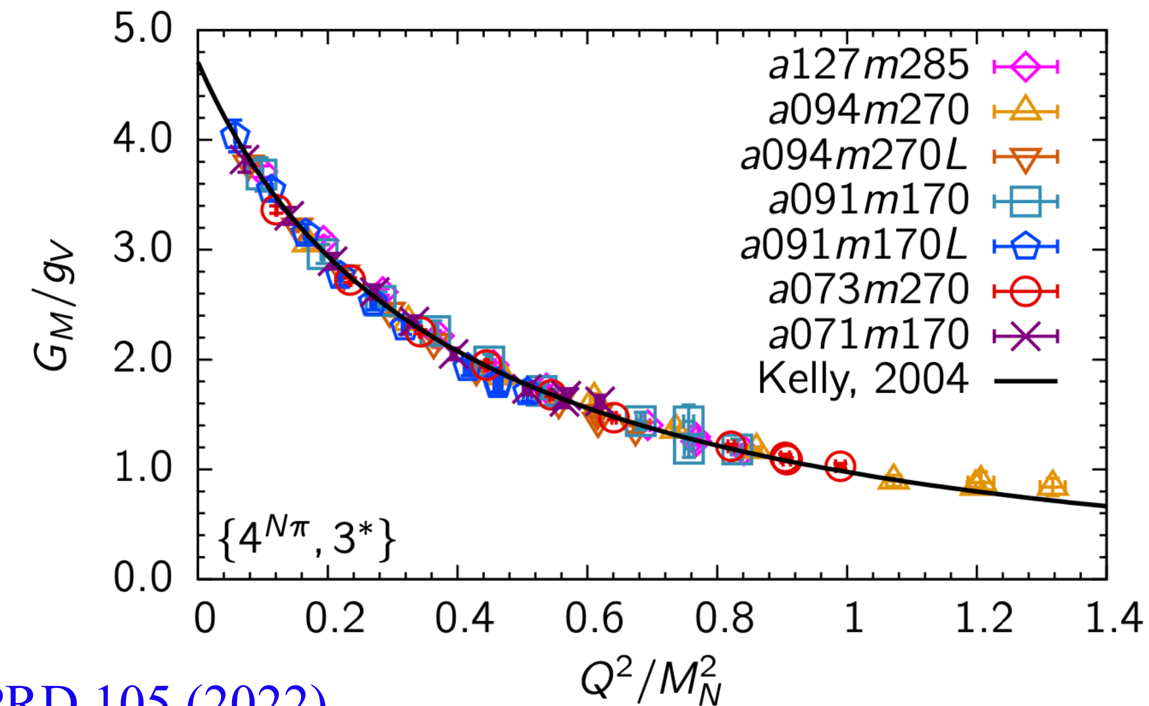
Nucleon electric and magnetic form factors recently calculated using LQCD with approximately physical quark masses



$V_\mu \sim \overline{q}\gamma_\mu q$

$$\langle N(\mathbf{p}+\mathbf{q})|V^\mu|N(\mathbf{p})\rangle = \overline{u}(\mathbf{p}+\mathbf{q})\left[F_1(q^2)\gamma^\mu + i\sigma^{\mu\nu}q_\nu\frac{F_2(q^2)}{2M_N}\right]u(\mathbf{p})$$





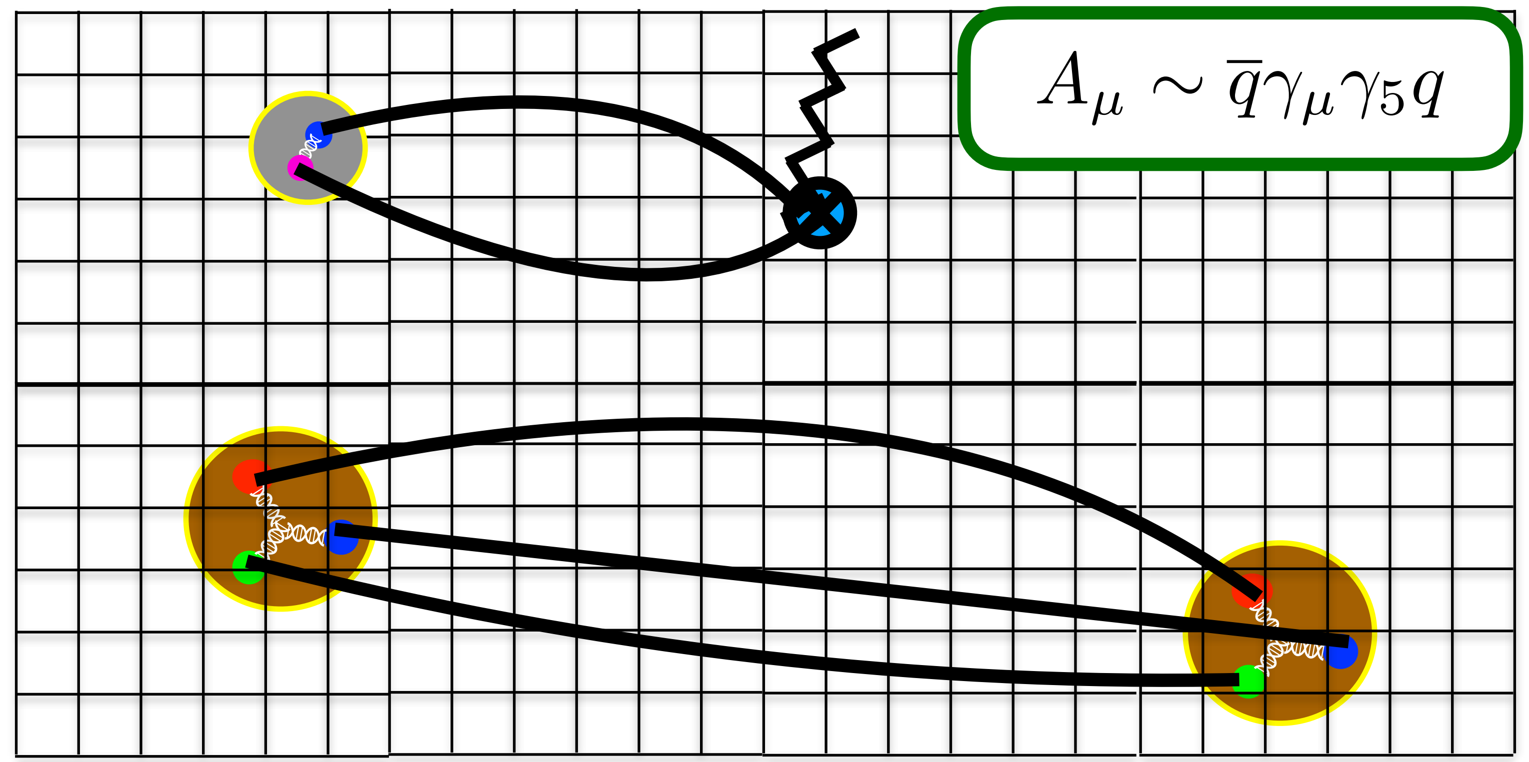
Park et al [NME], PRD 105 (2022)

LQCD results for nucleon electric and magnetic form factors (linear combinations of F_1 and F_2) show good consistency with phenomenological parameterizations

Excited states

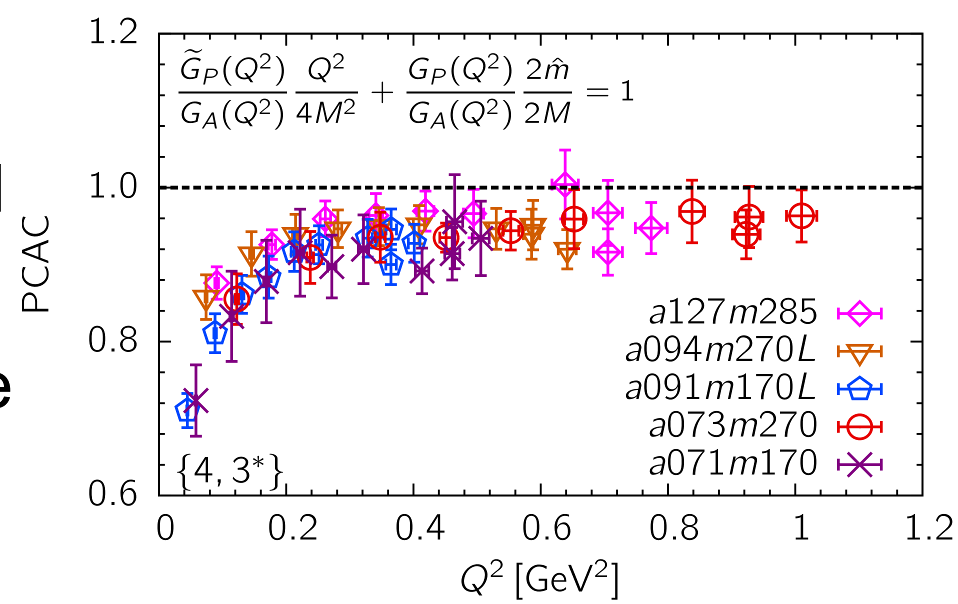
Axial form factor calculations have been performed using analogous methods

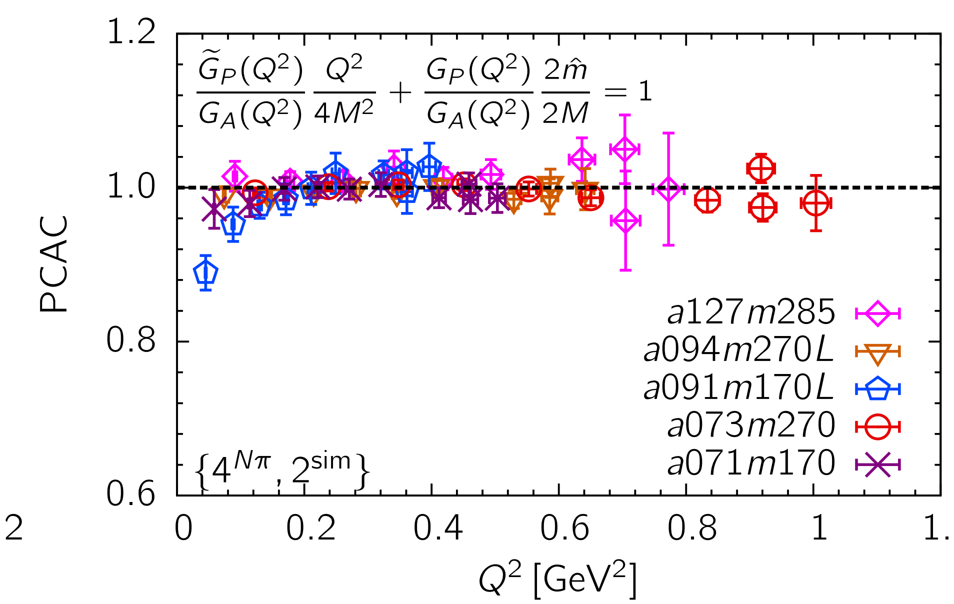
Additional excited-state effects arise from the fact that axial currents can act as pion sources



$$\langle N(\mathbf{p}+\mathbf{q})|A^\mu|N(\mathbf{p})\rangle = \overline{u}(\mathbf{p}+\mathbf{q})\left[G_A(q^2)\gamma^\mu\gamma_5 + q^\mu\gamma_5\frac{\widetilde{G}_P(q^2)}{2M_N}\right]u(\mathbf{p})$$

Careful treatment of $N\pi$ excited states required to reproduce known symmetry constraints assuming ground-state dominance of results





Barca, Bali, and Collins, *PoS* LATTICE 2022 (2022)

Park et al [NME], PRD 105 (2022)

Axial form factors

See talk tomorrow by Rajan Gupta

LQCD calculations of nucleon axial form factors with approximately physical quark masses and continuum extrapolations achieved by multiple groups

Bali et al [RQCD], JHEP 05 126 (2020)

Park et al [NME], PRD 105 (2022)

Djukanovic et al, PRD 103 (2021)

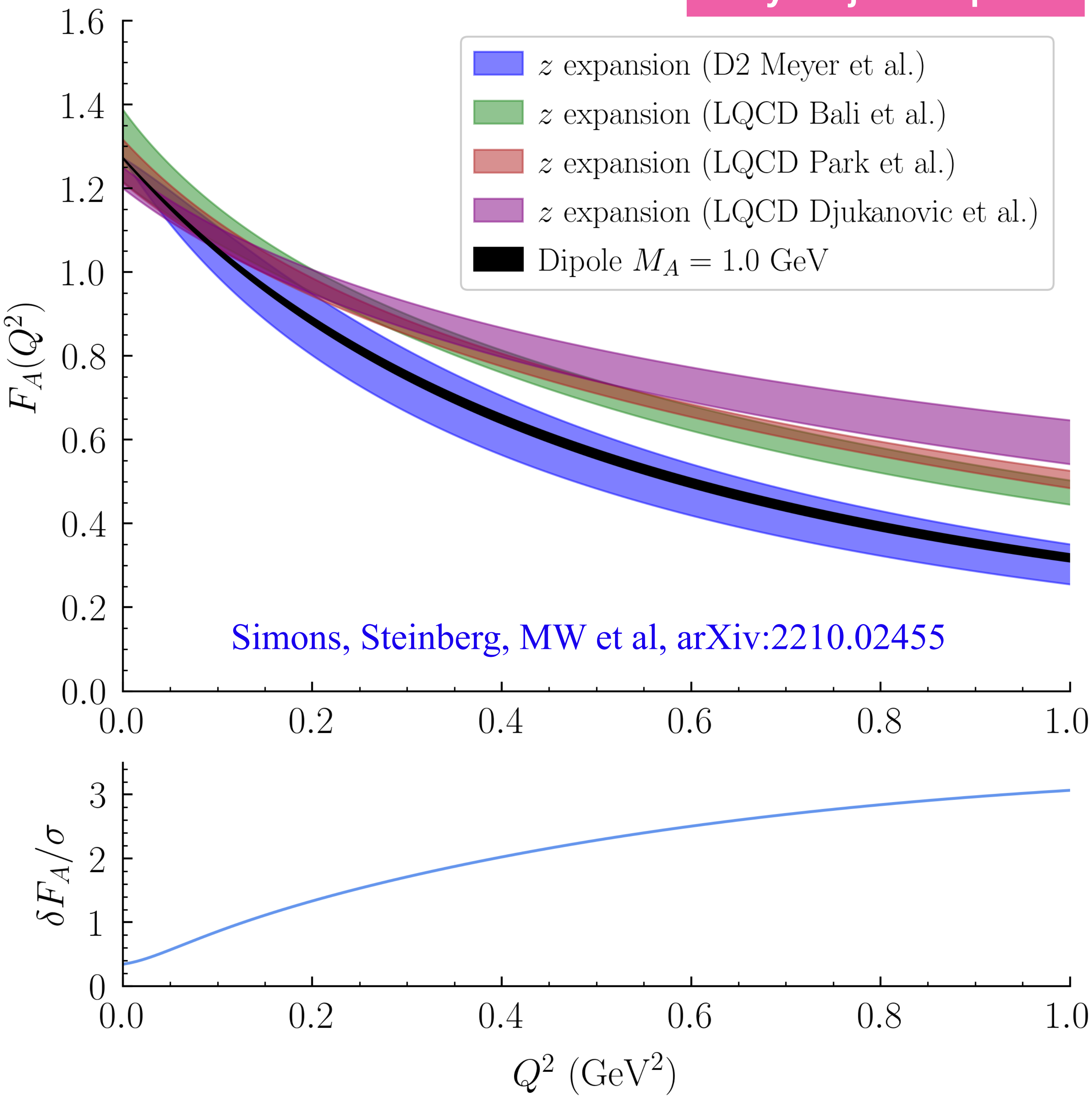
Alexandrou et al [ETMC], PRD 109 (2024)

Jang et al [NME], PRD 109 (2024)

Up to 3 sigma differences between LQCD and experimental axial form factor determinations, could arise from challenging LQCD systematic uncertainties

Differences could also arise from underestimated uncertainties in phenomenological form factor determinations using deuterium bubble chamber data

Meyer, Betancourt, Gran, and Hill, PRD 93 (2016)

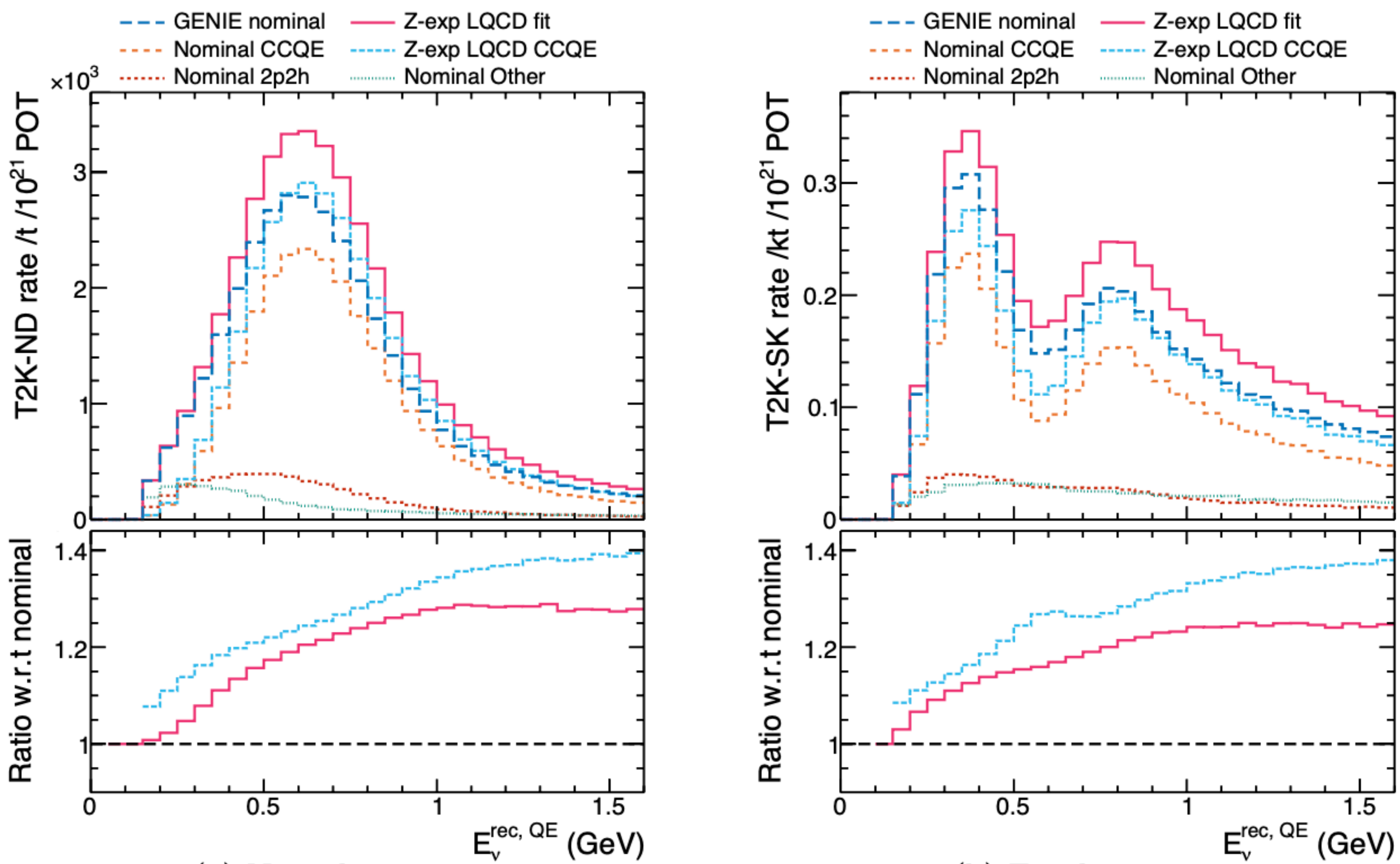


Axial form factor uncertainties

GENIE event generator predictions for T2K event rate using deuterium bubble chamber vs recent LQCD axial form factors differ by ~20%

- Effects on near and far detectors differ, understanding discrepancy essential for reliable neutrino oscillation analyses

Meyer, Walker-Loud, Wilkinson, Ann. Rev. Nucl. Part. Sci. 72 (2022)



(a) Near detector (b) Far detector

What precision do we need from LQCD for neutrino physics?

- **Quantitative precision targets** essential for high-performance computing campaigns
- **Cannot assume an overly restrictive form factor shape**, e.g. dipole; assumptions lead to underestimated precision needs
- **Theoretically consistent nuclear models** without tuning to neutrino data needed to disentangle axial form factor from multi-nucleon effects
- **Comparison between multiple nuclear models** needed to study nuclear uncertainties in the role of axial form factors in νA

Simons, Steinberg, Lovato, Meurice, Rocco, MW, arXiv:2210.02455

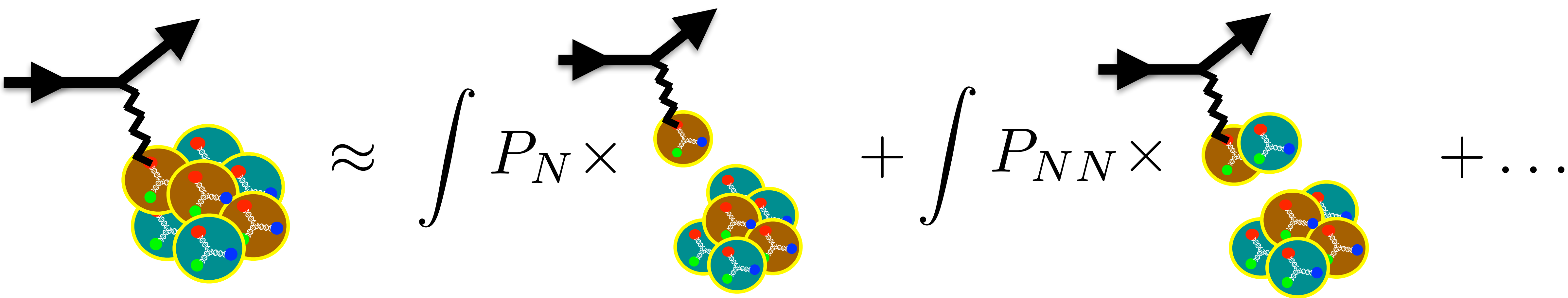
Predicting νA cross sections

Green's function Monte Carlo (GFMC) methods can accurately solve nuclear many-body problem given a Hamiltonian and electroweak current operators

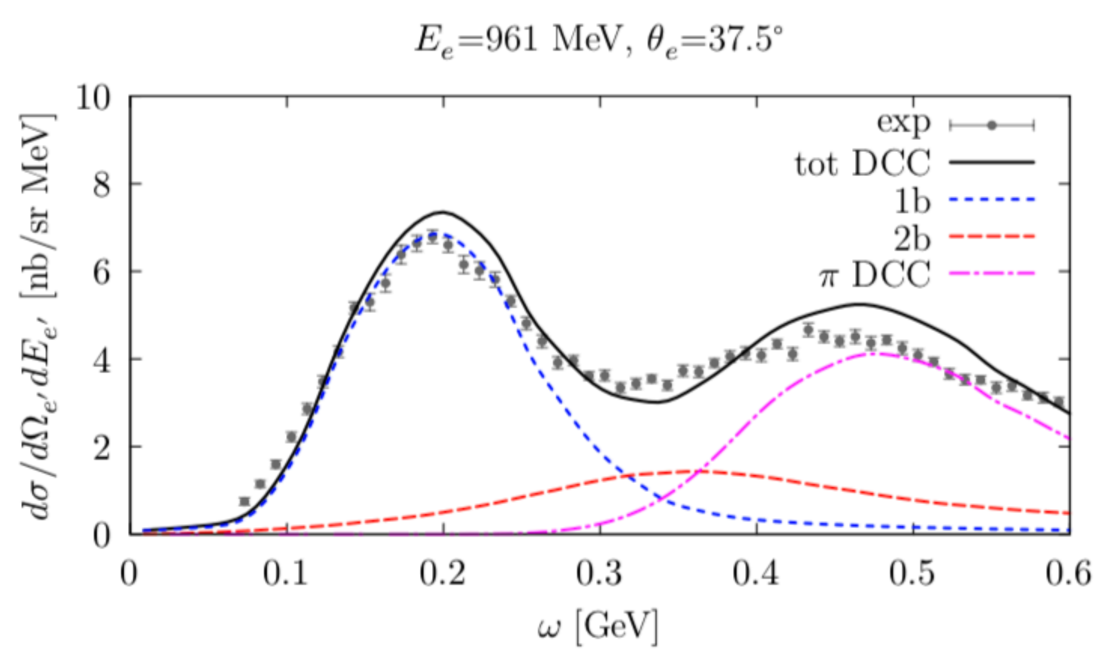
- Cost grows rapidly with nucleon number, computationally limited to $A \lesssim 12$

More computationally tractable: extended factorization scheme using approximate spectral functions — distributions of nucleons (+ *NN* pairs + …) in nucleus

Benhar et al, PRD (2005) Rocco, Lovato, Benhar PRL 116 (2016) Rocco et al, PRC 99 (2019) …



Allows inclusion of 2-body currents and resonance production, computationally feasible for medium-mass nuclei



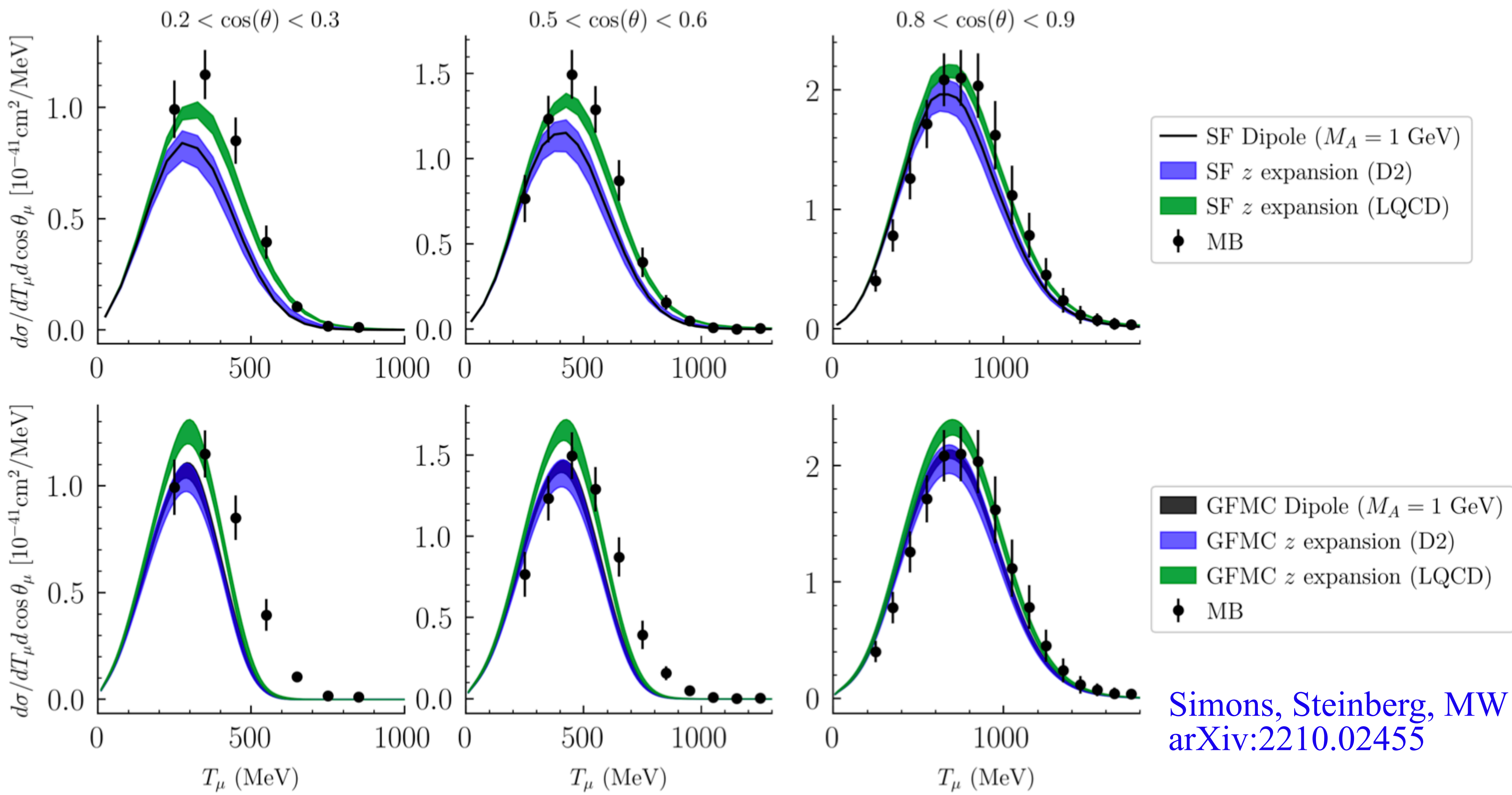
See talks by Jay, Steinberg

Rocco, Nakamura, Lee, Lovato, PRC 100 (2019)

MiniBooNE results

Comparison of GFMC and spectral function (SF) results for $^{12}\mathrm{C}$ with experimental data and one another provides validation of nuclear many-body methods

- 5-20% differences found between GFMC and SF predictions for MiniBooNE, largest at kinematics where relativistic effects neglected in GFMC are most significant
- 10-20% differences found between LQCD and D2 form factors



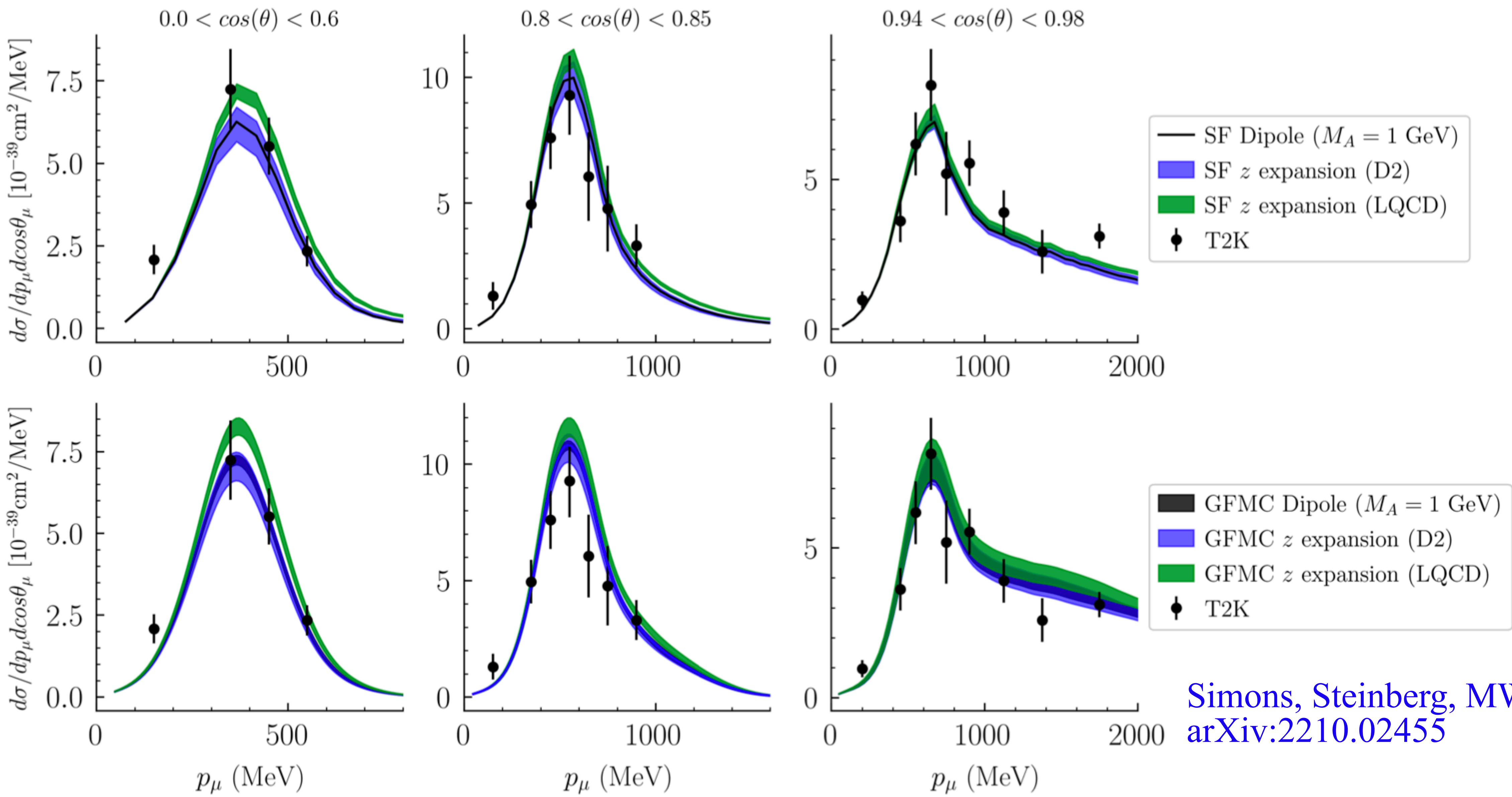
Simons, Steinberg, MW et al, arXiv:2210.02455

T2K results

Form factor and nuclear model differences both somewhat smaller with T2K kinematics (lower flux of high-energy neutrinos than MiniBooNE)

Changes in nuclear model vs changes in axial form factor are not easy to distinguish

- Consistent treatment of nucleon axial form factor and nuclear many-body effects essential when fitting to neutrino scattering data



Simons, Steinberg, MW et al, arXiv:2210.02455

Quantifying form factor uncertainties

z expansion — model independent parameterization of axial (and other) form factors that only assumes basic field theory / QCD properties

Hill, eConf C060409, 027 (2006) Hill and Paz, PRD 82 (2010) Bhattacharya, Hill, and Paz, PRD 84 (2011)

$$F_A(Q^2) = \sum_{k=0}^{\infty} a_k \, z(Q^2)^k$$

Free parameters

Known function

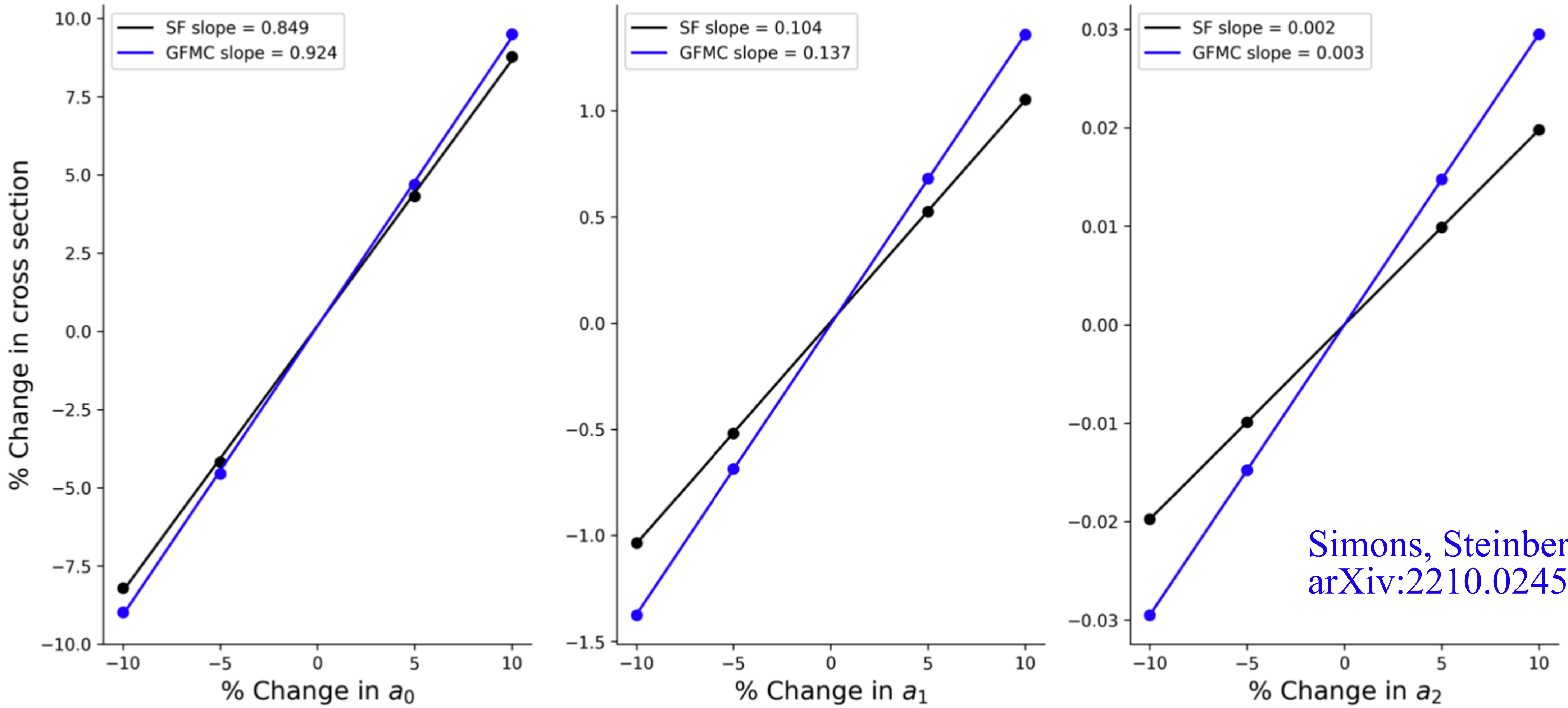
Can be used to quantify relations between nucleon axial form factor uncertainties and neutrino-nucleus cross section uncertainties

$$\delta\sigma = \sum_k \frac{\partial \sigma}{\partial a_k} \delta a_k + \dots$$

Straightforward to determine from calculations with varying a_k

Axial FF uncertainty needs

Uncertainty relations calculated for MiniBooNE cross sections



Simons, Steinberg, MW et al, arXiv:2210.02455

Achieving 1% cross-section precision for MiniBooNE kinematics requires:

- ~ 1% precision in a_0
- ~ 10% precision in a_1
- Relatively little knowledge of $a_2, \ldots$

LQCD target

DUNE will be more sensitive to higher coefficients, further dedicated studies needed

Beyond the axial form factor

Process	Neutrino Energy Range	Example Final State
Coherent Elastic Scattering	$\lesssim 50$ MeV	$\nu + A$
Inelastic Scattering	$\lesssim 100$ MeV	$e + {}^{A}(Z+1)^{*}(\to {}^{A}(Z+1) + n\gamma)$
Quasi-Elastic Scattering	100 MeV–1 GeV	$l + p + X$
Two-Nucleon Emission	1 GeV	$l + 2N + X$
Resonance Production	1–3 GeV	$l + \Delta(\to N + \pi) + X$
Shallow Inelastic Scattering	3–5 GeV	$l + n\pi + X$
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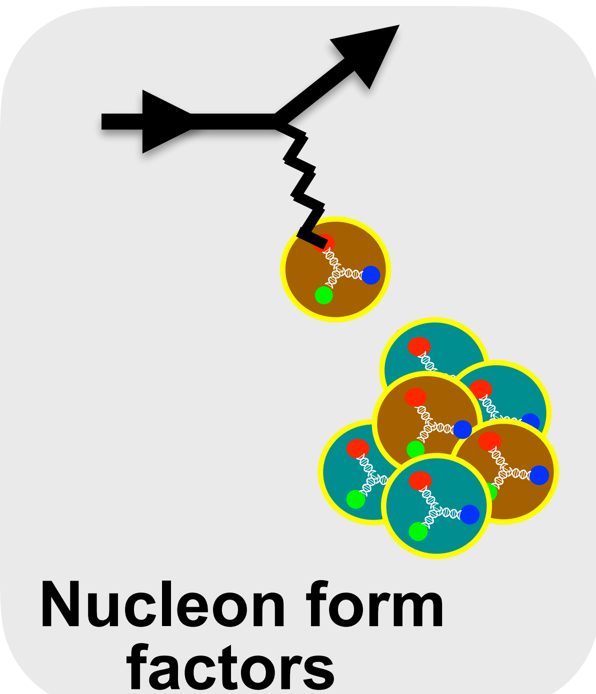
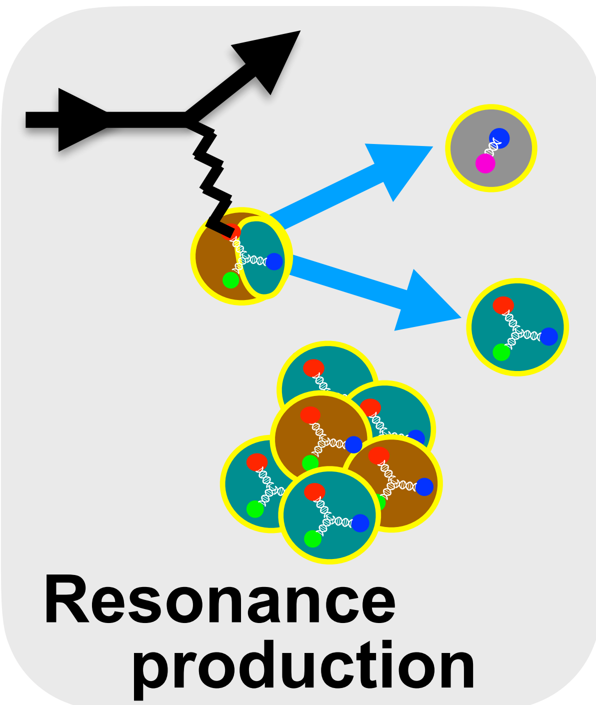
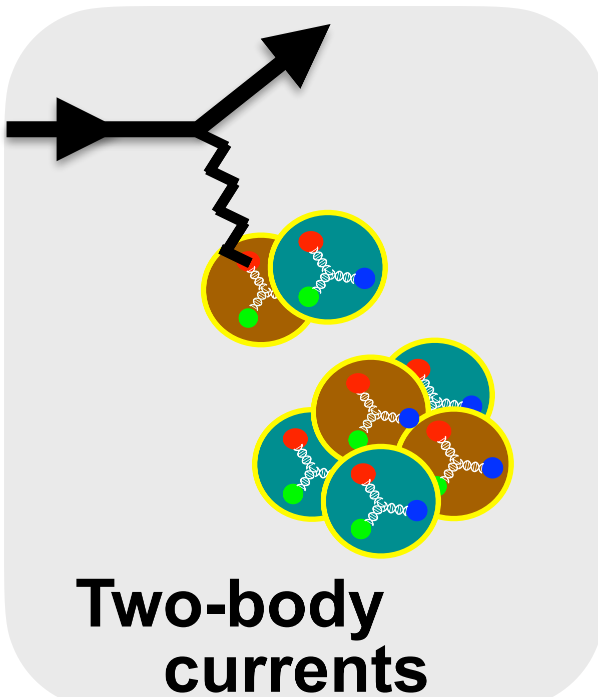
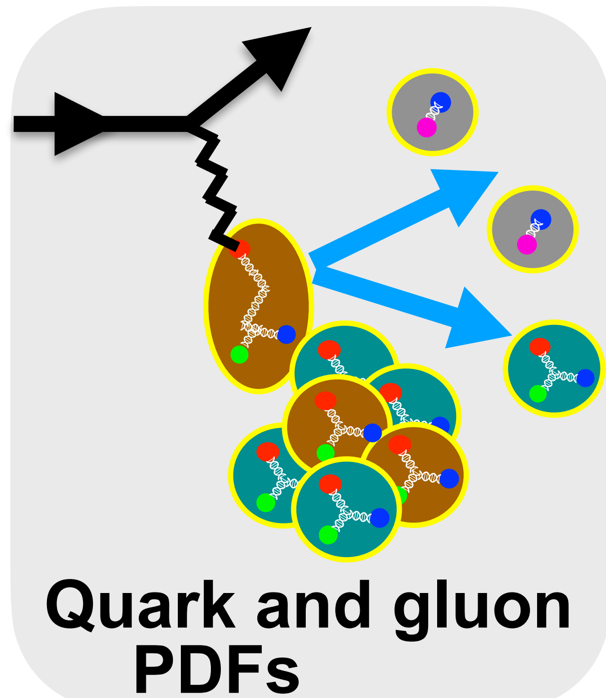
Alvarez-Ruso, MW et al, arXiv:2203.09030





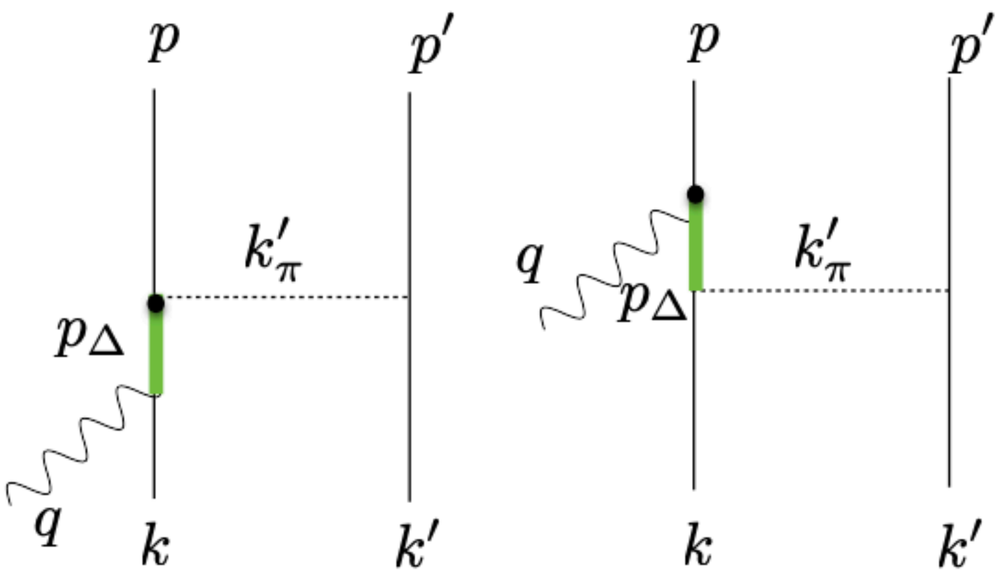




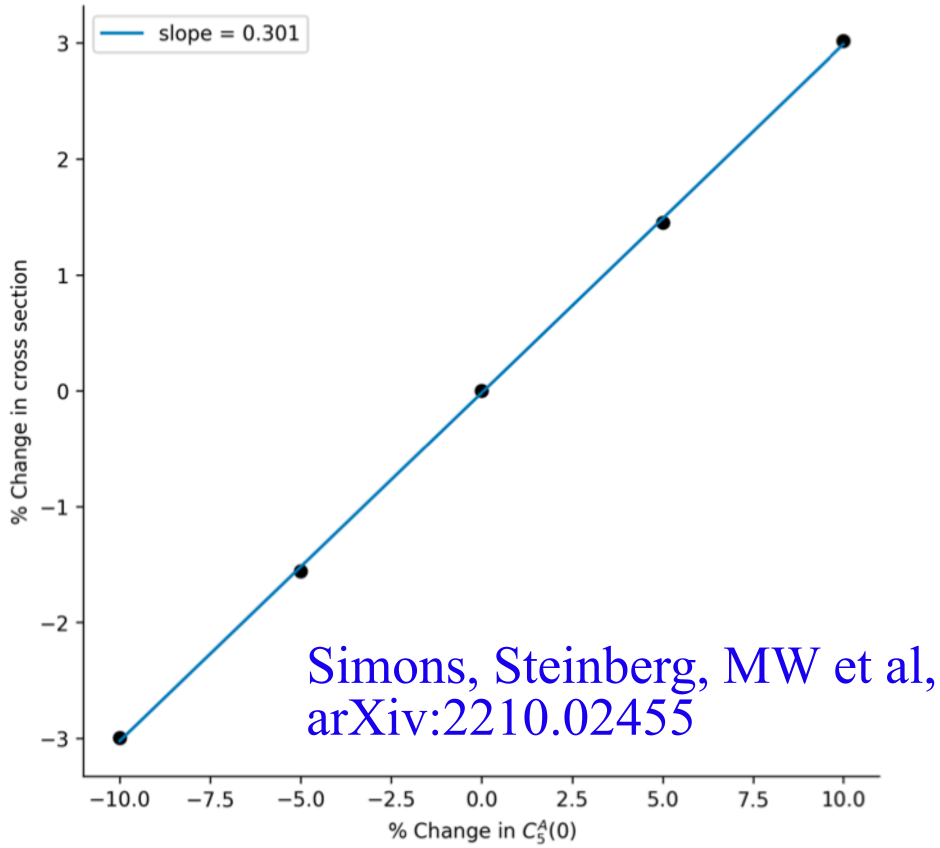
Resonance uncertainty needs

Similar uncertainties quantification can be studied for other cross-section pieces

The largest contributions to two-body currents arise from resonant $N \to \Delta$ transitions in conjunction with pion exchange



The normalization of the dominant $N \to \Delta$ transition form factor must be known to 3% precision to achieve 1% cross-section precision for MiniBooNE kinematics



Simons, Steinberg, MW et al, arXiv:2210.02455

State-of-the-art determinations of this form factor from experimental data on pion electroproduction achieve 10-15% precision (under some assumptions)

Hernandez et al, PRD 81 (2010)

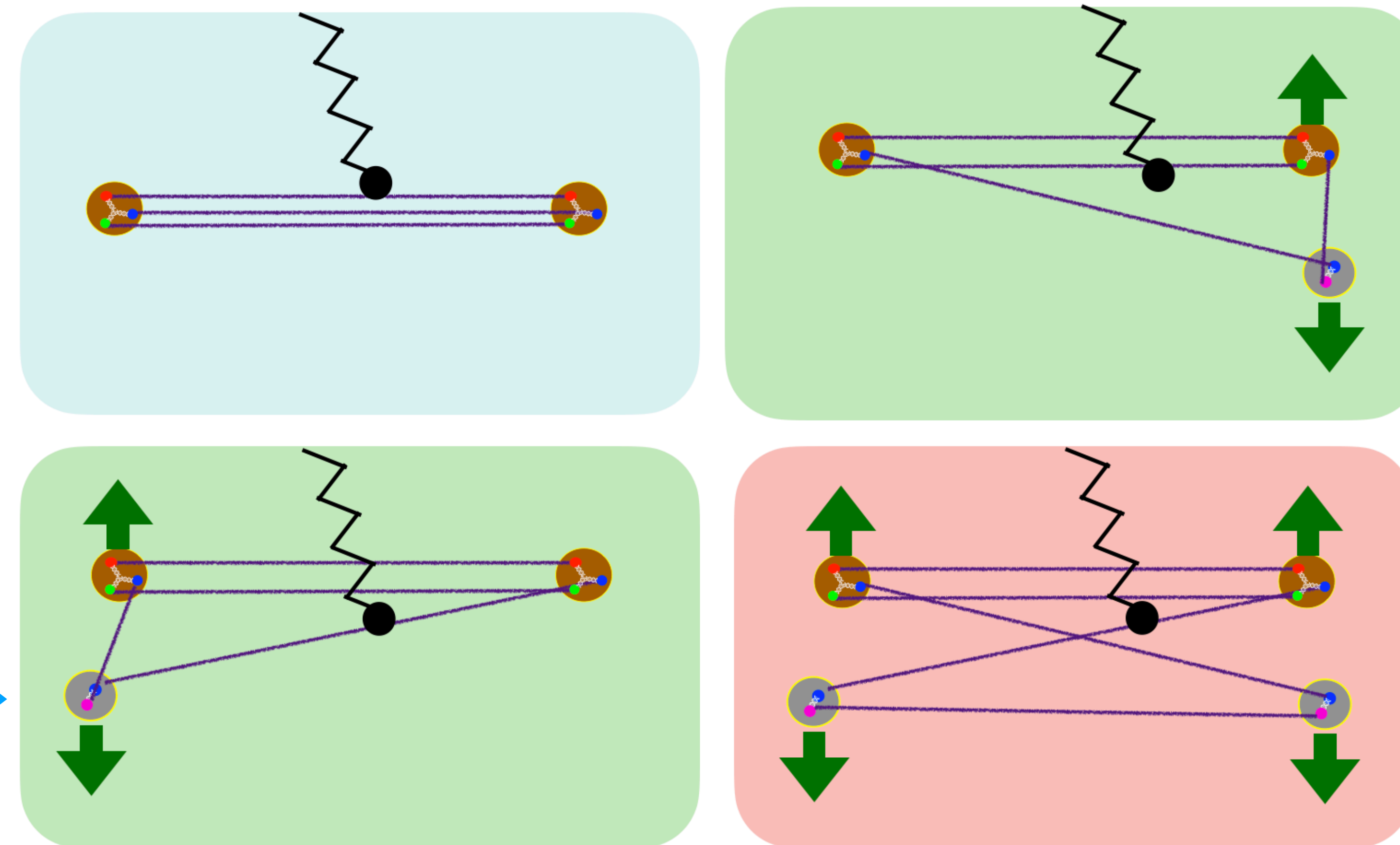
Further constraints on $N \to \Delta$ transitions and two-body currents will be necessary to achieve few-percent cross-section precision

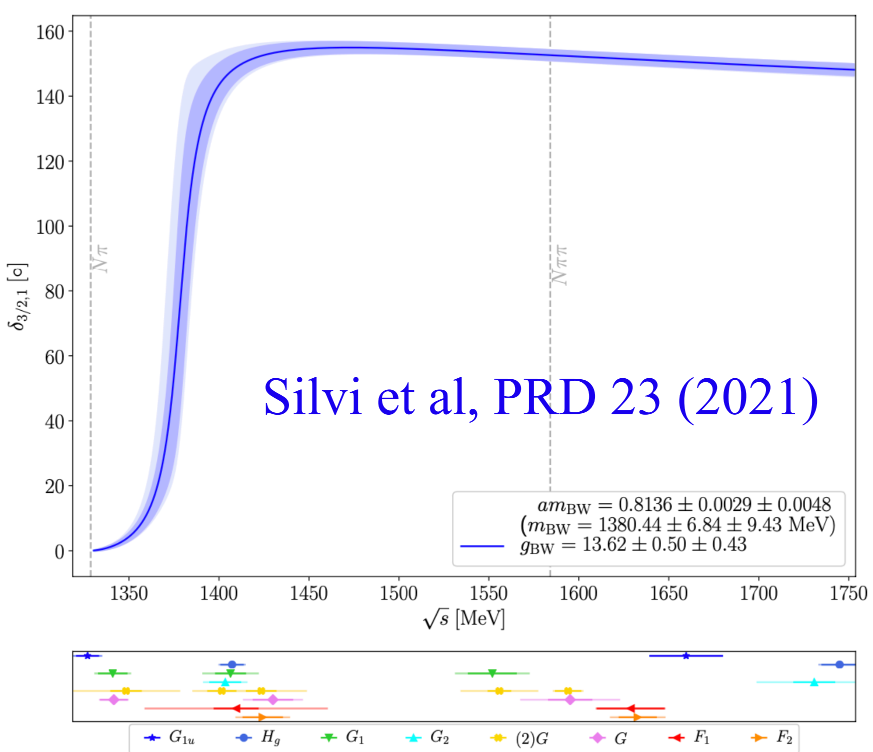
$N\pi$ systems in LQCD

The Δ is not a stable particle in QCD

Need to disentangle resonant and non-resonant form factors from transition matrix involving all QCD energy eigenstates

Barca, Bali, and Collins, PRD 107 (2023)





Silvi et al, PRD 23 (2021)

Variational methods can be used to compute analogous correlation function matrices describing $N\pi$ scattering

Andersen, Bulava, Hörz, Morningstar, PRD 97 (2018)

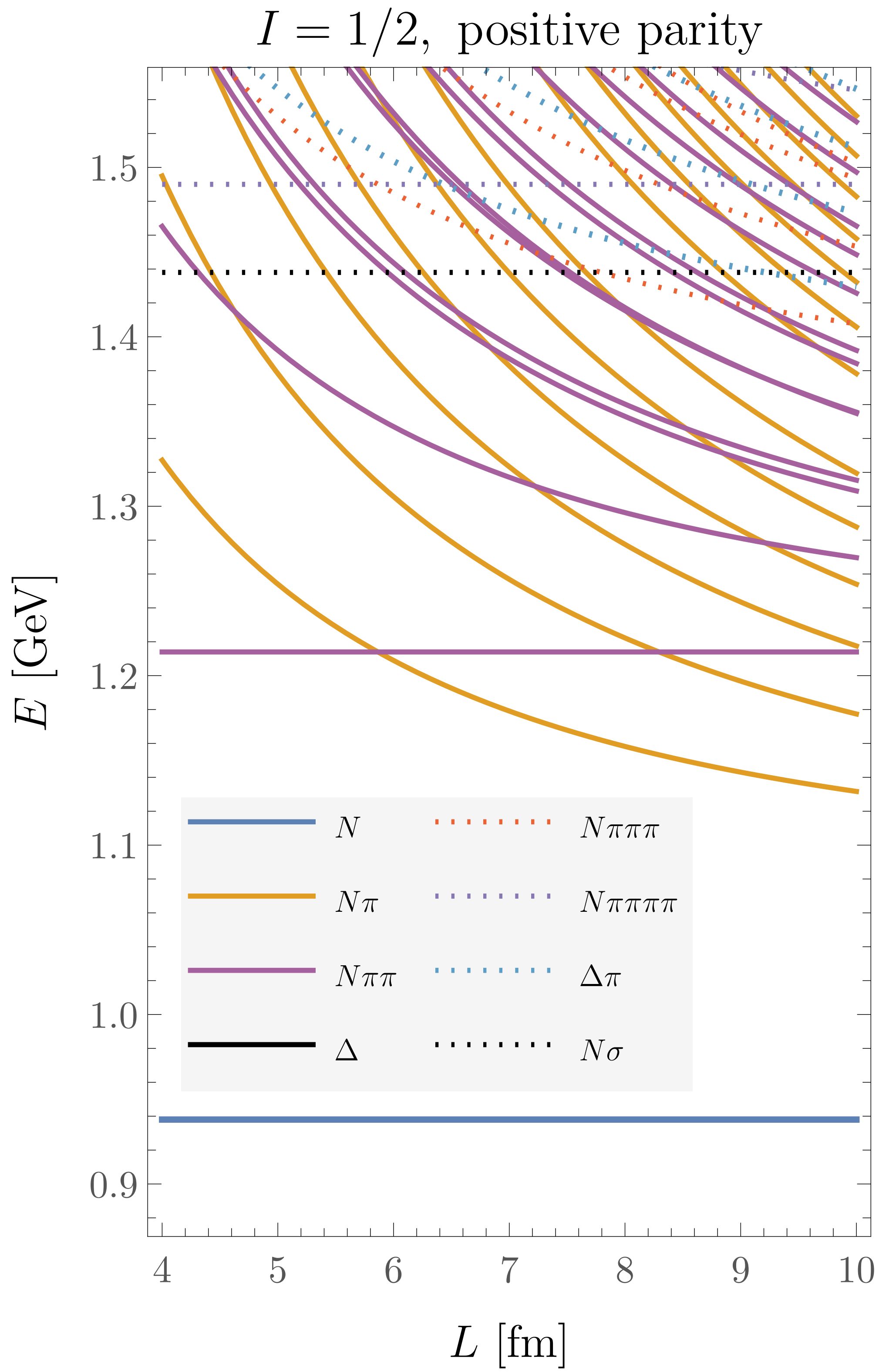
Silvi et al, PRD 23 (2021)

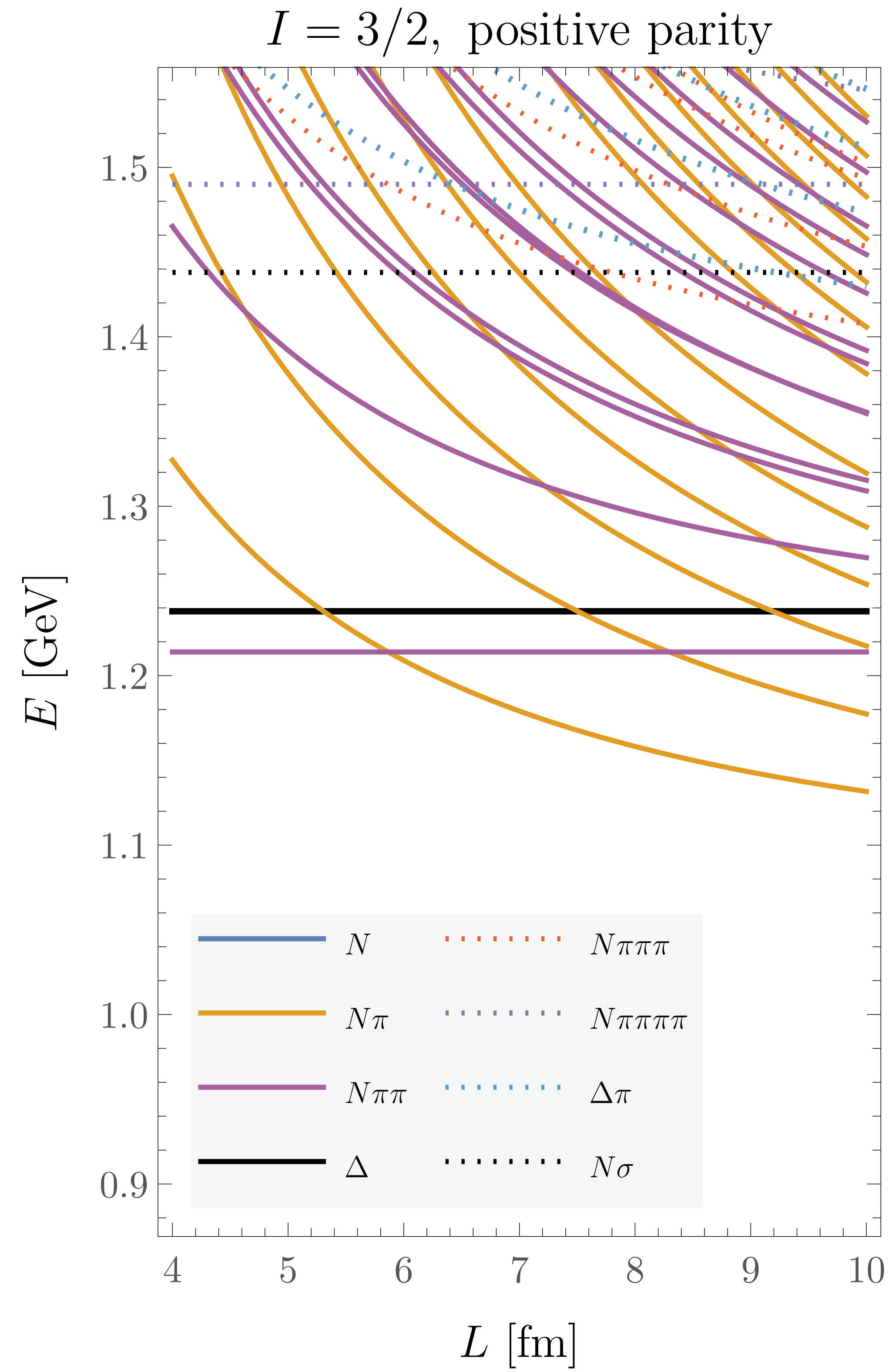
Bulava et al, Nucl. Phys. B 987 (2023)

Alexandrou et al, PRD 109 (2024)

Δ , $N\pi$, and $N\pi\pi$

Three-particle states close to the Δ threshold, must be explicitly included

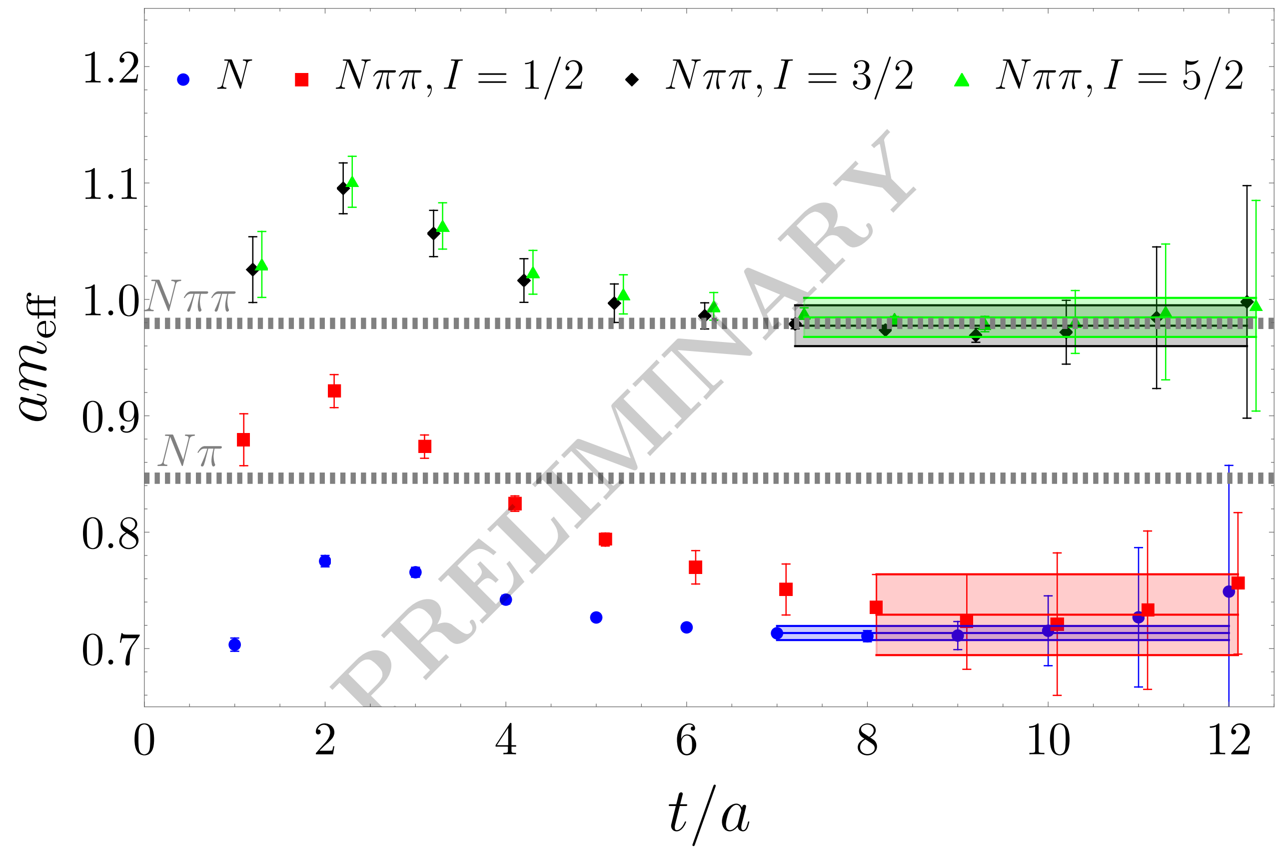




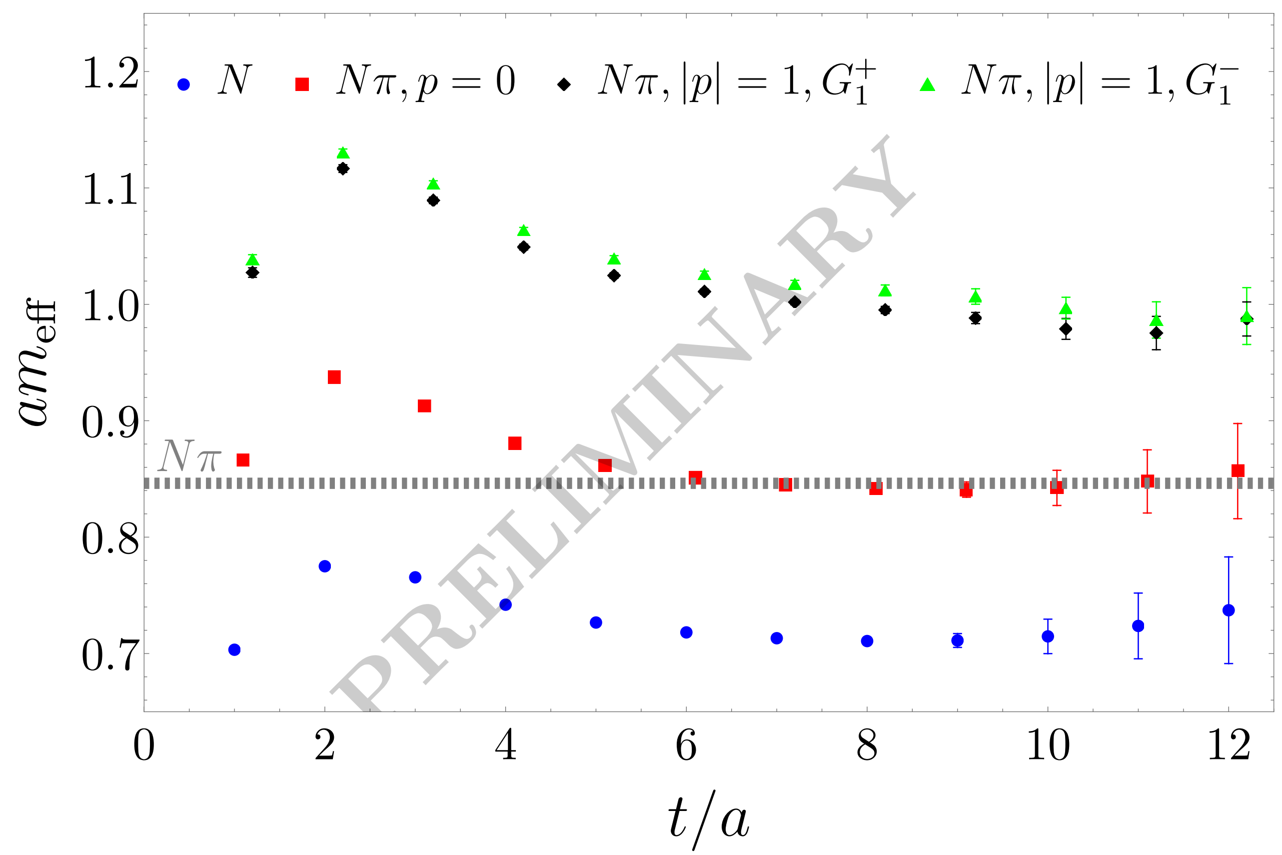
$N\pi\pi$ systems in LQCD

Proof-of-principle calculations of $N\pi\pi$ systems demonstrated

Grebe and MW, PoS LATTICE 2023



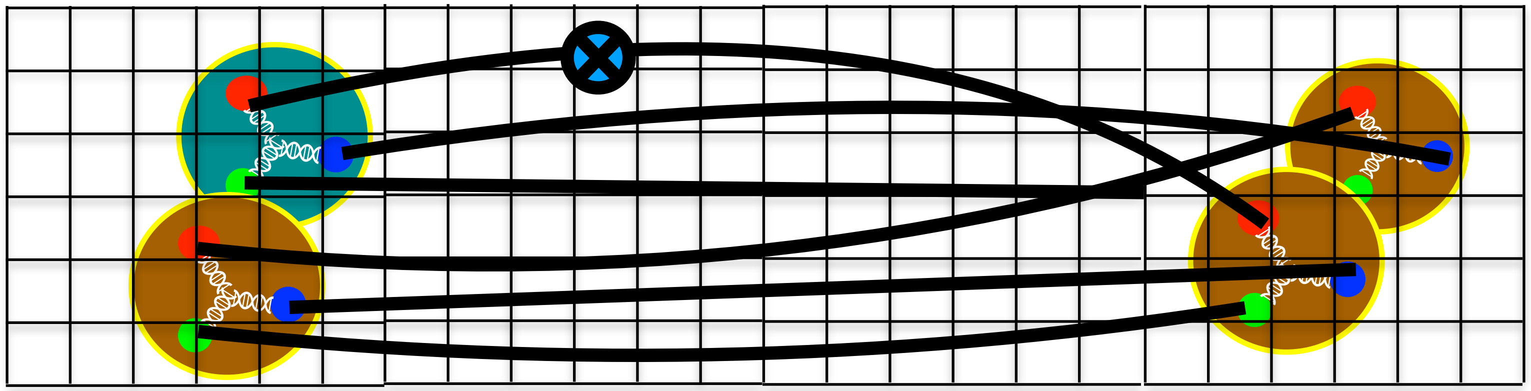
- Overlap between single-nucleon and $N\pi\pi$ states appears larger than for $N\pi$



Extensions to compute full matrices of resonant and non-resonant correlation functions in progress

Two-body currents in LQCD

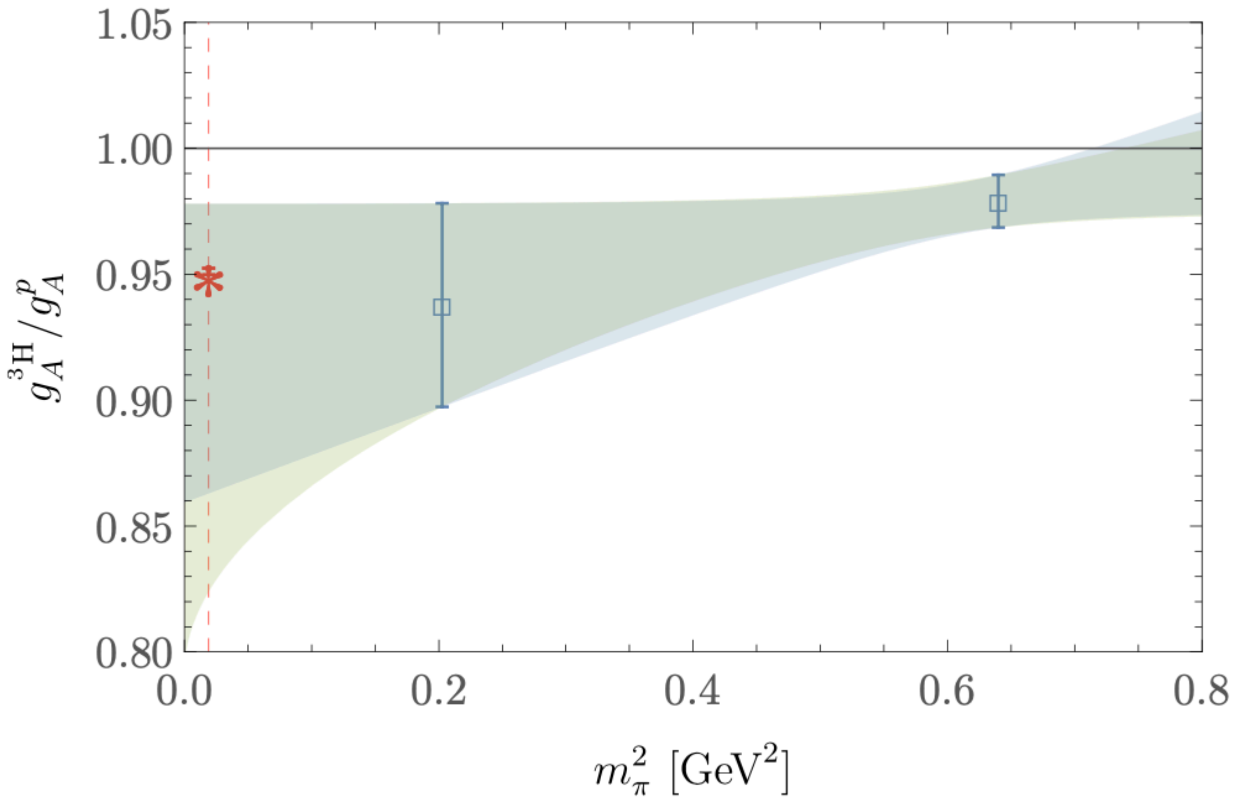
Two-nucleon axial matrix elements relevant for proton-proton fusion computed, used to constrain two-body currents



Savage, MW et al [NPLQCD], PRL 119 (2017)

Flavor decomposition of axial matrix elements of two and three nucleon systems computed with $m_\pi = 806\ \mathrm{MeV}$

Chang, MW et al [NPLQCD], PRL 120 (2018)



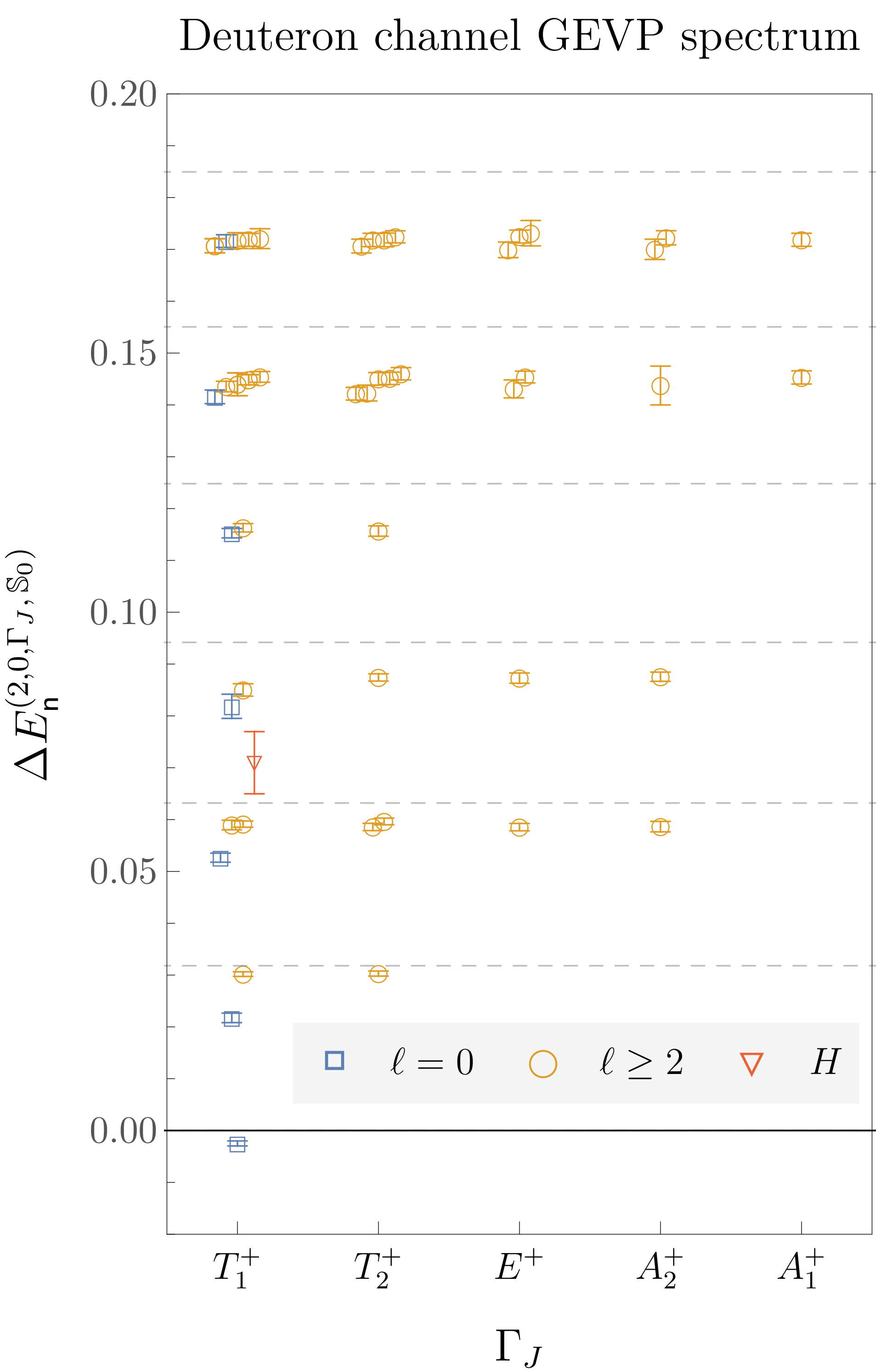
Axial current matrix element calculations with $m_\pi = 450\ \mathrm{MeV}$ permit preliminary extrapolations to physical quark masses

Several systematic uncertainties remain, but encouraging agreement with experiment seen

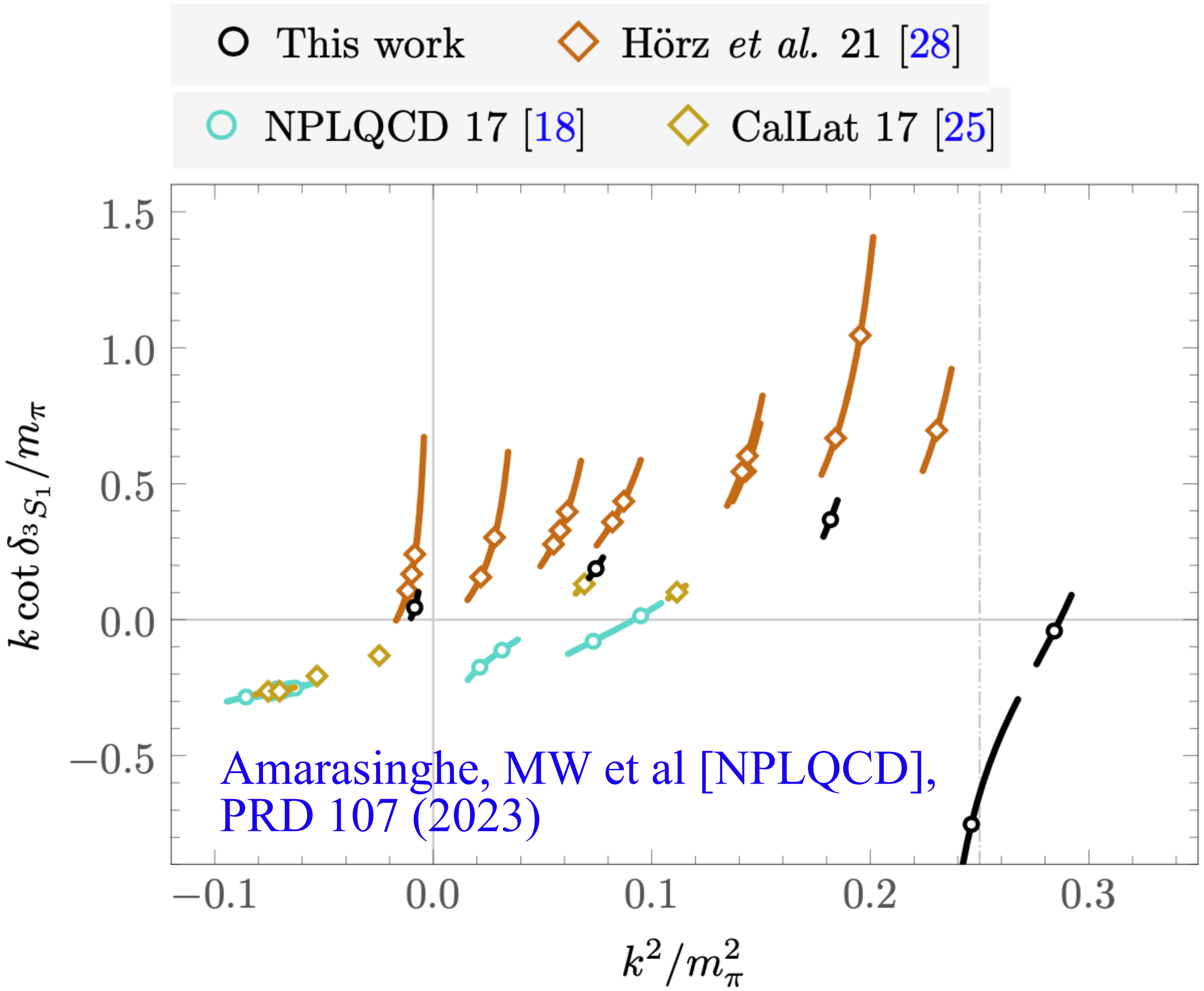
Parreño, MW et al [NPLQCD] PRD 103 (2021)

Towards *NN* scattering from LQCD

Small energy gaps to unbound scattering states make excited-state effects challenging for two-nucleon systems. Progress using variational calculations:



Lüscher quantization condition



- Consistency among studies with similar interpolating operators
- Significant discrepancies between calculations with different operators

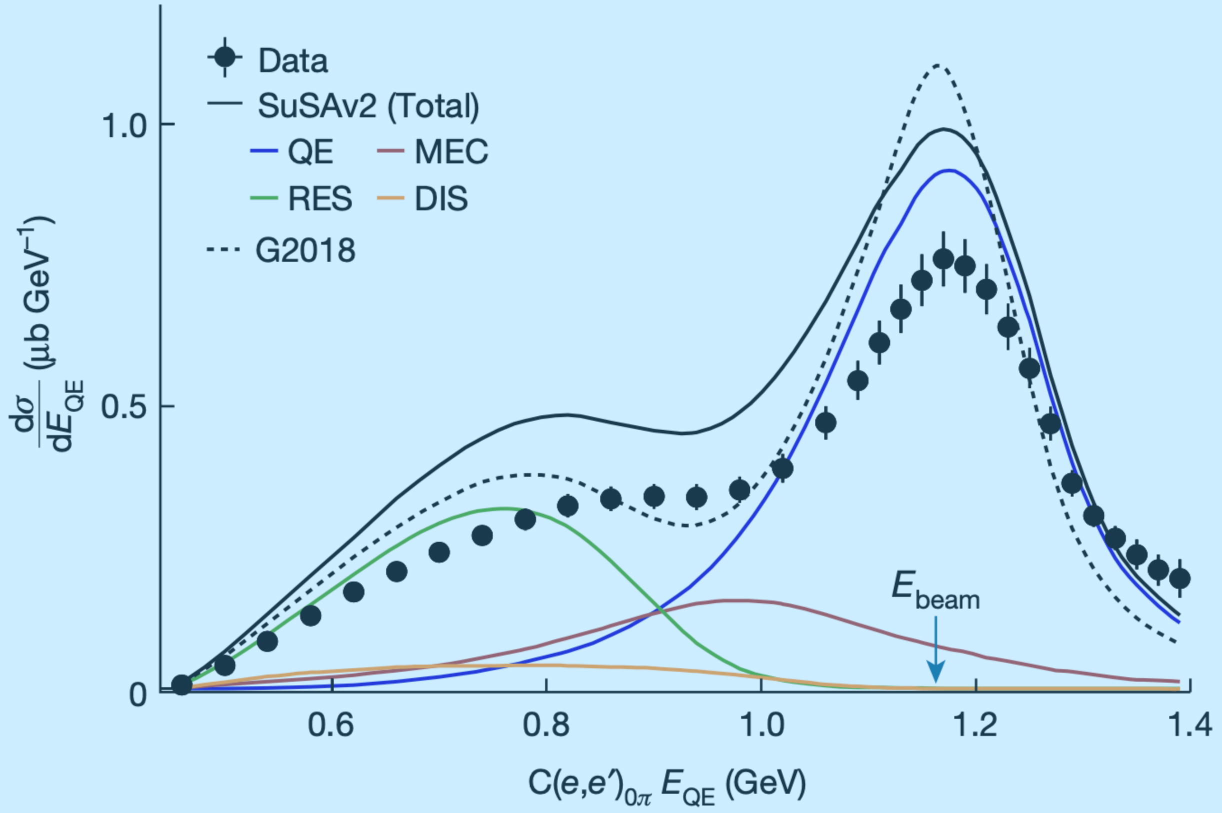
Further study needed!

Data-driven vs Theory-driven

Data

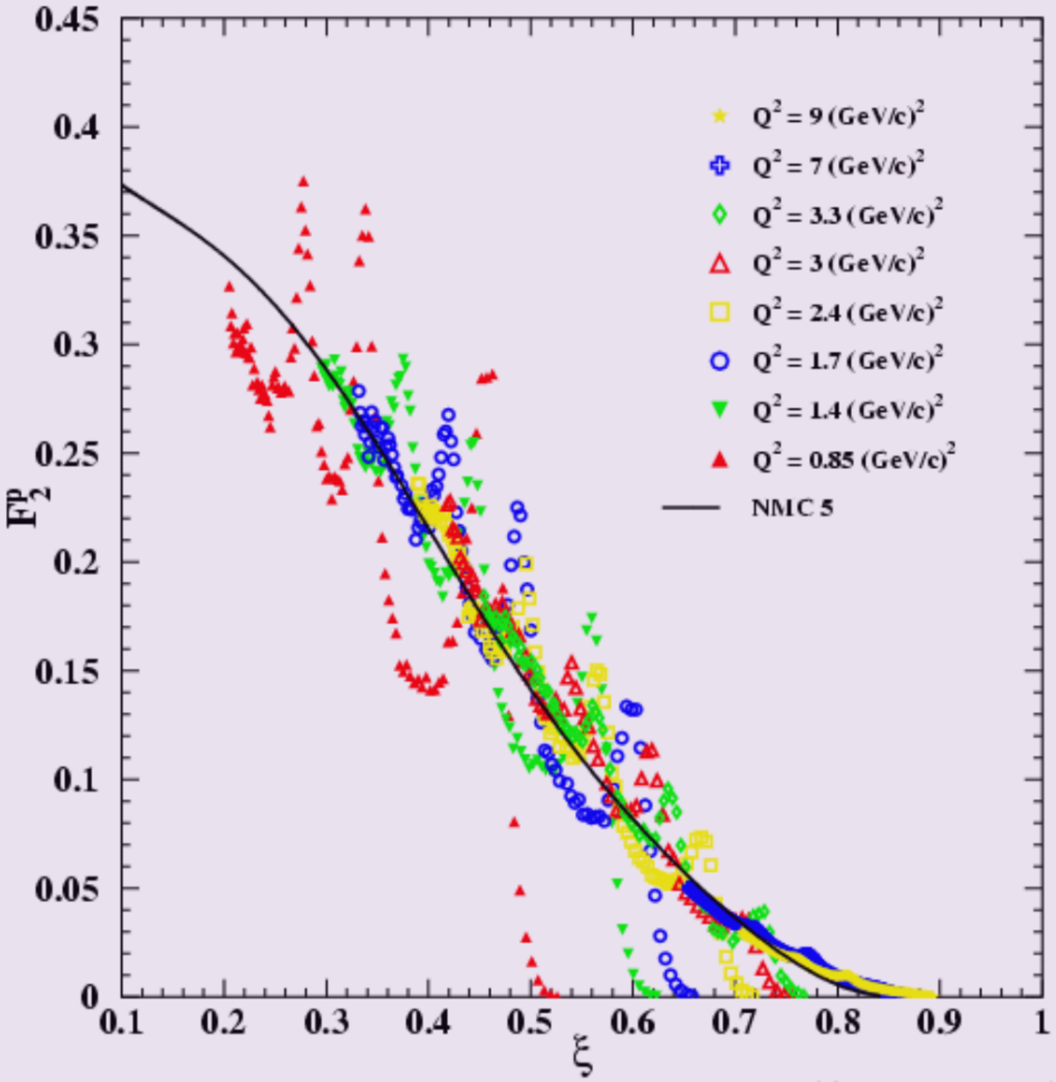
Tuning for one flux doesn't mean other kinematics will be modeled correctly

Khachatryan et al [Clas and e4v] Nature 599 (2021)



Theory

QCD is complicated at scales ~ 1 GeV



Alvarez-Ruso, MW et al, arXiv:2203.09030

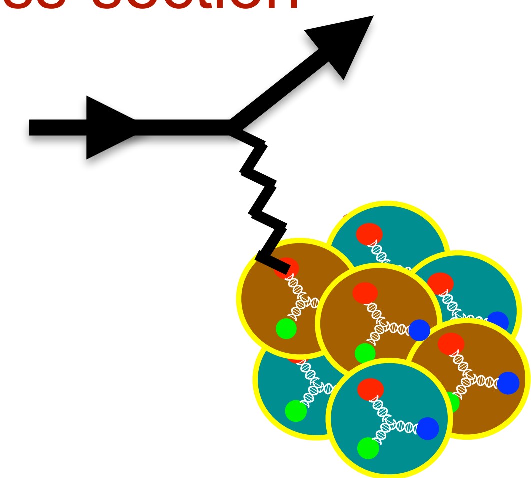
Tuning can obscure new physics

New physics and νA cross-section both affect shape of far detector flux

Near-detector neutrino flux and acceptance

$$\frac{N_{\text{near}}}{N_{\text{far}}} = \frac{\int dE_\nu \Phi_{\text{near}}(E_\nu)\sigma(E_\nu)}{\int dE_\nu \Phi_{\text{far}}(E_\nu)\sigma(E_\nu)}$$

Cross-section



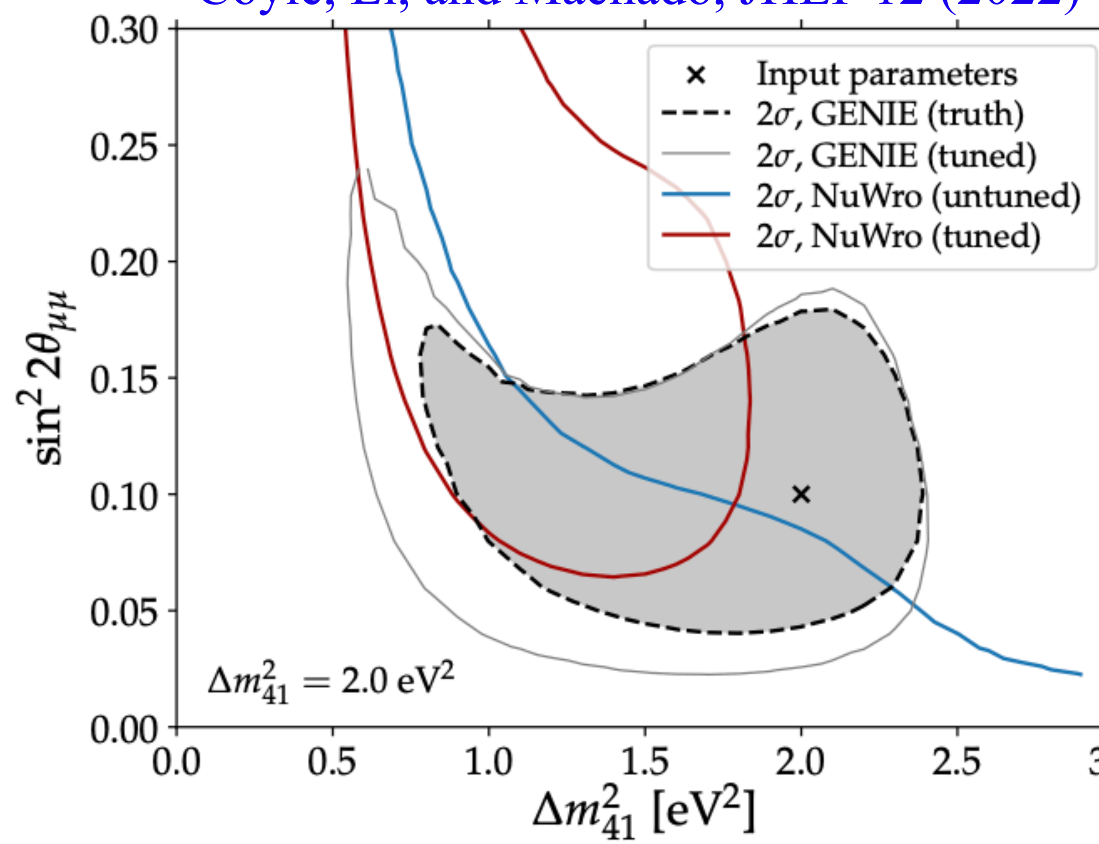
Experimentally measured event rates

Far-detector flux (depends on oscillation parameters)

Inaccuracies in cross-section modeling can distort signatures of new physics such as oscillations into sterile neutrinos

Near-detector tuning is not always sufficient for correctly extracting new physics signals (due to assumptions in cross-section models)

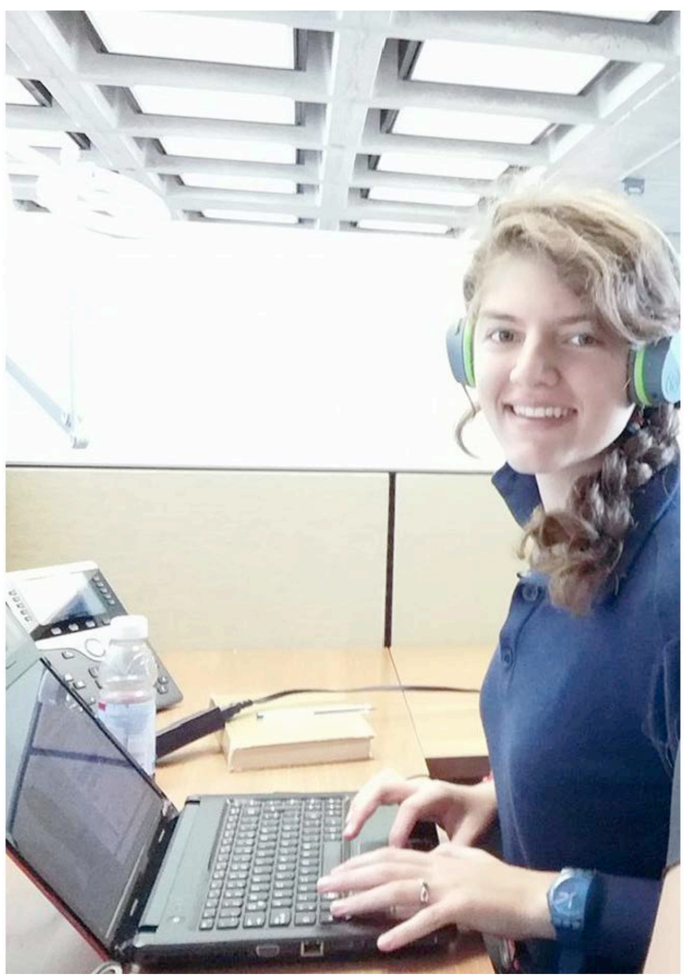
Coyle, Li, and Machado, JHEP 12 (2022)

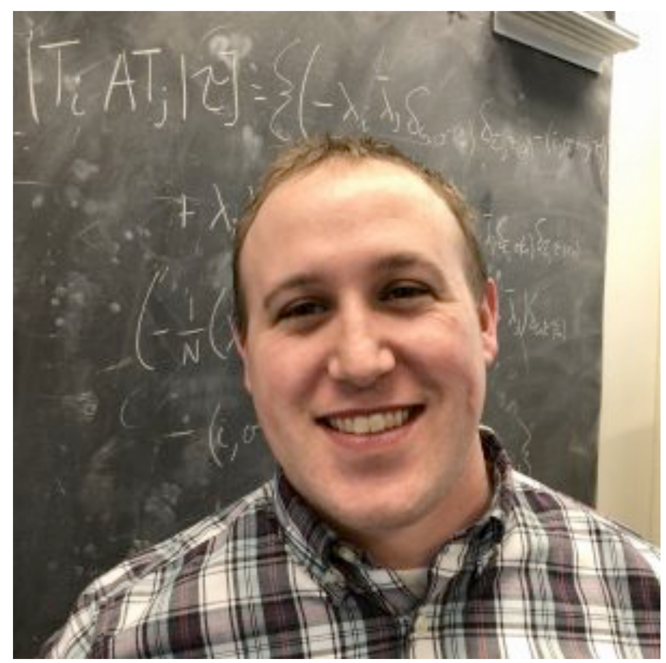


Theorist-approved tuning?

Work in progress

with Karla Tame Narveaz, Josh Isaacson, and Shirley Li





Theorist-approved tuning?

- Cross section can be decomposed into lepton / hadron tensors

$$\frac{d^2\sigma^{(\nu A)}}{dE' d\cos\theta} = L^{\mu\nu}(E_\nu, x, Q^2) W_{\mu\nu}(x, Q^2)$$

Theorist-approved tuning?

- Cross section can be decomposed into lepton / hadron tensors

$$\frac{d^2\sigma^{(\nu A)}}{dE' d\cos\theta} = L^{\mu\nu}(E_\nu, x, Q^2) W_{\mu\nu}(x, Q^2)$$

- Structure function decomposition of hadron tensor assumes nothing beyond symmetries of Standard Model

$$\frac{d^2\sigma^{(\nu A)}}{dE' d\cos\theta} = \frac{|V_{ud}|^2 G_F^2 E' \sqrt{1 - \frac{m_{\ell'}^2}{E'^2}}}{\pi\,(1 + q^2/M_W^2)^2} \Bigg[\left(y + \frac{m_{\ell'}^2}{2M_A E x} \right) W_1^{(\nu A)}(x, q^2) + \frac{E}{M_A}\left(1 - y - \frac{M_A x y}{2E} - \frac{m_{\ell'}^2}{4E^2}\right) W_2^{(\nu A)}(x, q^2) + \left(1 - \frac{y}{2} - \frac{m_{\ell'}^2}{4M_A E x}\right) W_3^{(\nu A)}(x, q^2) + \left(\frac{m_{\ell'}^2}{2M_A E x} + \frac{m_{\ell'}^4}{4M_A^2 E^2 x^2 y}\right) W_4^{(\nu A)}(x, q^2) - \left(\frac{m_{\ell'}^2}{M_A E x y}\right) W_5^{(\nu A)}(x, q^2) \Bigg],$$

- Spin 0 nucleus like argon described by 5 structure functions

- (muon mass non-negligible!)

Albright and Jarlskog, Nucl. Phys. B 84 (1975)

Machine learning structure functions

$$\frac{d^2\sigma^{(\ell A)}(E_\nu, E', \cos\theta)}{dE' d\cos\theta}$$

3D probability distribution

- Parameterize via normalizing flows
- Loss function: KL divergence between data and model distributions

Machine learning structure functions

$$\frac{d^2\sigma^{(\ell A)}(E_\nu, E', \cos\theta)}{dE'\,d\cos\theta}$$

3D probability distribution

- Parameterize via normalizing flows
- Loss function: KL divergence between data and model distributions
- Wrong dimensionality!!

Near detector data constrains 2D marginal of cross-section distribution

$$Q_{ND}(E', \cos\theta) = \frac{1}{\mathcal{N}_{ND}} \int dE_\nu \Phi_{ND}(E_\nu) \frac{d^2\sigma^{(\ell A)}(E_\nu, E', \cos\theta)}{dE'\,d\cos\theta}$$

Machine learning structure functions

Near detector data constrains 2D marginal of cross-section distribution

$$Q_{ND}(E', \cos\theta) = \frac{1}{\mathcal{N}_{ND}} \int dE_\nu \Phi_{ND}(E_\nu) \frac{d^2\sigma^{(\ell A)}(E_\nu, E', \cos\theta)}{dE' d\cos\theta}$$

$$\frac{d^2\sigma^{(\ell A)}(E_\nu, E', \cos\theta)}{dE' d\cos\theta}$$

3D probability distribution

- Parameterize via normalizing flows
- Loss function: KL divergence between data and model distributions
- Wrong dimensionality!!

$$W_1^{(\nu A)}(x, q^2)$$

$$\frac{d^2\sigma^{(\ell A)}(E_\nu, E', \cos\theta; \lambda)}{dE' d\cos\theta} = \sum_i k_i(E_\nu, x, Q^2) W_i(x, Q^2; \lambda_i)$$

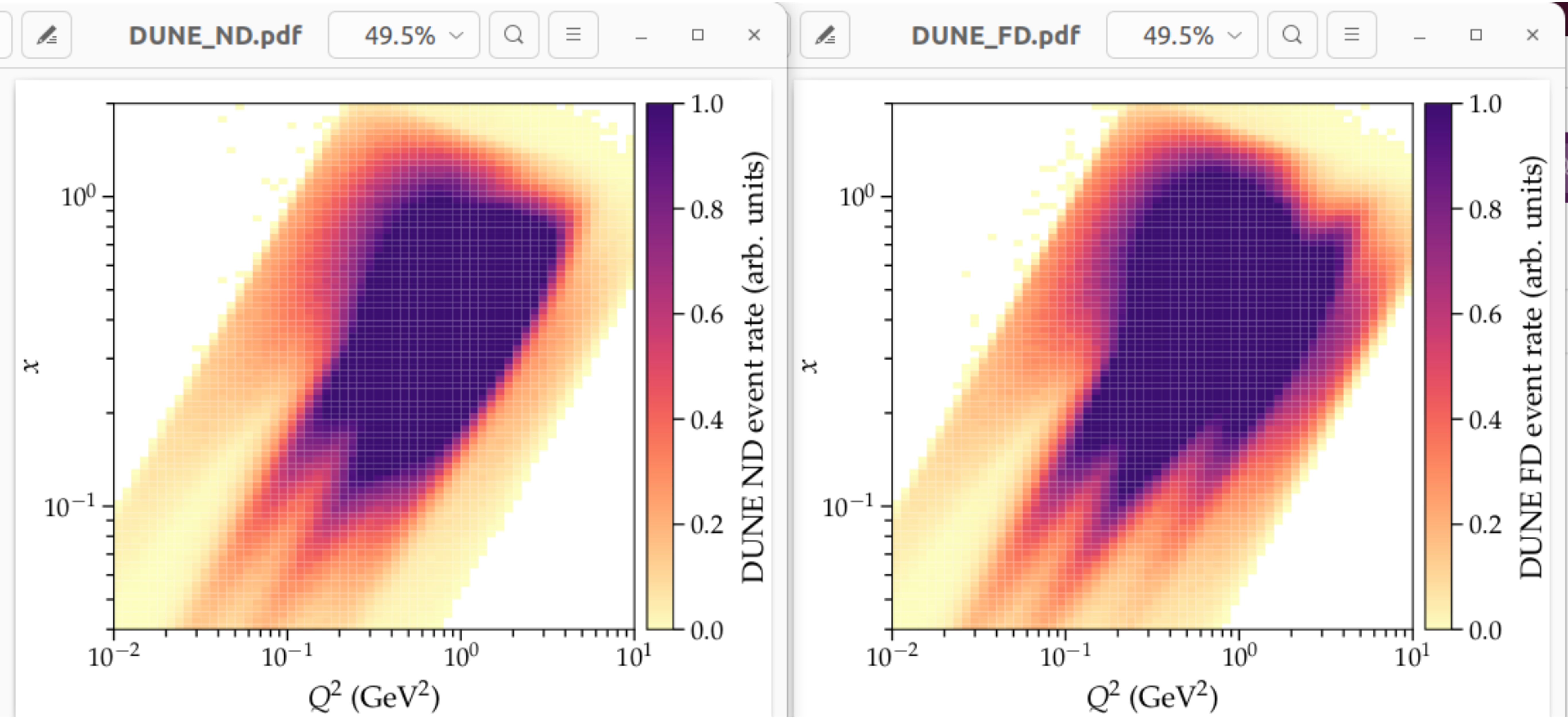
2D continuous function

- Parameterize as 2D neural net
- Loss function: same KL divergence applicable to cross section

Far detector extrapolation

DUNE phase space distributions for ND and FD almost completely overlap

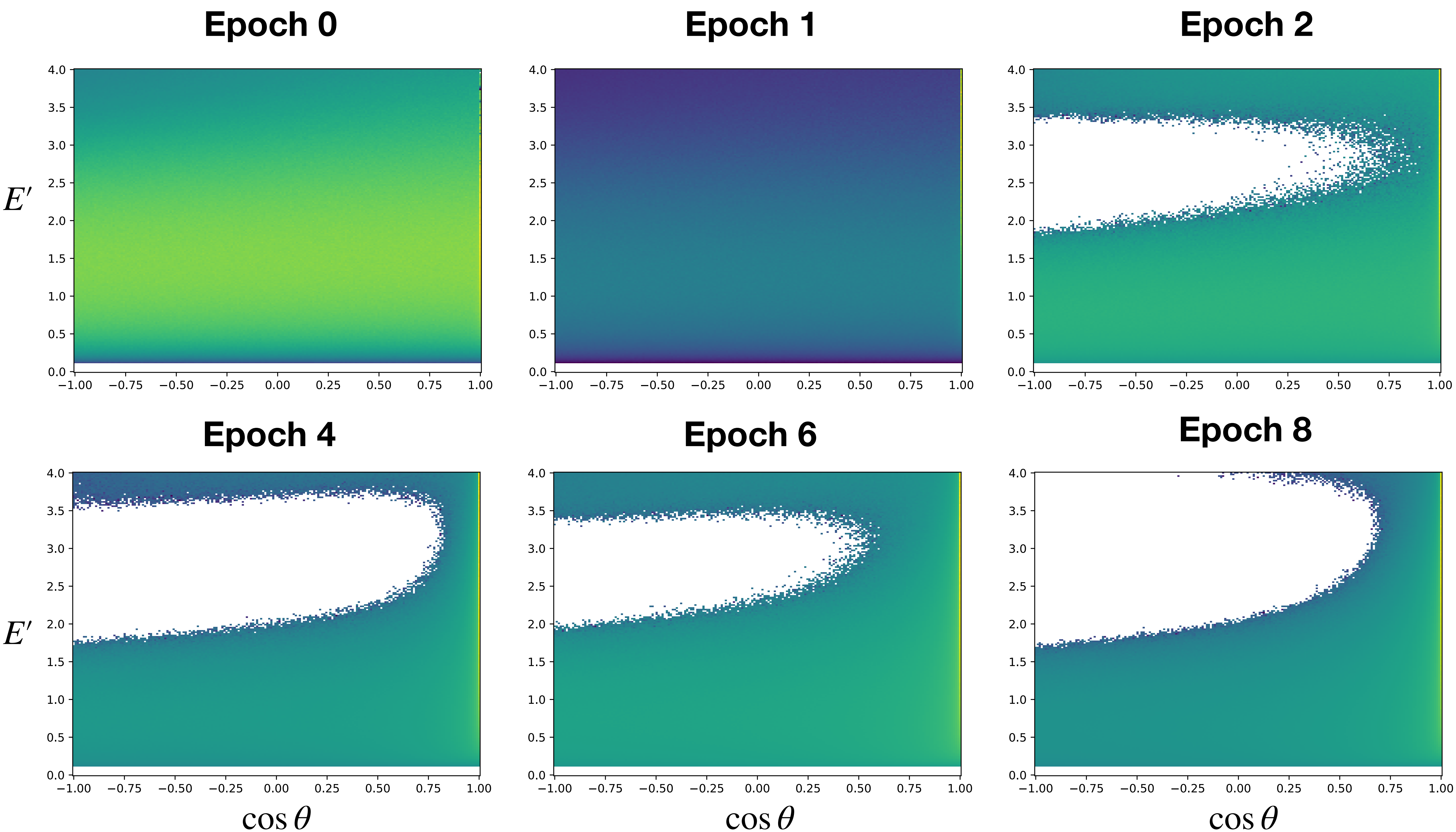
→ Learning at ND and extrapolating to FD possible



Li, Isaacson, Tame, MW, *in preparation*

Near detector training

Fake data study using DUNE flux estimates and GENIE cross sections

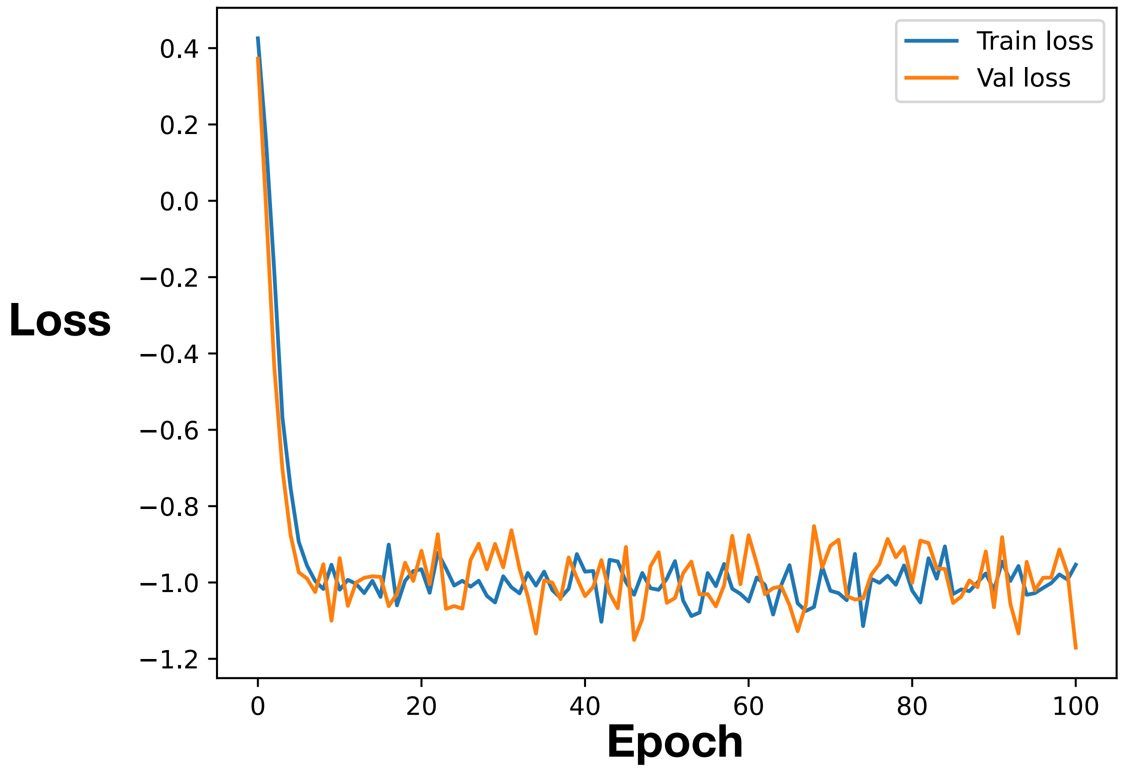


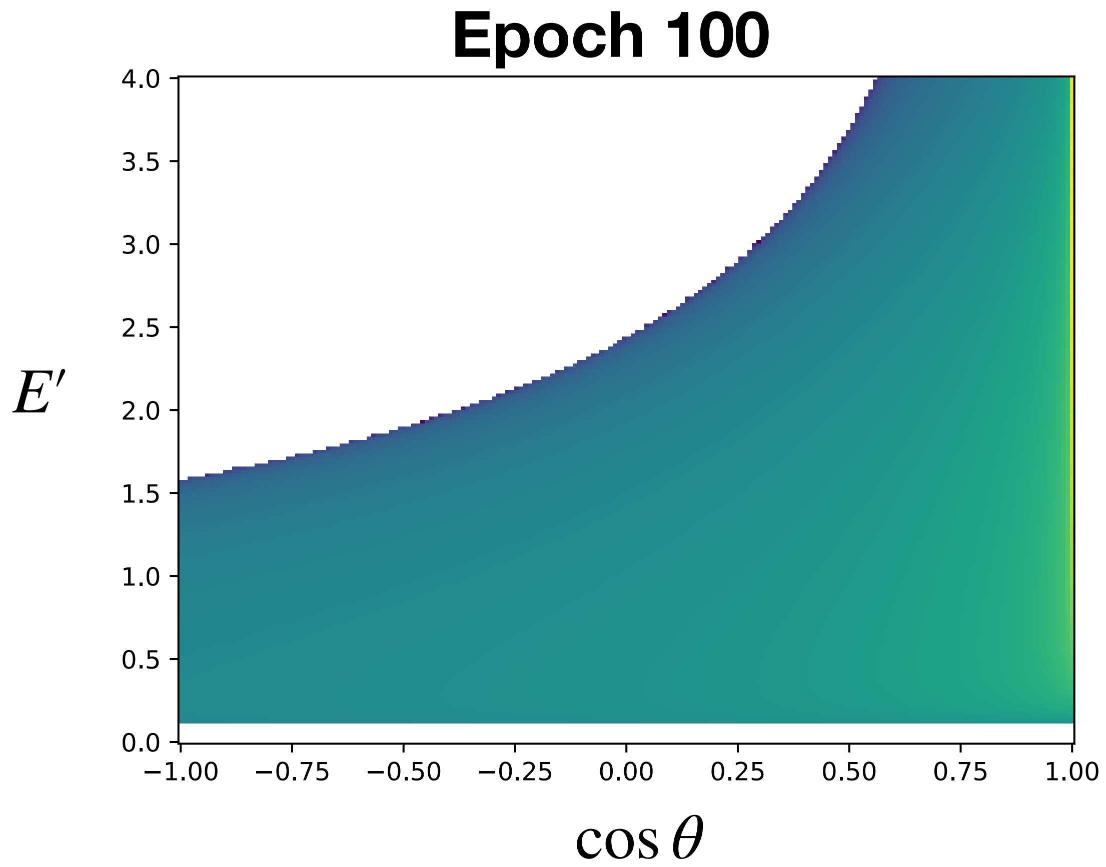
Li, Isaacson, Tame, MW, *in preparation*

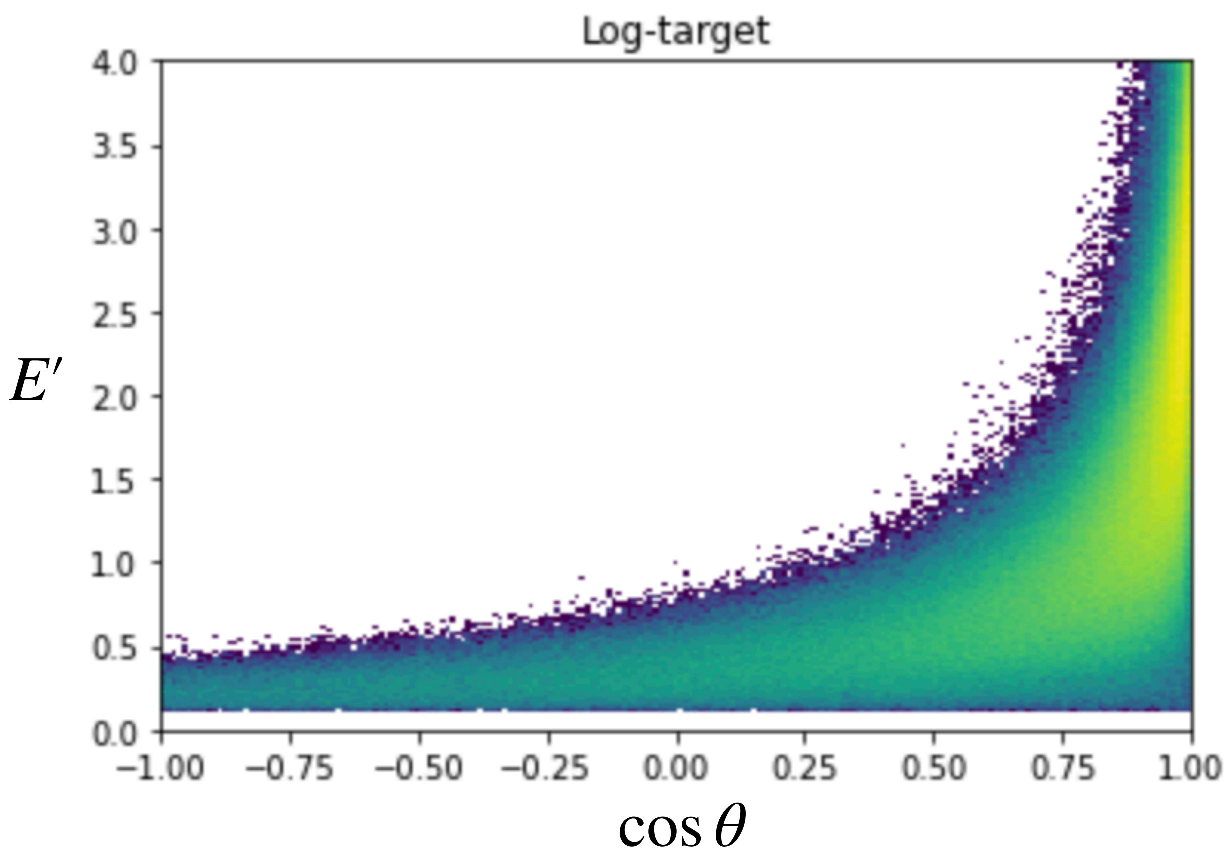
Near detector training

After 10s of training iterations, network has captured rough features of ND data distribution

More sophisticated training likely required to capture fine details







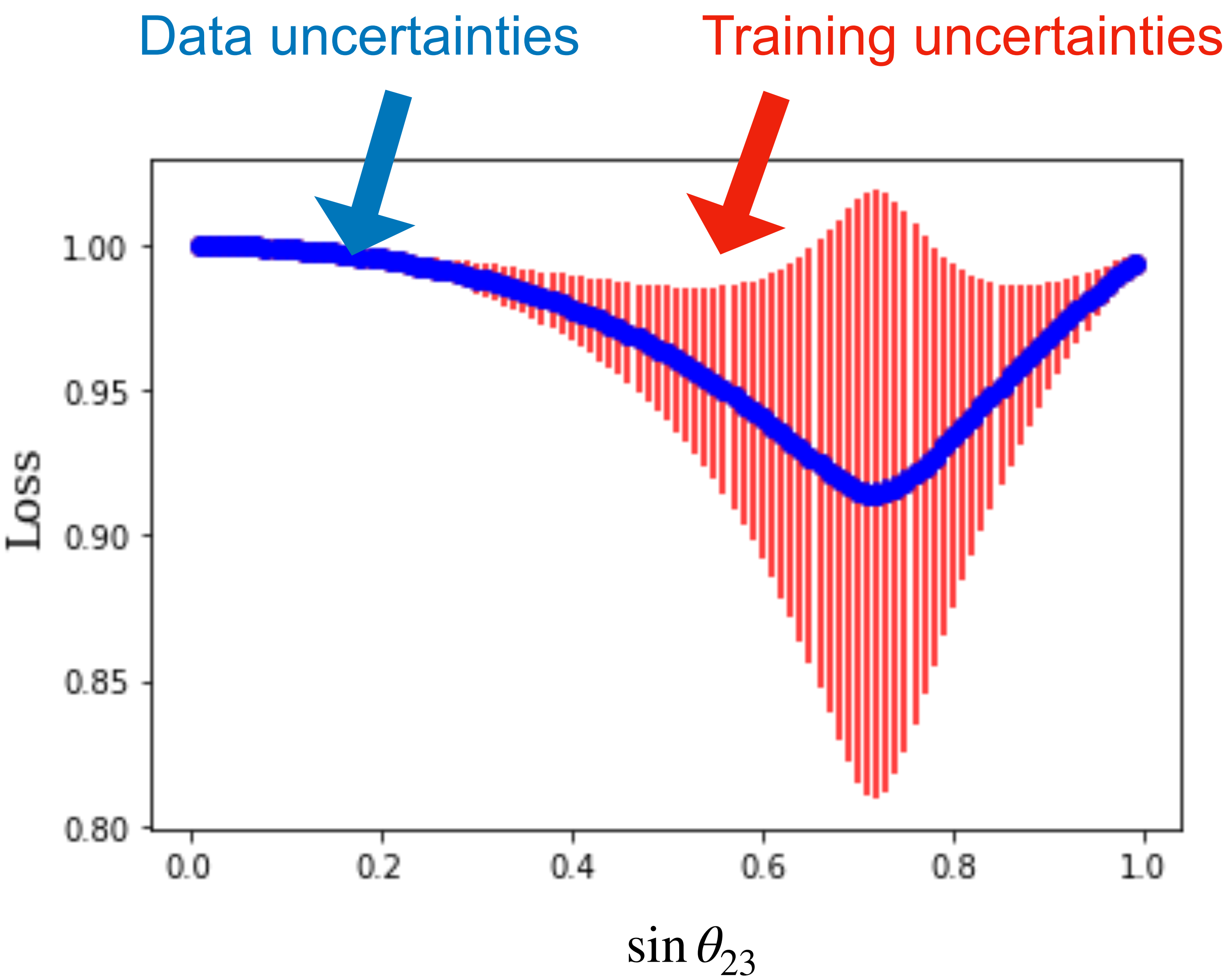
Li, Isaacson, Tame, MW, *in preparation*

Far detector extrapolation

After training on ND data, model used to predict FD distribution after oscillations

- Same KL divergence loss can be used to compare model predictions after oscillations to FD data

- Oscillation parameters that minimize loss are “best-fit” oscillation parameters



Li, Isaacson, Tame, MW, *in preparation*

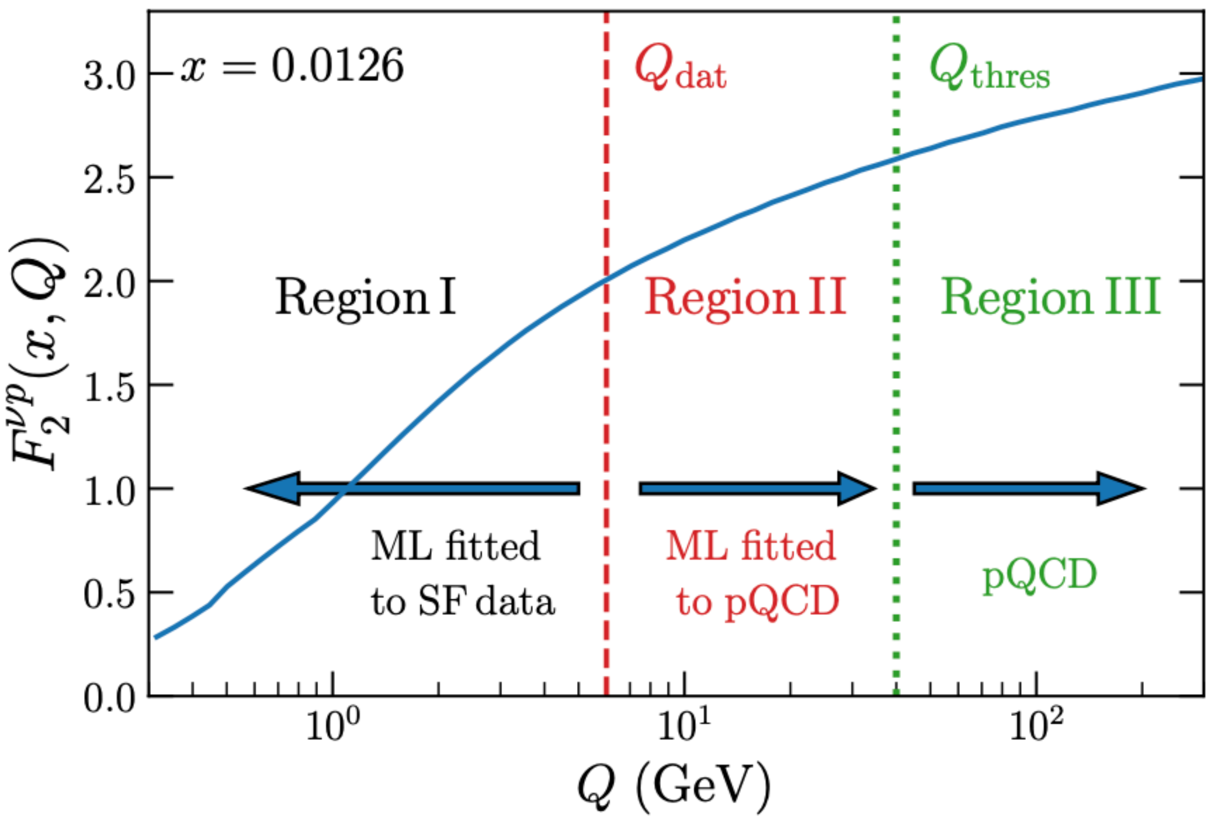
(Very) preliminary results encouraging, training uncertainties under investigation!

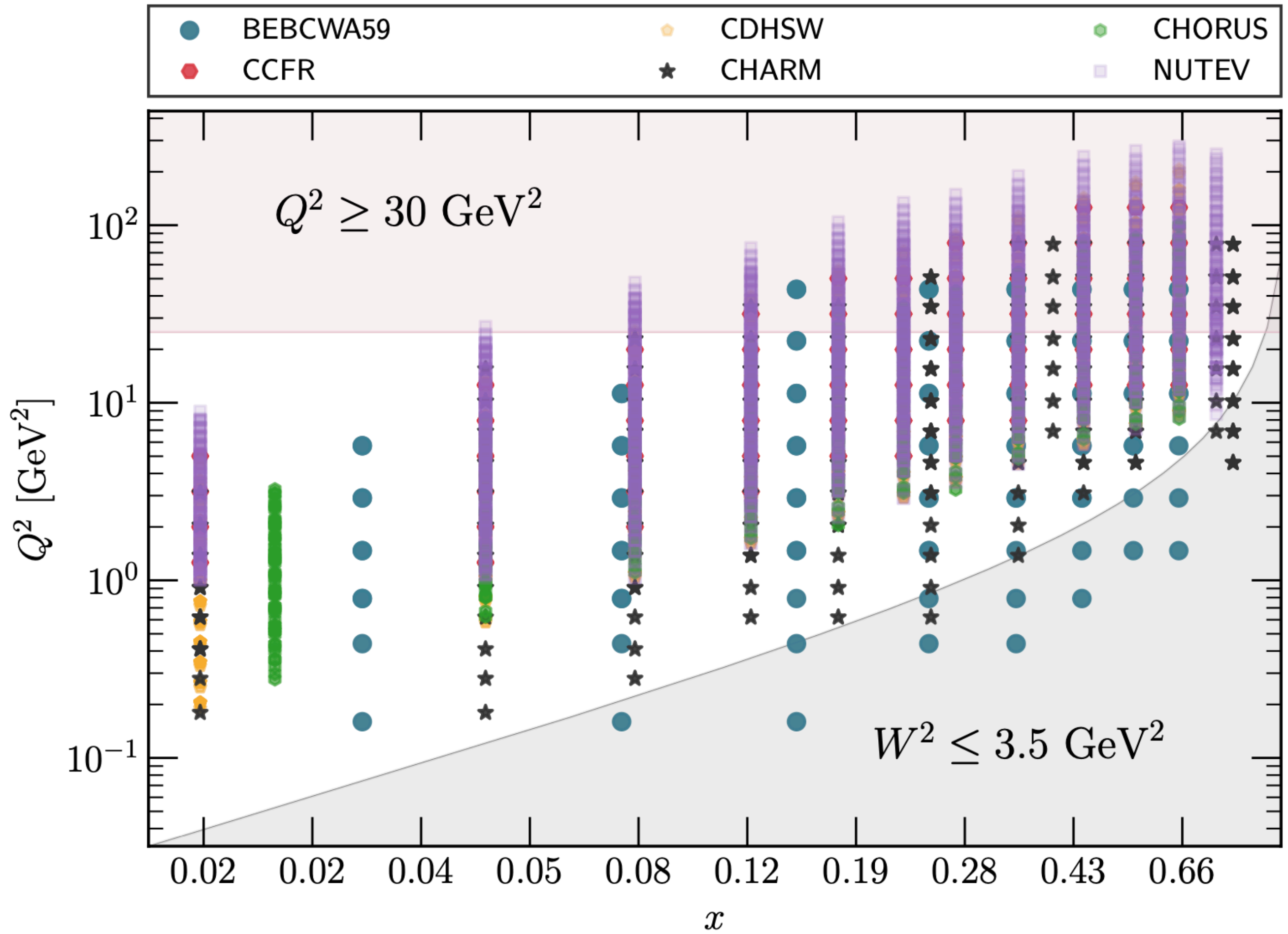
Incorporating global constraints

Neural net structure function parameterizations have been studied by NNSFν Collaboration

Candido et al, arXiv:2302.08527

Structure functions (assuming massless charged leptons) parameterized by neural nets





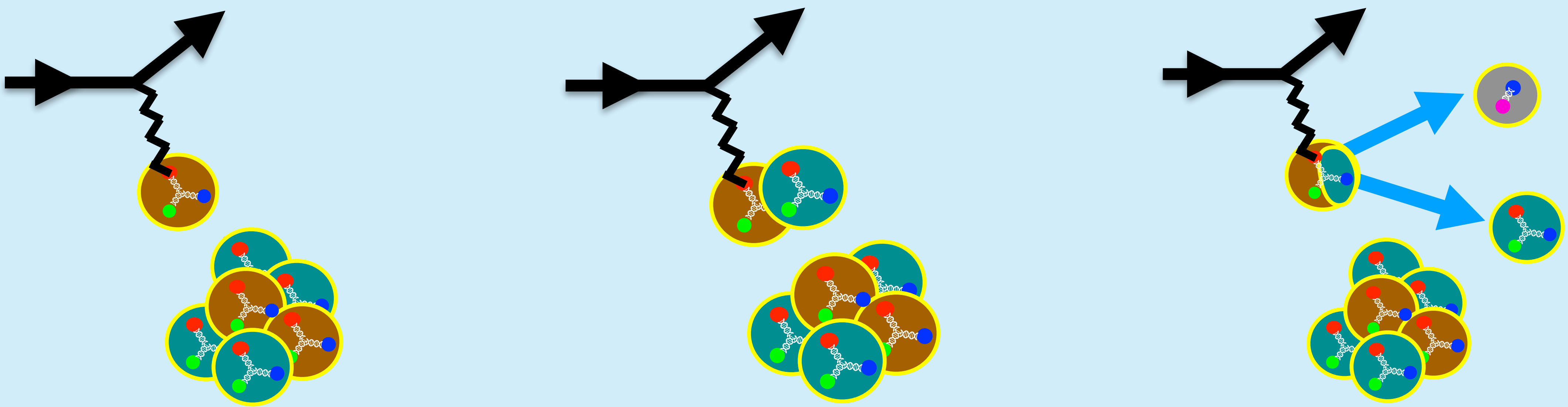
Global data on DIS and lower-energy processes used to constrain structure functions

Can these constraints be combined with ND constraints for more refined data-driven oscillation analyses?

Conclusions

Theory-driven cross sections are required to search for new physics that could be absorbed into cross-section model

Lattice QCD can provide nucleon-level form factors to ground nuclear models in the Standard Model



Data-driven cross sections can reduce (remove?) theory uncertainties in oscillation analyses, and provide valuable complementary cross checks