2nd Short-Baseline
Experiment-Theory Workshop
Santa Fe, NM

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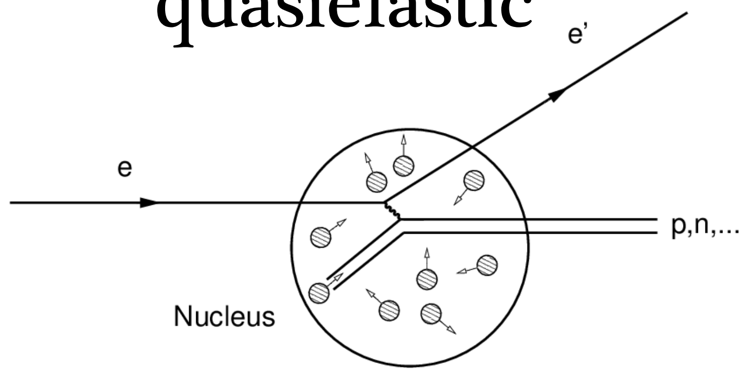
Radiative corrections to neutrino-nucleon/nucleus cross sections

Oleksandr (Sasha) Tomalak
LA-UR-24-22363

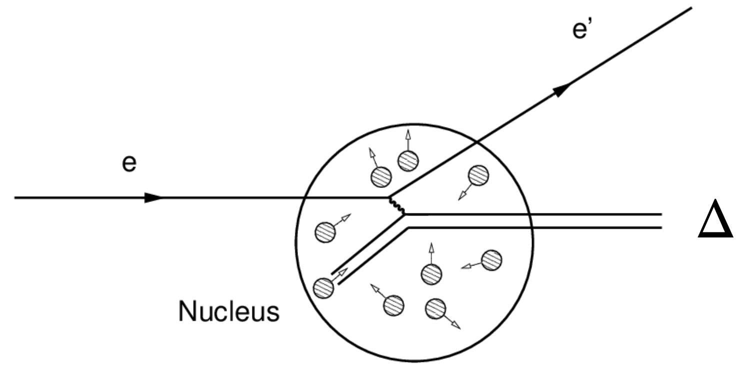
Los Alamos NATIONAL LABORATORY

CCQE. Why should we care?

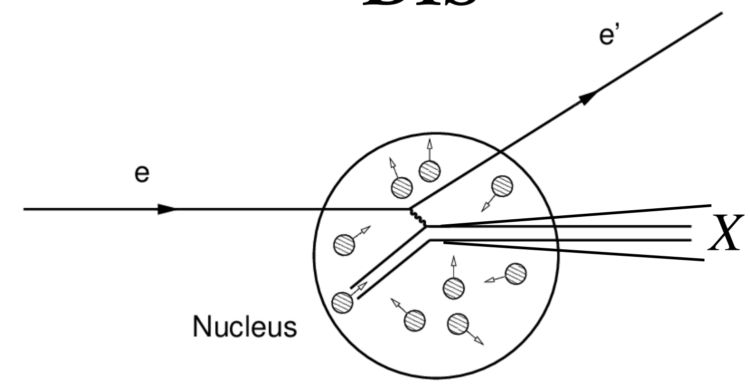
quasielastic

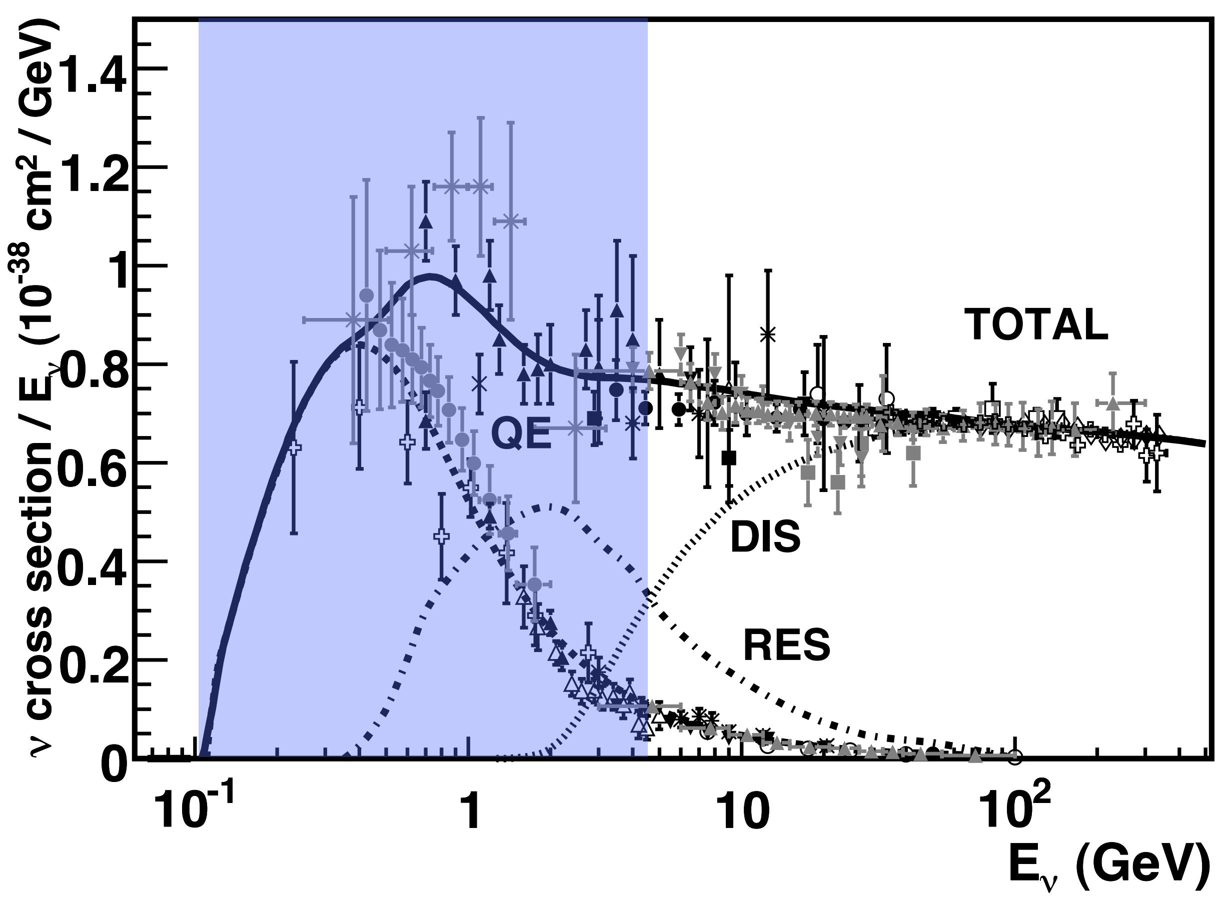


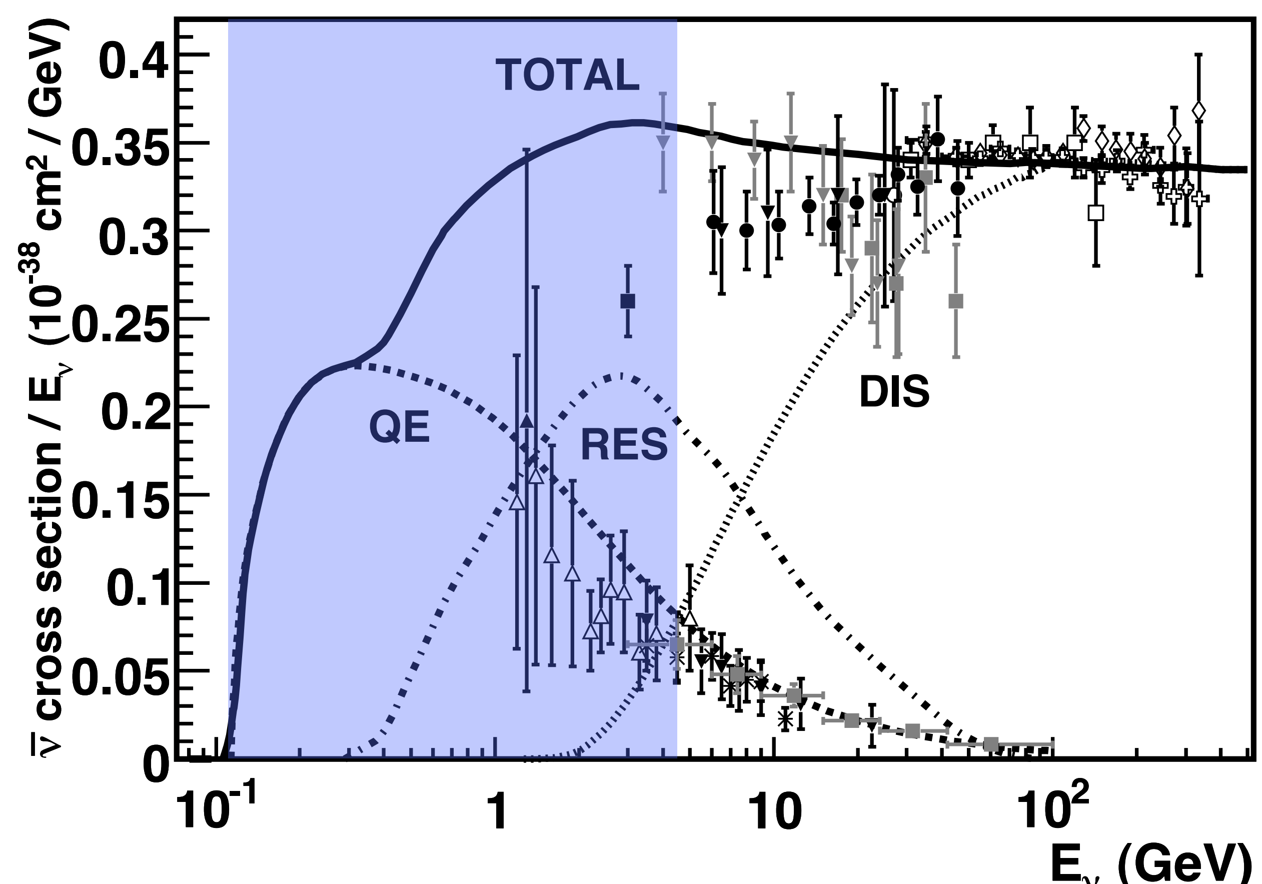
resonance



DIS



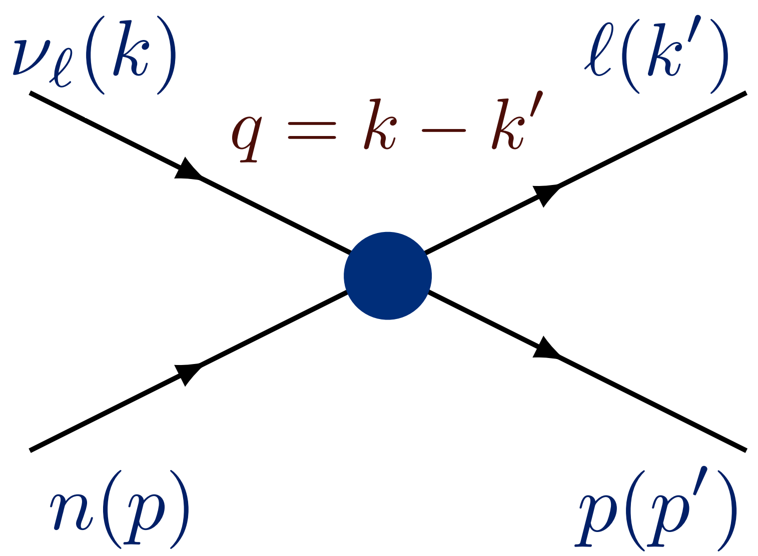




Formaggio and Zeller (2013)

- basic process @ SBN

CCQE scattering on free nucleon



neutrino energy E_ν

momentum transfer $Q^2 = -q^2$

contact interaction at GeV energies

- assuming isospin symmetry, nucleon current:

$$\Gamma^\mu\left(Q^2\right) = < p|\bar{u}\left(\gamma^\mu - \gamma^\mu\gamma_5\right)d|n >$$

$$\Gamma^\mu(Q^2) = \gamma^\mu F_D^V(Q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M}F_P^V(Q^2) + \gamma^\mu\gamma_5 F_A(Q^2) + \frac{q^\mu}{M}\gamma_5 F_P(Q^2)$$

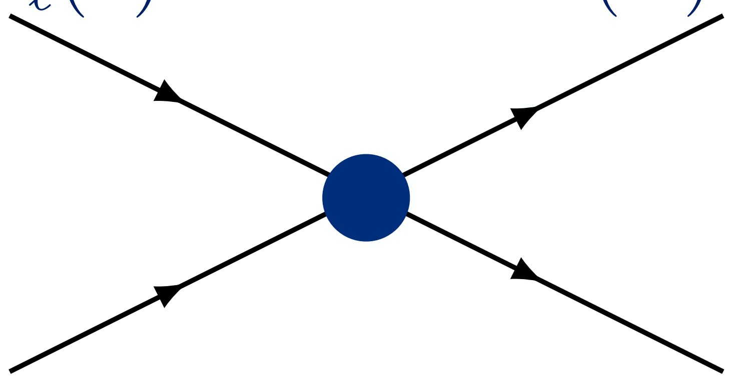
form factors: isovector Dirac and Pauli axial and pseudoscalar

$$F_{D,P}^V = F_{D,P}^p - F_{D,P}^n$$

tree-level amplitude

$$\mathrm{T} = \frac{\mathrm{G_F} V_{ud}}{\sqrt{2}}\left(\bar{\ell}(k')\gamma_\mu\left(1-\gamma_5\right)\nu_\ell(k)\right)\left(\bar{p}(p')\Gamma^\mu(Q^2)n(p)\right)$$

CCQE scattering on free nucleon



$$\nu = E_\nu/M - \tau - r^2$$

$$r = \frac{m_\ell}{2M} \qquad \tau = \frac{Q^2}{4M^2}$$

unpolarized cross section

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2} \sim \frac{M^2}{E_\nu^2}\left(\left(\tau + r^2\right) A(Q^2) - \nu B(Q^2) + \frac{\nu^2}{1+\tau} C(Q^2)\right)$$

Llewellyn Smith (1972)

- structure-dependent functions

$$A = \tau\left(G_M^V\right)^2 - \left(G_E^V\right)^2 + (1+\tau)F_A^2 - r^2\left(\left(G_M^V\right)^2 + F_A^2 - 4\tau F_P^2 + 4F_A F_P\right)$$

$$B = \pm 4\tau F_A G_M^V \qquad C = \tau\left(G_M^V\right)^2 + \left(G_E^V\right)^2 + (1+\tau)F_A^2$$

- pseudoscalar form factor contribution is suppressed by lepton mass
- cross section is sensitive to both vector and axial contributions

Elastic scattering on free nucleon

- only 3 experiments performed with deuterium bubble chamber

direct access to form-factor shape

ANL 1982: 1737 events

BNL 1981: 1138 events

FNAL 1983: 362 events

world data: ~3200 events

Fermilab bubble chamber, Richard Drew

- axial form factor extracted based on electromagnetic structure

A.S. Meyer, M. Betancourt, R. Gran and R.J. Hill (2016)

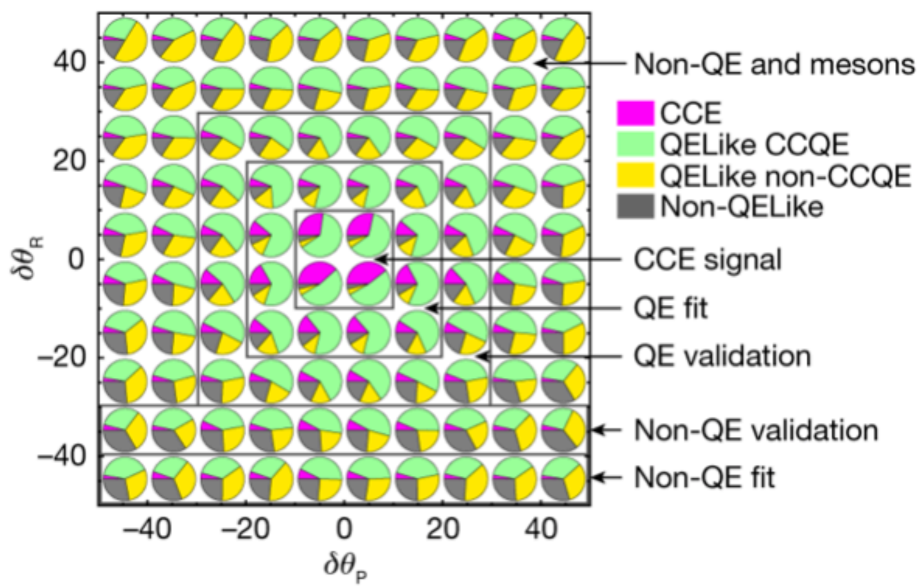
MINERvA result with free protons

- idea of scattering on molecular hydrogen realized !!!

talk of K. McFarland

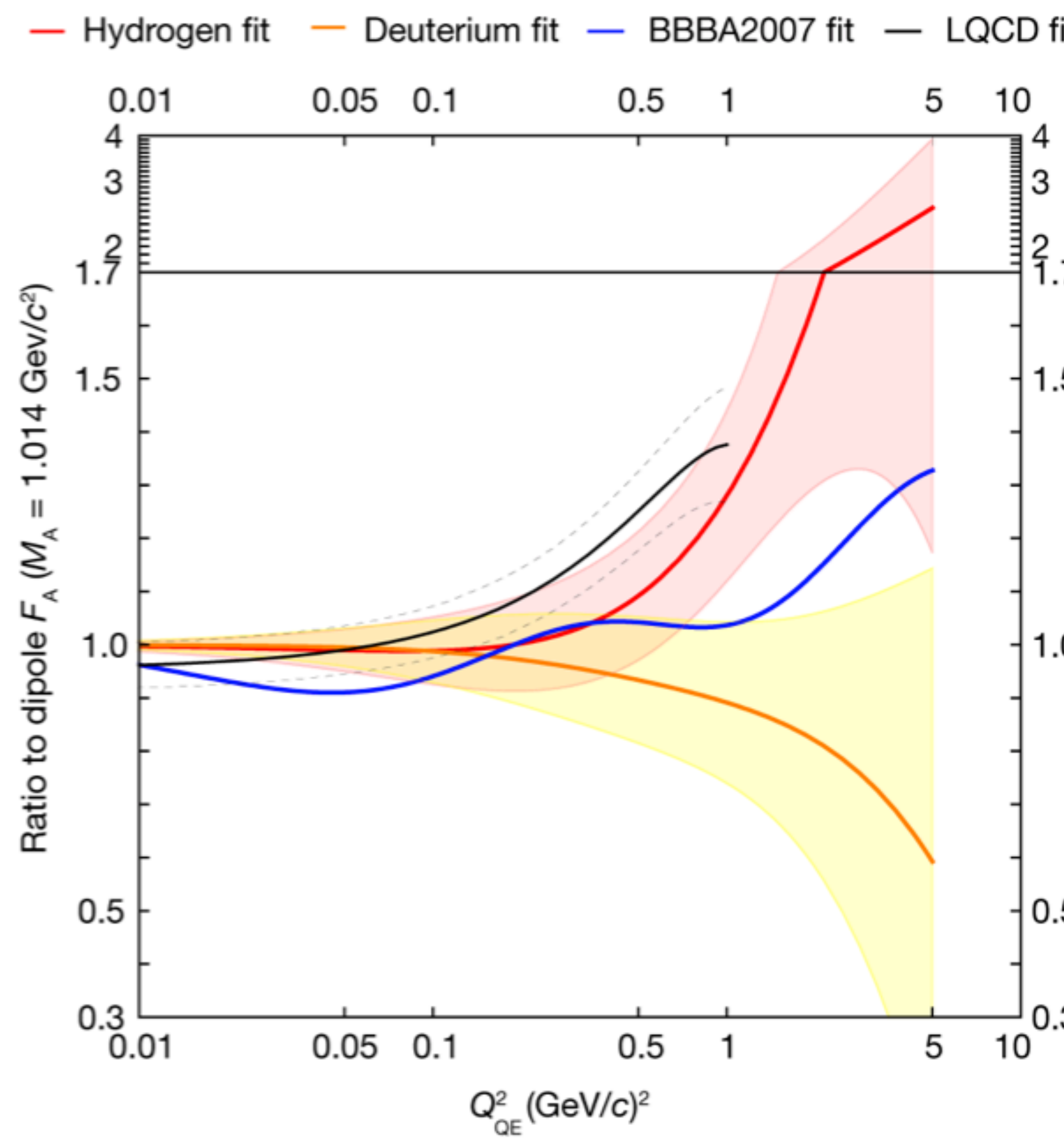
$\overline{\nu}_\mu p \to \mu^+ n$

muon kinematic selection



5580 events over
12500 background

background nuclear events constrained by scattering of ν



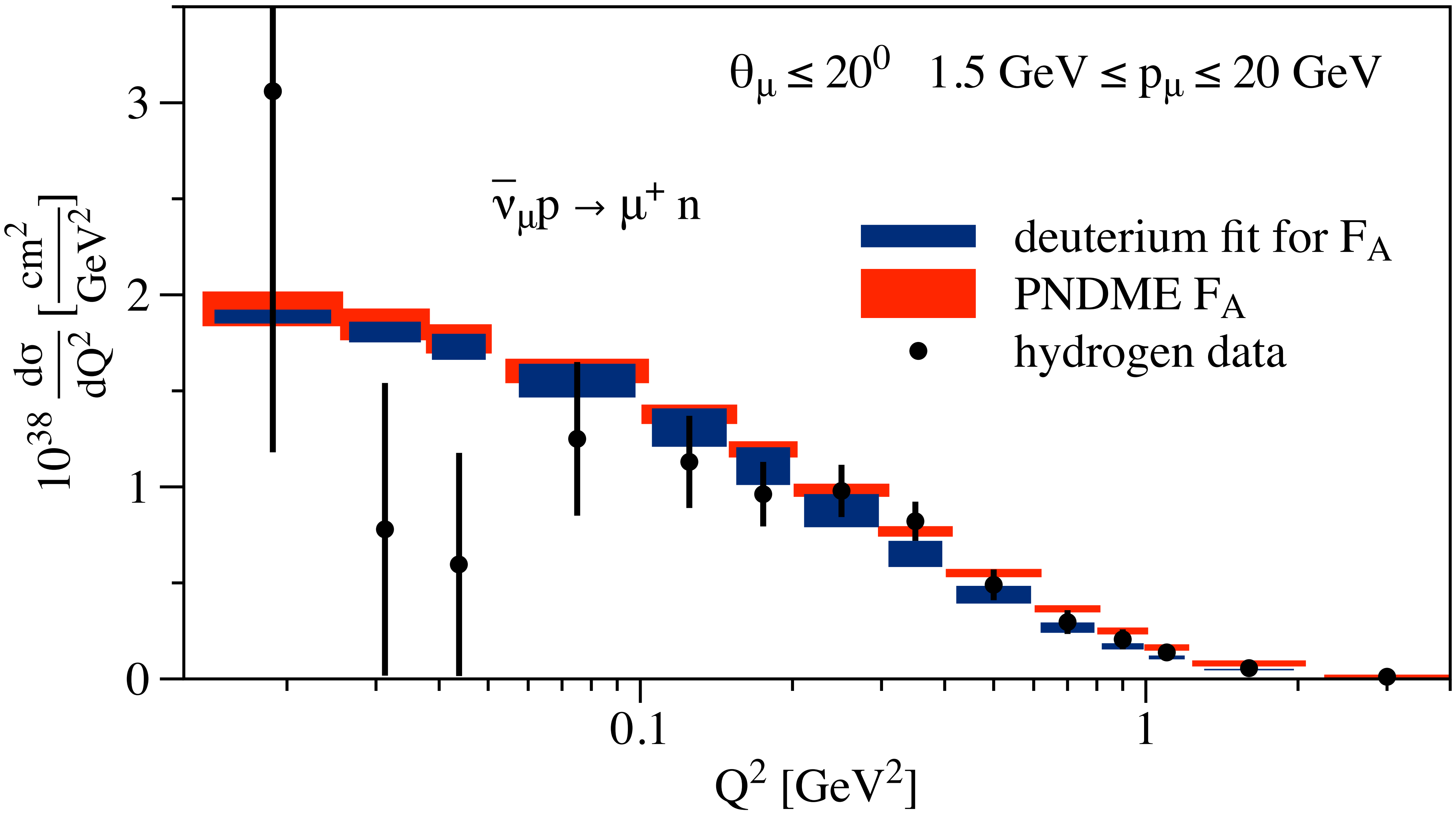
- 1st measurement of axial form factor on "free" protons $\overline{\nu}_\mu p \to \mu^+ n$

T. Cai et al., MINERvA Collaboration, Nature (2023), 614, 48-53

Lattice QCD vs MINERvA

talk of R. Gupta

- PNDME 2023 axial-vector form factor as representative of lattice QCD



- $\lesssim 1\sigma$ agreement for each bin besides two at small Q^2

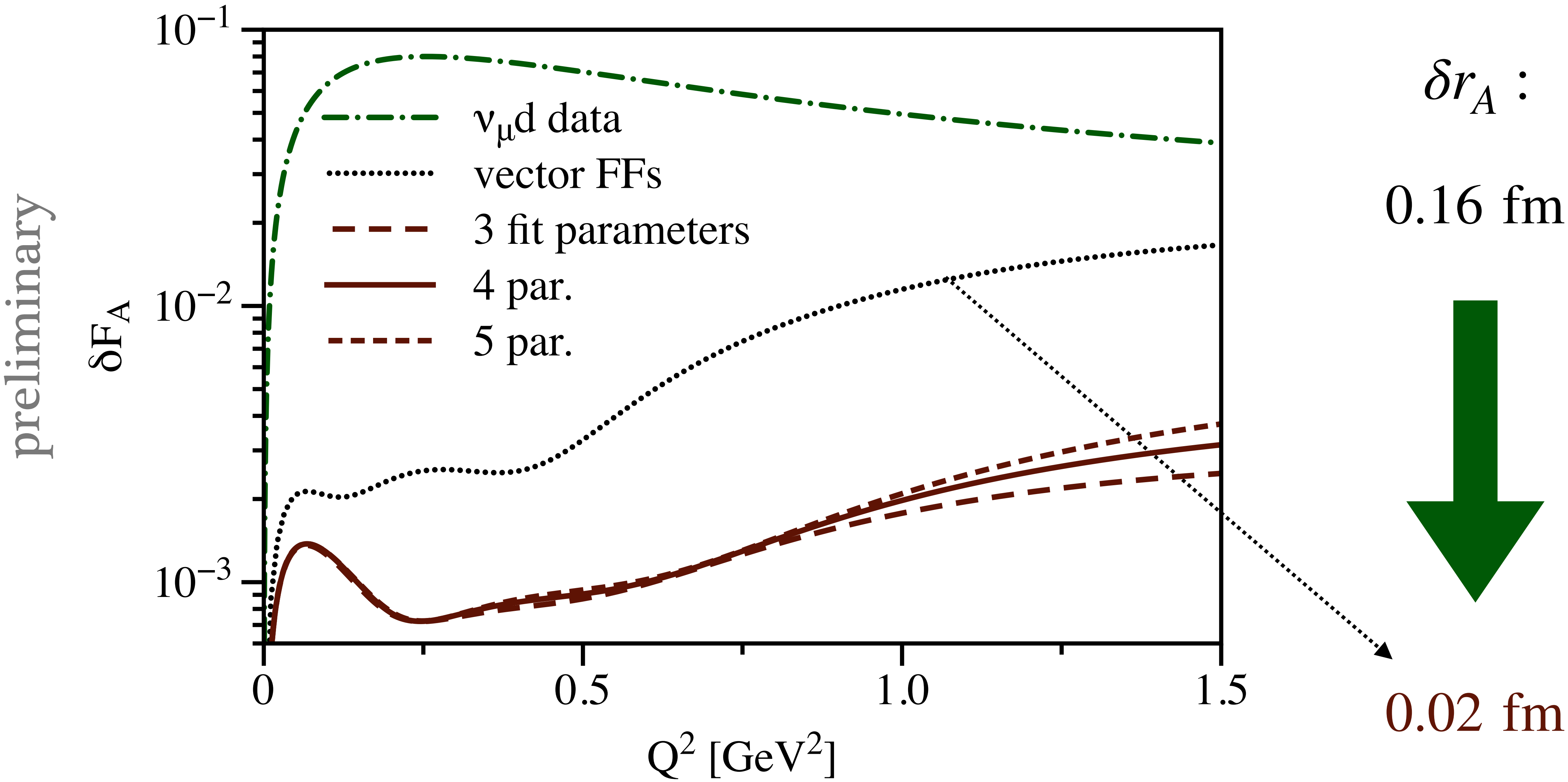
- 2-3σ tension between lattice QCD and deuterium data
- MINERvA hydrogen data consistent with LQCD and deuterium

O. T., Rajan Gupta, and Tanmoy Bhattacharya, Phys.Rev.D 108 (2023) 7, 074514

DUNE projections

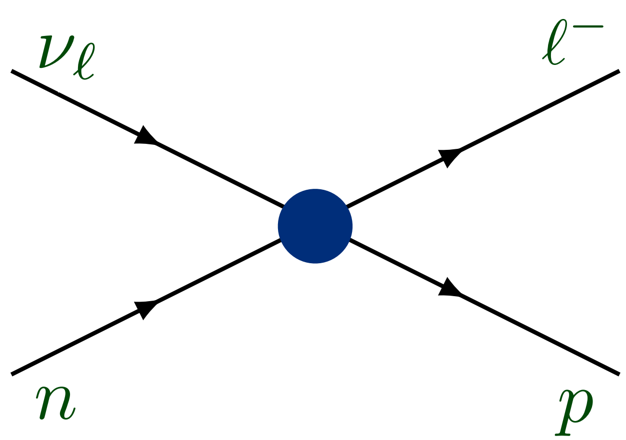
- estimates for 700 kg of H in Straw Tube Tracker at near detector

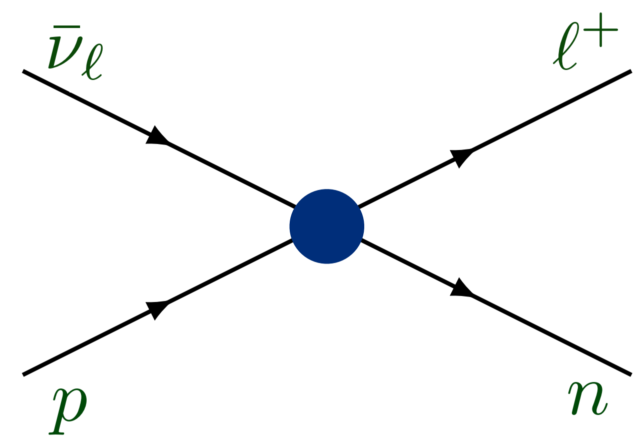
H. Duyang, B. Guo, S. R. Mishra, and R. Petti (2016)



- order of magnitude improvement in axial form factor and radius
- DUNE will probe vector form factors and isospin symmetry

Roberto Petti, O. T., and Richard J. Hill, letter in Phys. Rev. D 109 5 (2024)

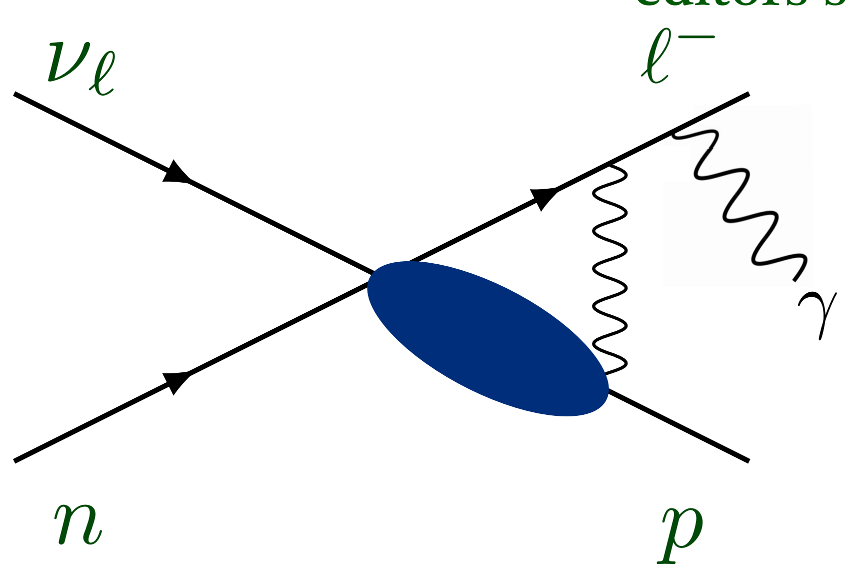




O. T., Qing Chen, Richard J. Hill and Kevin S. McFarland, Nature Commun. 13 (2022), 1, 5286

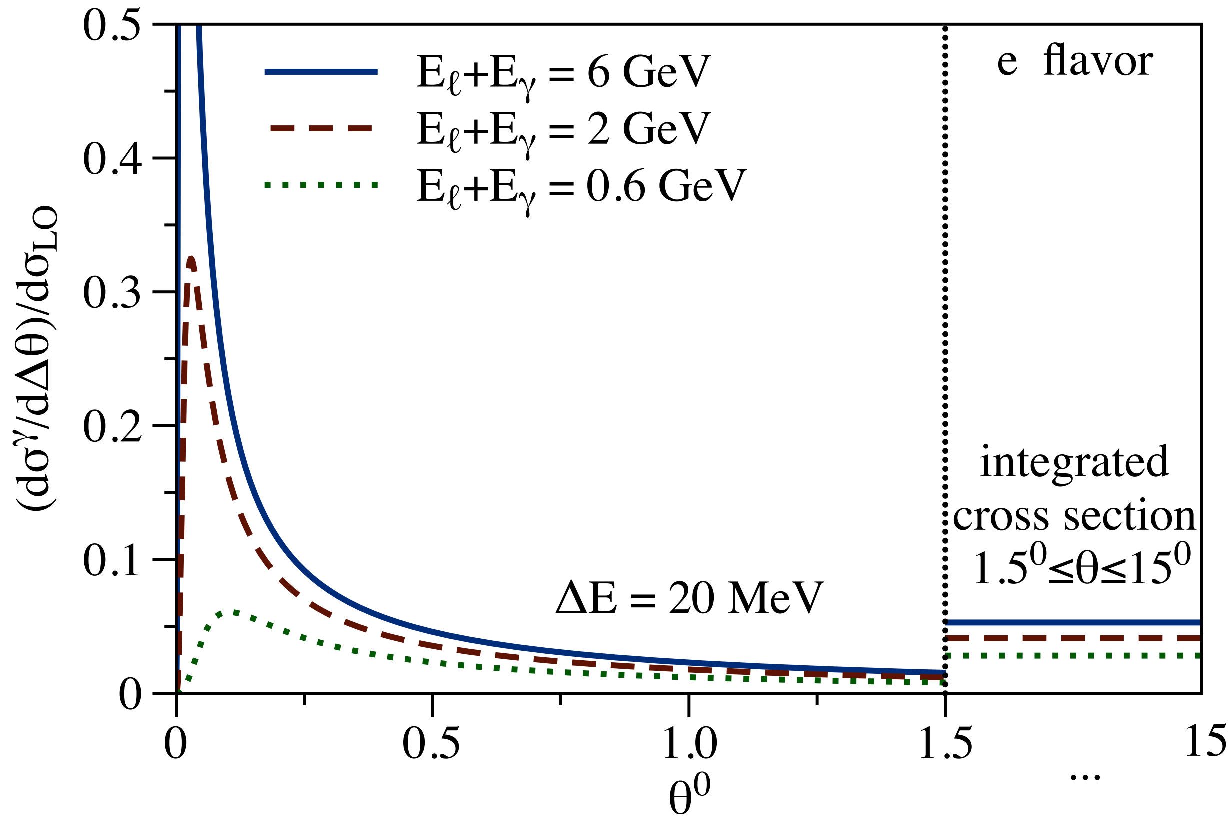
Radiative corrections in charged-current elastic scattering on free nucleons

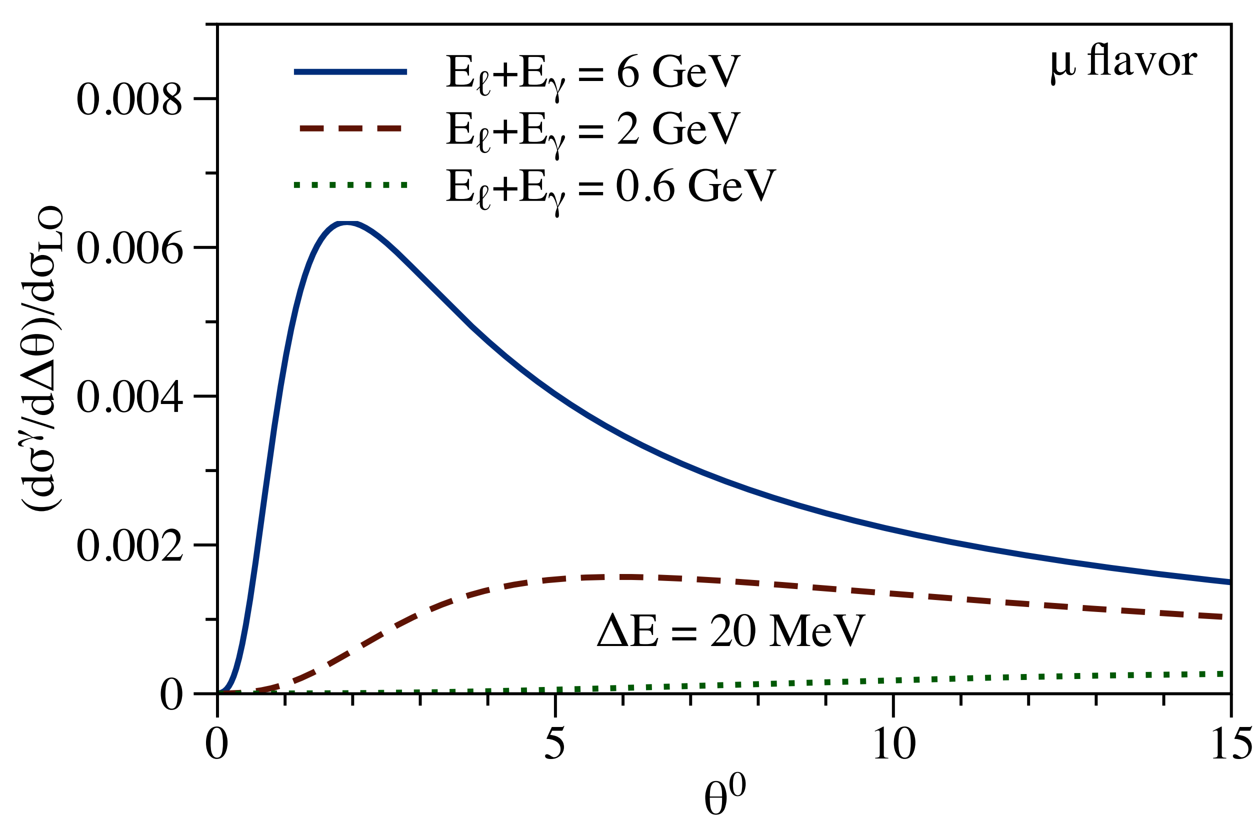
O. T., Qing Chen, Richard J. Hill, Kevin S. McFarland and Clarence Wret
editors suggestion in Phys. Rev. D (2022)



Electron vs muon jets

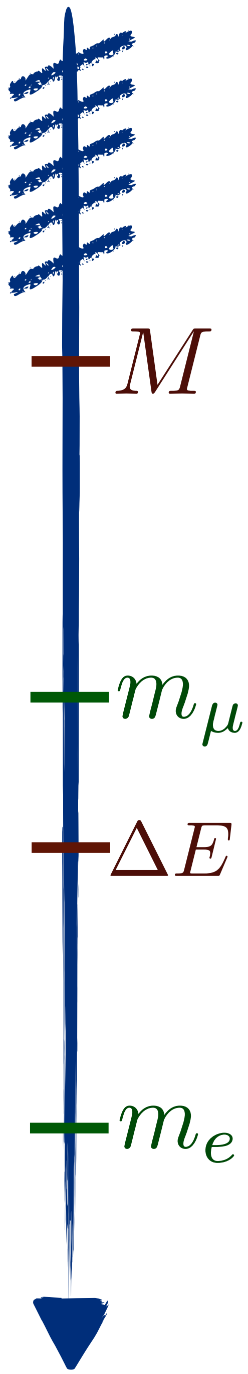
- electron flavor: factorization for radiation of collinear photons
- cone angle is defined to lepton direction
- photons of energy > 20 MeV, fixed energy in the cone





- flavor-dependent effect, same for $\nu_\ell n \to \ell^- p$ vs $\bar{\nu}_\ell p \to \ell^+ n$
- forward-peaked radiation for electron flavor
- negligible radiation for muons with shifted peak position

Factorization approach



- cross section is given by factorization formula

$$\mathrm{d}\sigma \sim S\left(\frac{\Delta E}{\mu}\right) J\left(\frac{m_\ell}{\mu}\right) H\left(\frac{M}{\mu}\right)$$

- determine hard function at hard scale by matching experiment or hadronic model to the theory with heavy nucleon

- soft and collinear functions are evaluated perturbatively

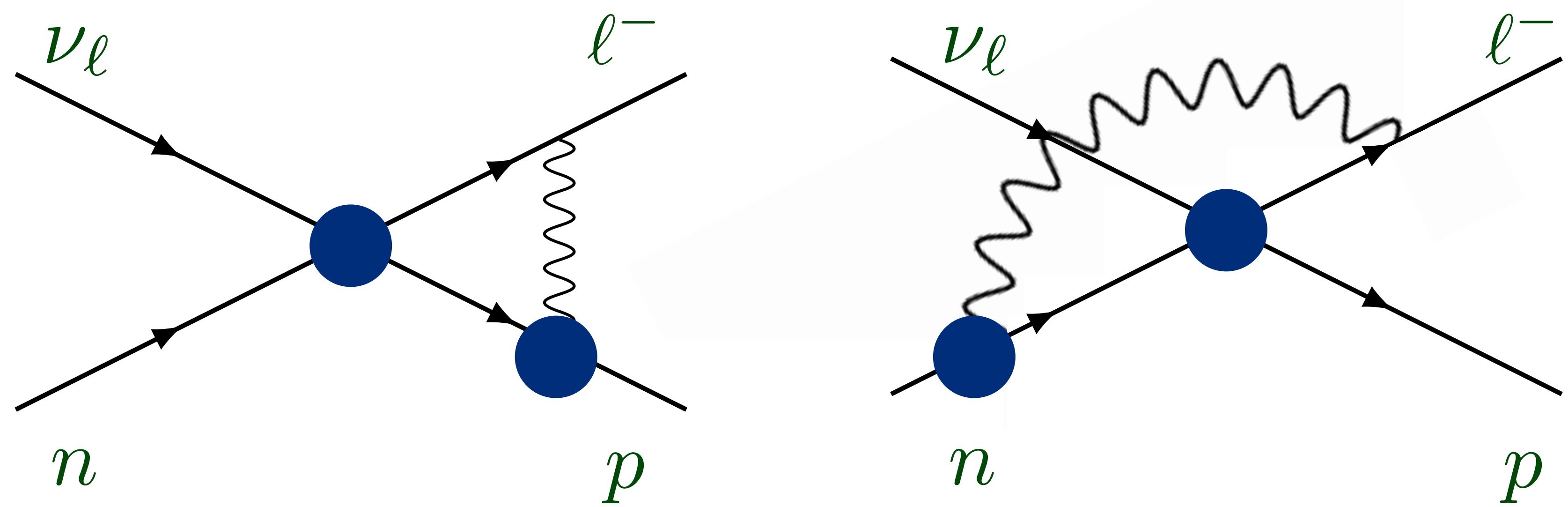
SCET power expansion parameter

$$\lambda \sim \frac{m_\mu^2}{E_\nu^2} \sim (\Delta\theta)^2 \sim \frac{\Delta E}{E_\nu}$$

$\ln \lambda$ enhancements

power corrections are large at lowest muon (anti)neutrino E @ SBND

Hadronic model at GeV scale



- exchange of photon between the charged lepton and nucleons

- assume onshell form for each interaction with dipole form factors

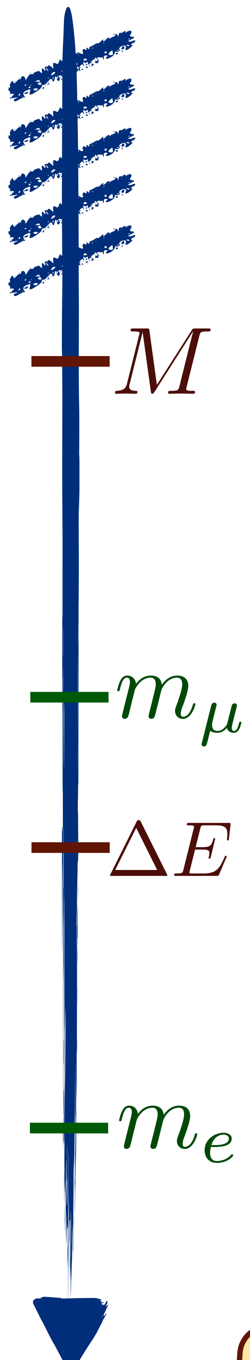
  discussed for neutrino-nucleon scattering: Graczyk (2013)

- add self energy for charged particles

- reproduce soft and collinear regions of SCET

- best determination of hard function

Factorization approach



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- determine hard function at hard scale by matching experiment or hadronic model to the theory with heavy nucleon

- RGE evolution of the hard function to scales $\Delta E, m_\ell$

- soft and collinear functions are evaluated perturbatively

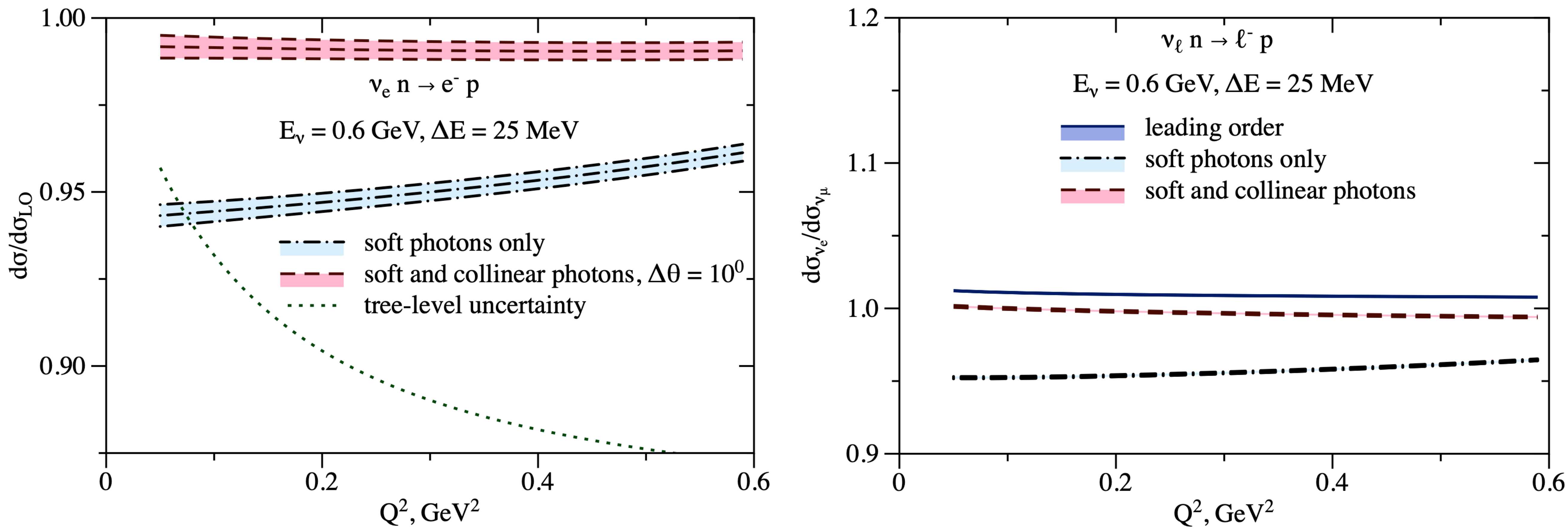
- calculate cross section at low energies accounting for all large logs

ep scattering with soft radiation only: Richard J. Hill (2016)

- soft and collinear functions determined analytically
- hard function describes physics at GeV energies

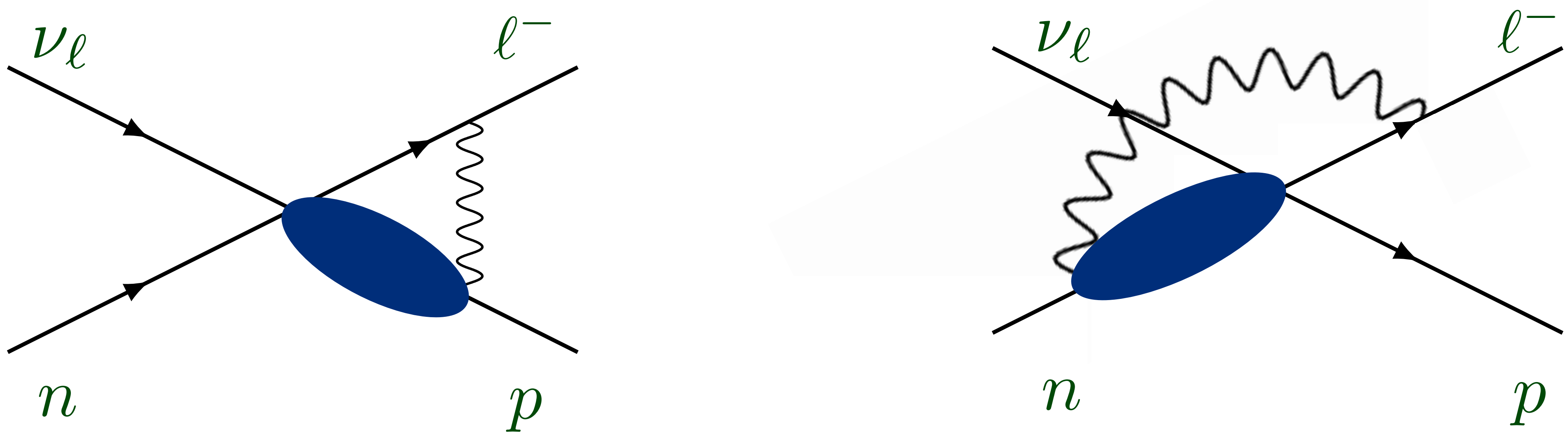
Exclusive observables

- cancellation of uncertainties from hard function for e/μ and ratio to LO



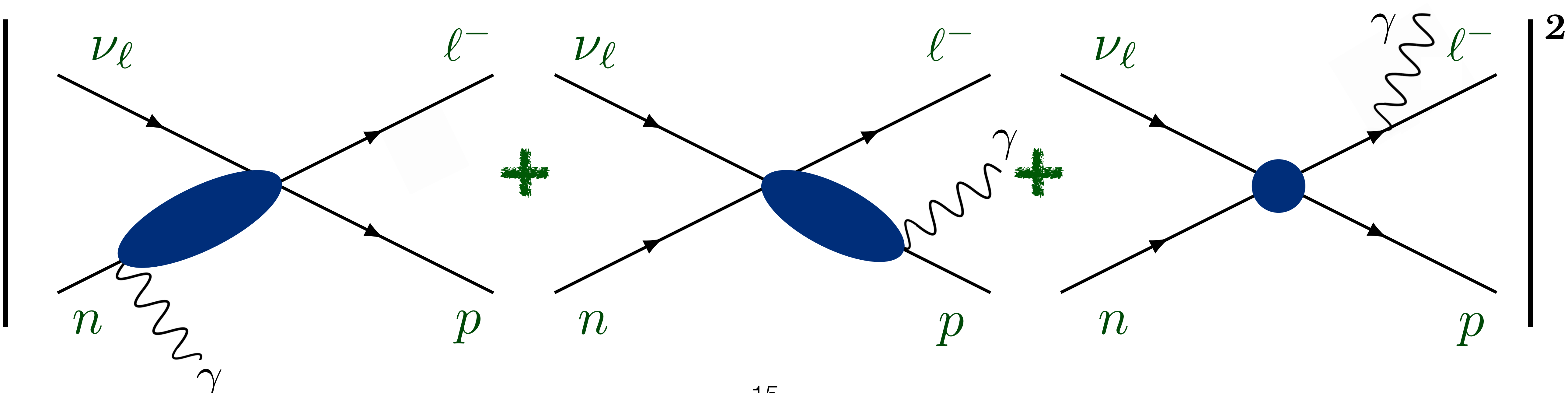
- ratios: cancellation of uncertainty from hard function

Inclusive observables



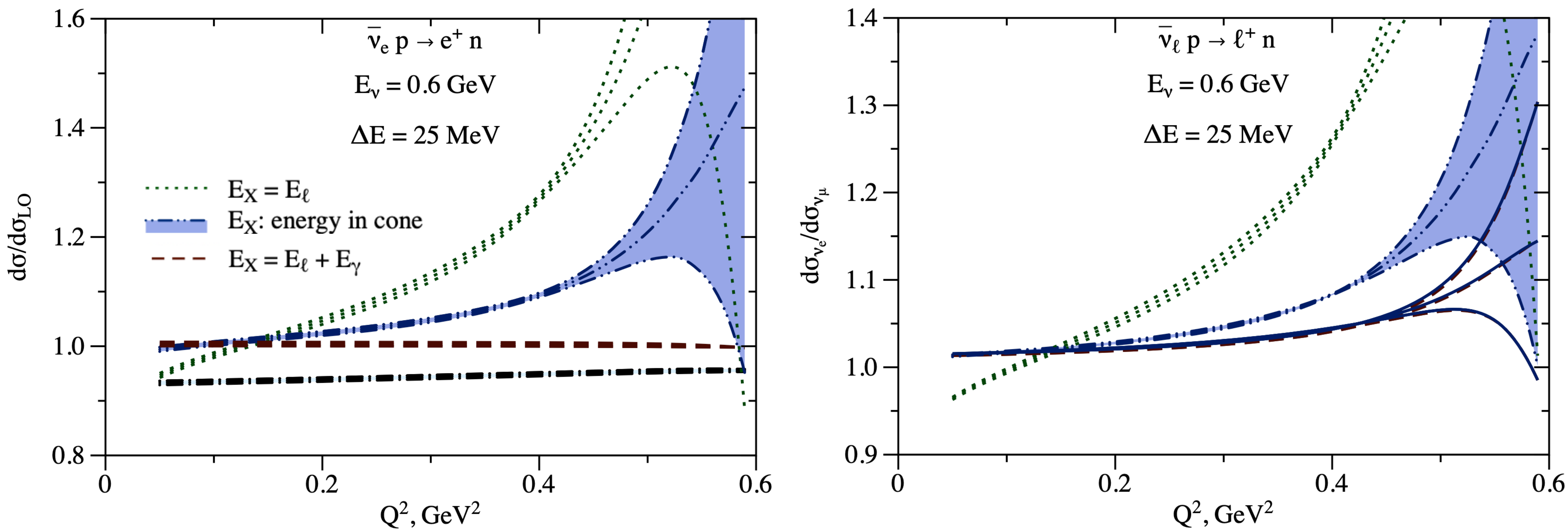
- the same gauge-invariant model for the real radiation

- arbitrary hard photons are part of the observable



Inclusive observables

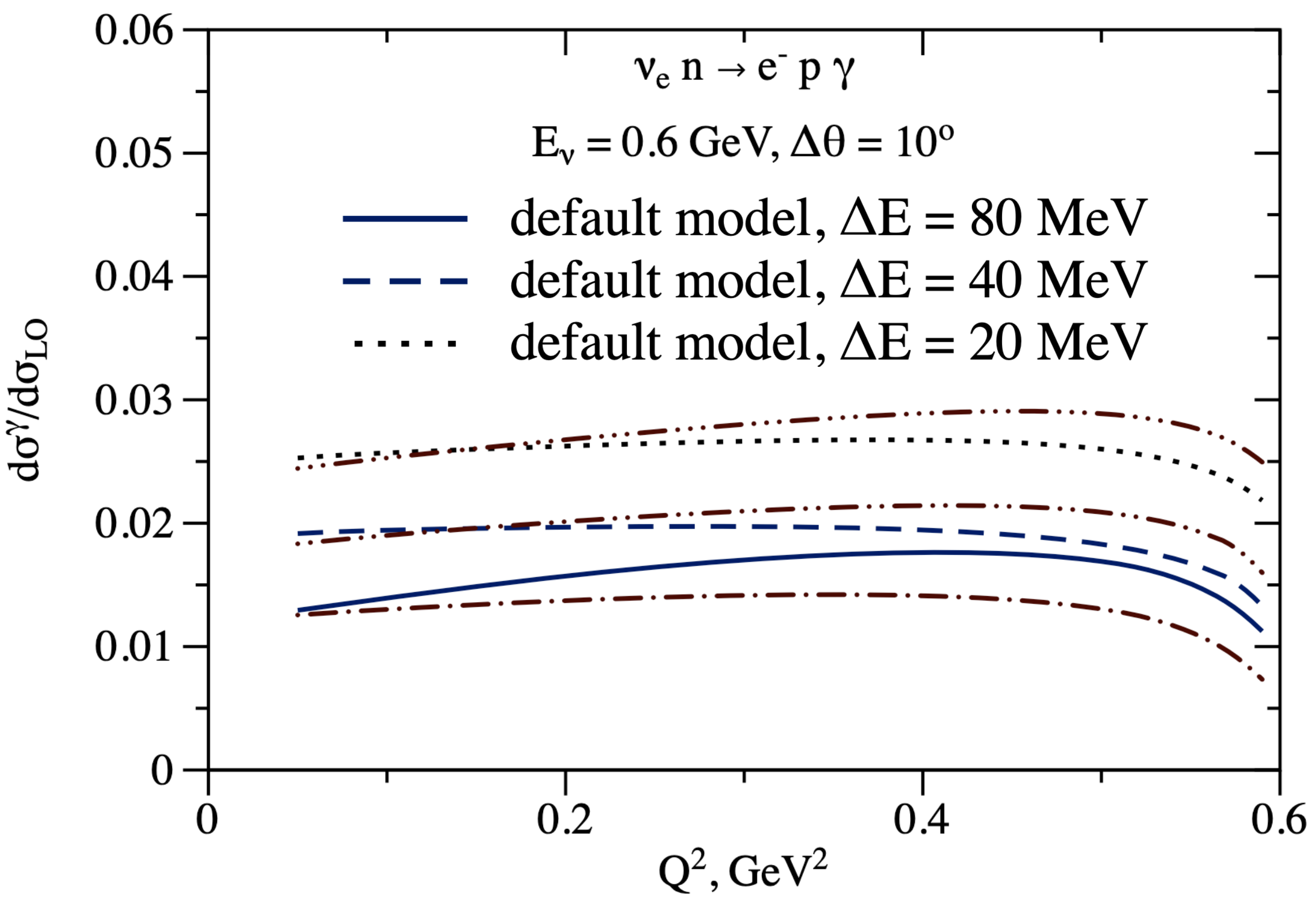
- kinematics $Q^2 = 2M\left(E_\nu - E_X\right)$ is reconstructed with 3 different E_X

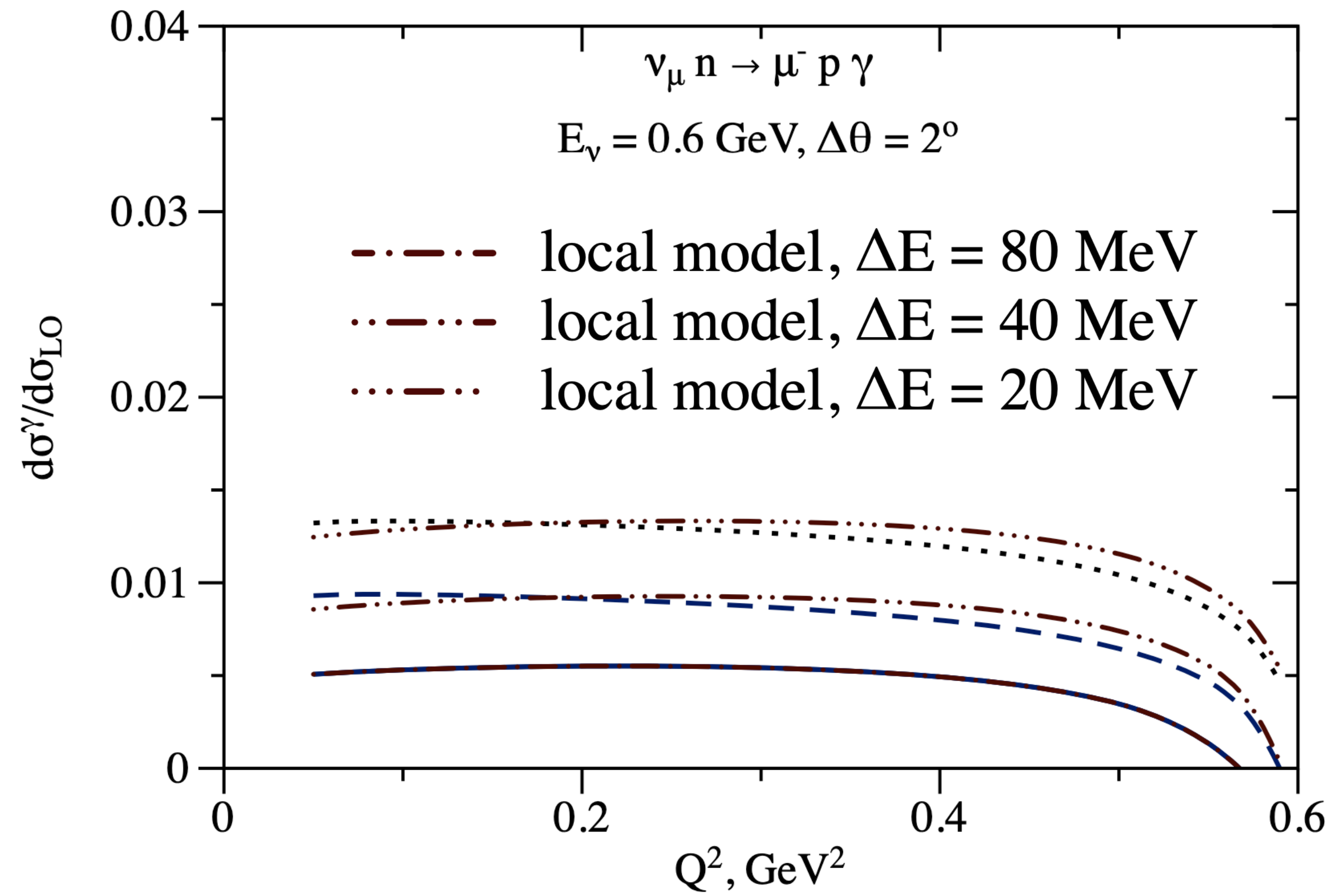


- dependence on reconstruction of kinematics and cuts
- precise prediction for ratios $\sigma_{\nu_e}/\sigma_{\nu_\mu}$

Radiation of hard photons

- model-dependent description for radiation of hard photons





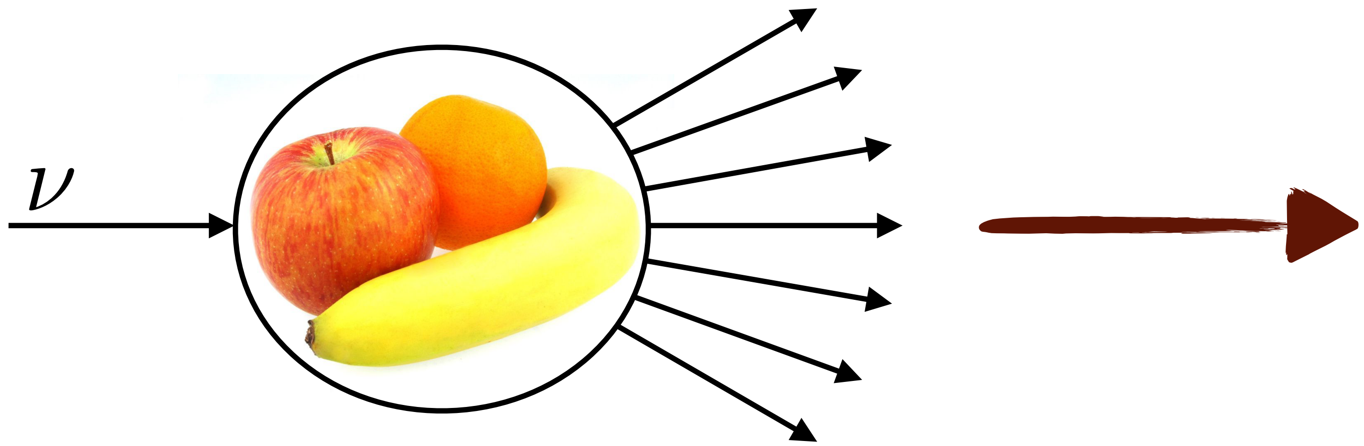
"Blunden calculation"

- photon energies are above 20, 40, and 80 MeV: default vs "SIFF"

"hadronic model"

- %-level radiation of non-collinear hard photons
- 10^{-4} flavor misidentification rate for NOvA&T2K kinematics

Conclusions



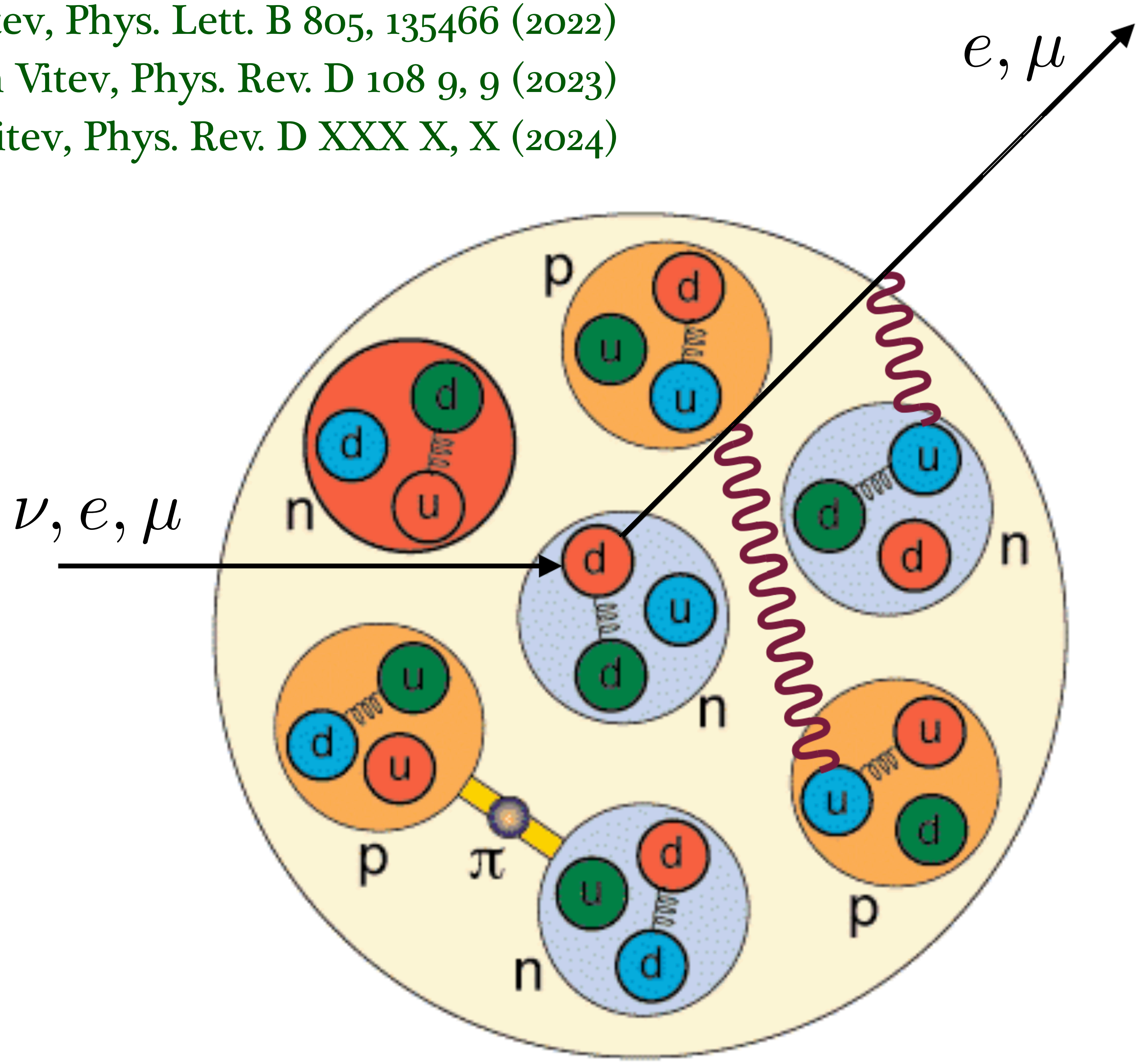
radiative corrections
in EFT framework

- radiative corrections to neutrino-nucleon cross sections formulated in factorization framework

- charged-current elastic electron vs muon cross-section ratios evaluated from theory with sub-percent uncertainty

QED medium effects

O. T. and Ivan Vitev, Phys. Lett. B 805, 135466 (2022)
O. T. and Ivan Vitev, Phys. Rev. D 108 9, 9 (2023)
O. T. and Ivan Vitev, Phys. Rev. D XXX X, X (2024)

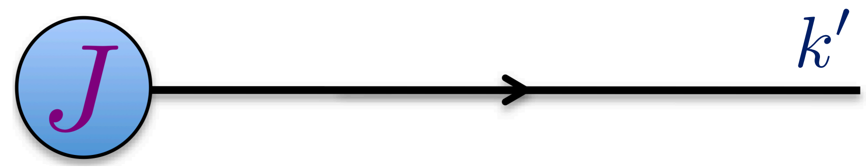


- charged lepton exchanges photons with nuclear medium

SCET$_G$ formulation

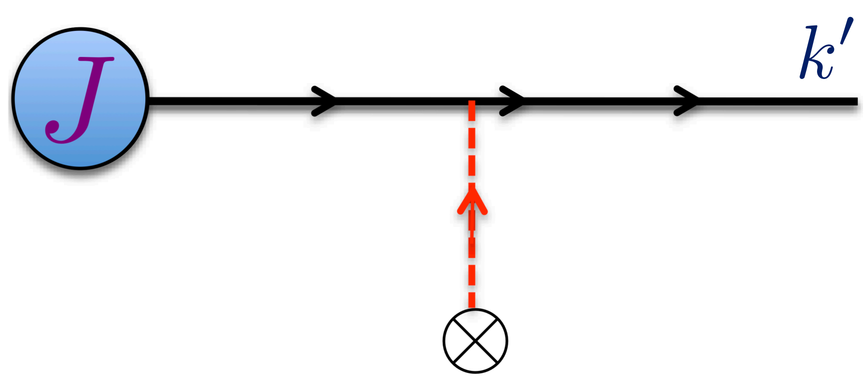
- forward scattering is dominant process
- Glauber photons exchanged with a nuclear charge distribution

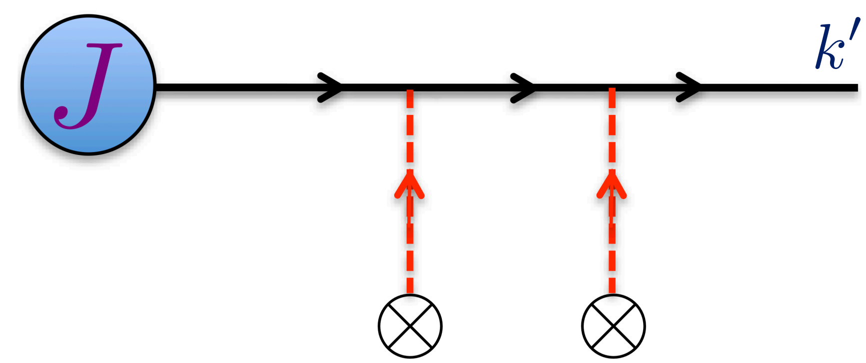
QCD: G. Ovanesyan and I. Vitev, JHEP (2011)



$q_\gamma \sim \left(\lambda, \lambda, \sqrt{\lambda}\right)$

$k' \sim \left(\lambda, 1, \sqrt{\lambda}\right)$





- change: integral along final lepton direction over charge and potential

$$\delta\sigma_f \sim \int_{\text{lepton line}}^{\text{final}} \rho(z)\,\mathrm{d}z \int \frac{\mathrm{d}^2\vec{q}_\perp}{(2\pi)^2} |v(\vec{q}_\perp)|^2 \left(\sigma_0\left(\vec{k}, \vec{k}' - \vec{q}_\perp\right) - \sigma_0\left(\vec{k}, \vec{k}'\right)\right)$$

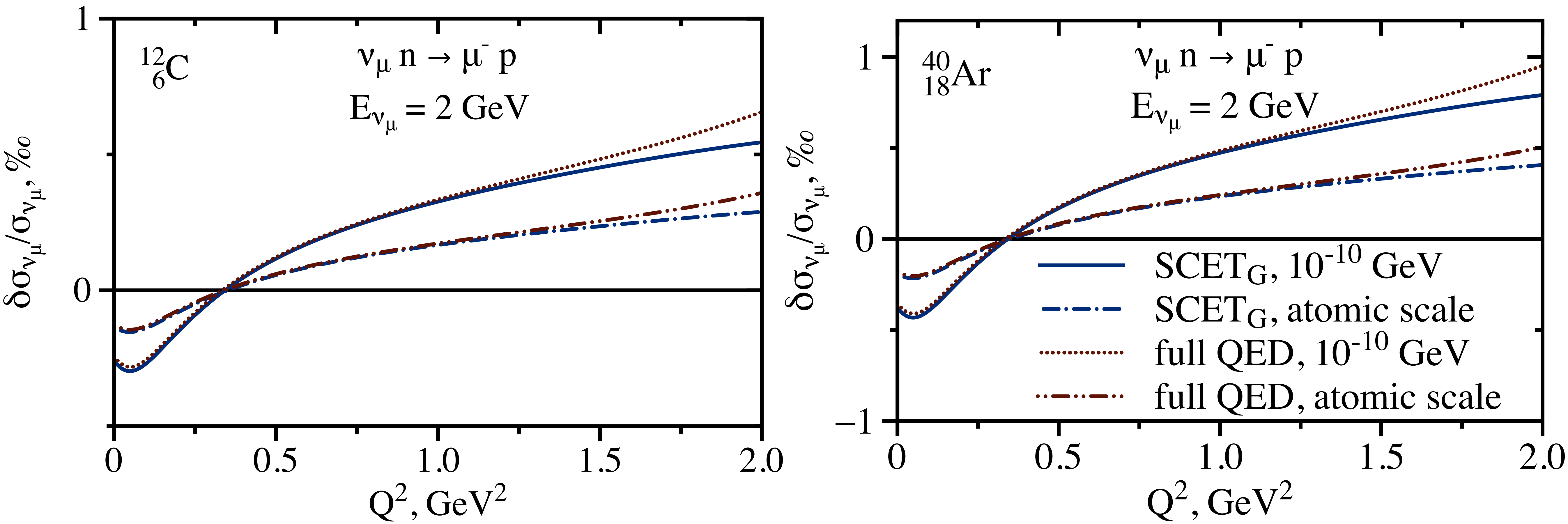
- leading-order cross sections are distorted
- EFT and full QED calculations are performed

Neutrino scattering

- relative correction per nucleon

IR regularization

$$v\left(q_\perp^2\right) = \frac{e^2}{q_\perp^2 + \lambda^2}$$



flavor-independent at GeV energies

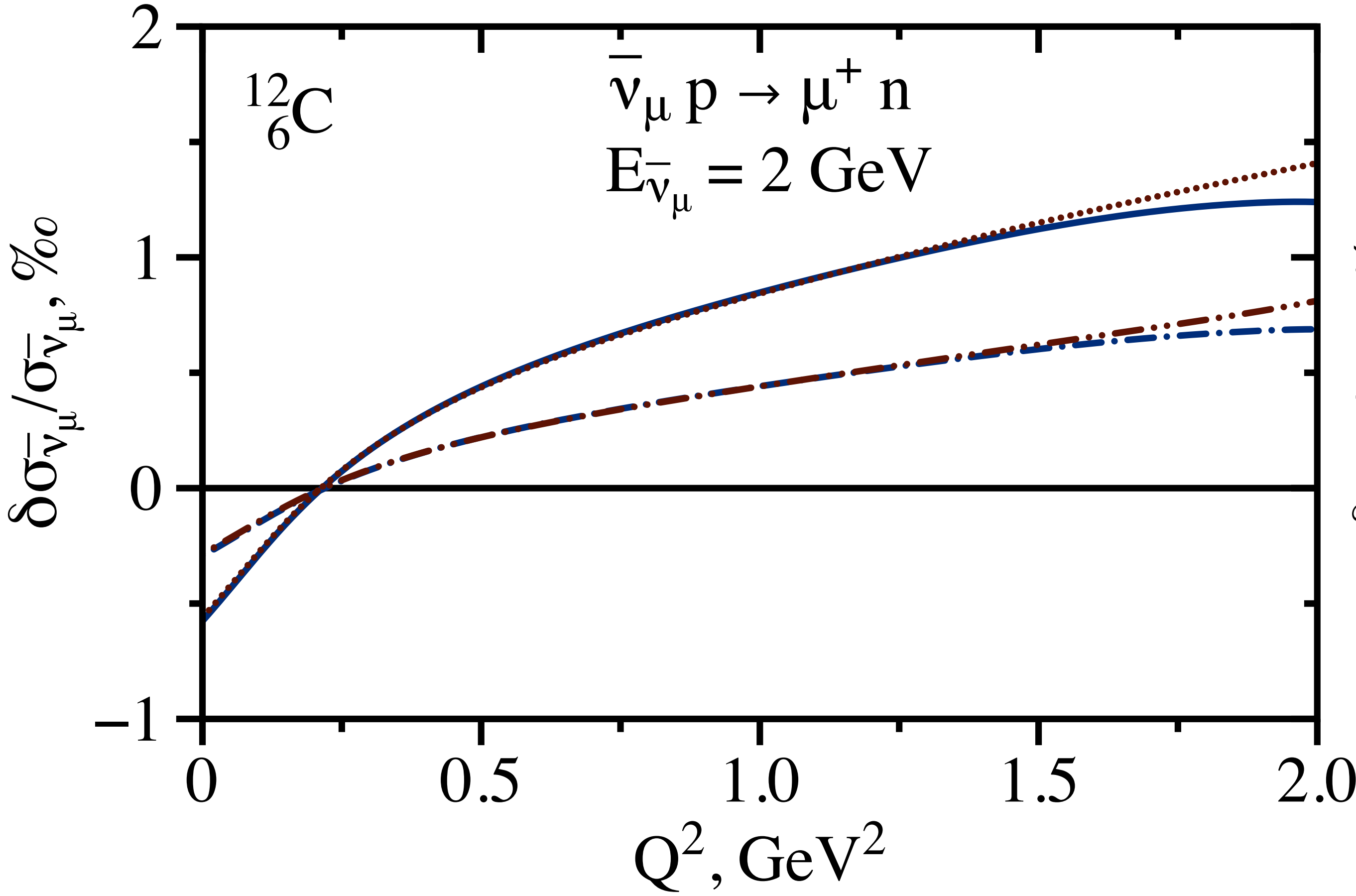
- permille-level distortion of cross sections: $O(\alpha^2)$ correction
- smaller correction to inclusive cross section

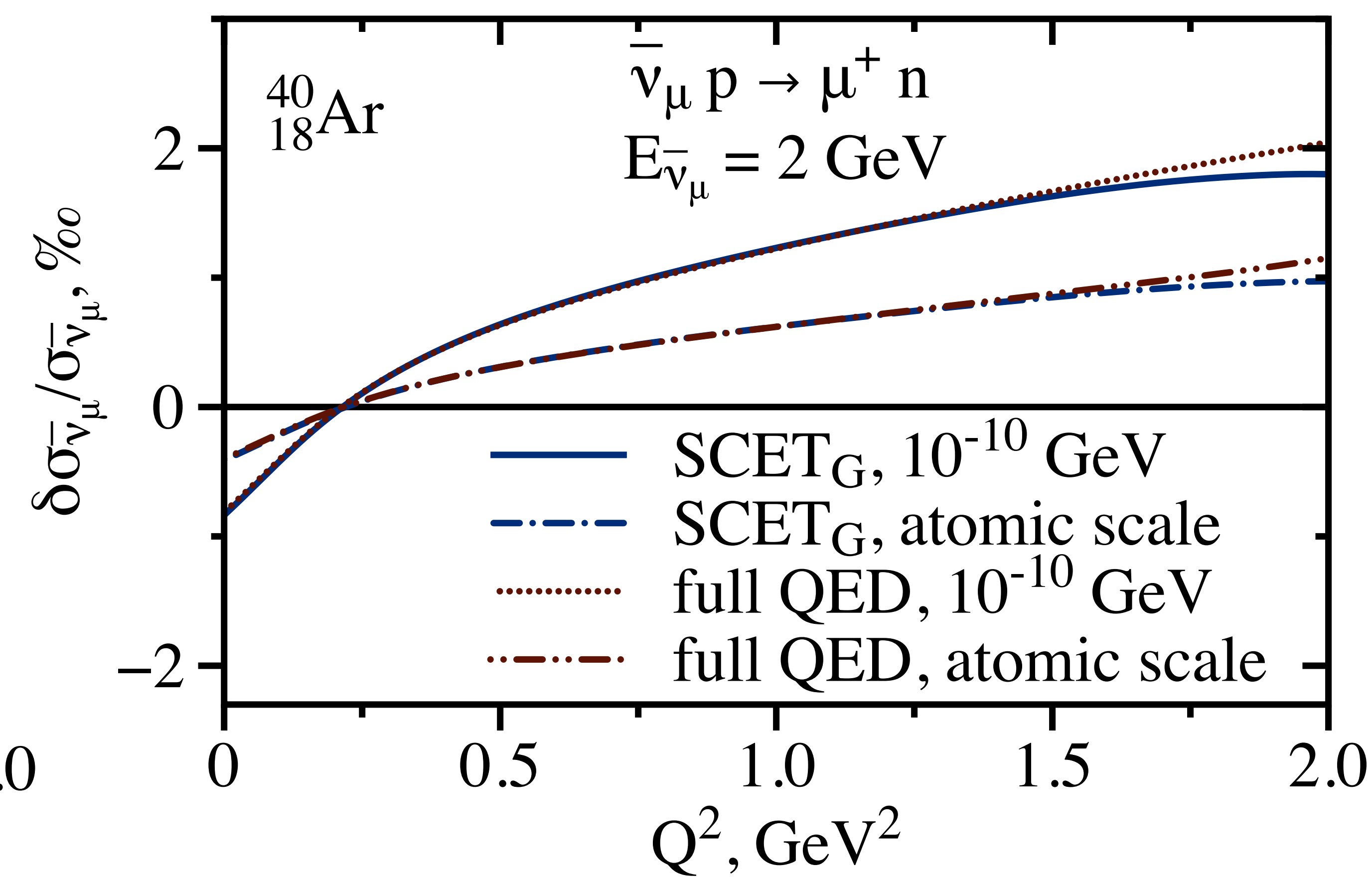
Antineutrino scattering

- relative correction per nucleon

IR regularization

$$v\left(q_\perp^2\right) = \frac{e^2}{q_\perp^2 + \lambda^2}$$





flavor-independent at GeV energies

- permille-level distortion of cross sections: $O(\alpha^2)$ correction
- larger correction than for neutrino scattering

SCET$_G$ formulation

- forward scattering is dominant process

- Glauber photons exchanged with a nuclear charge distribution

- add initial-state exchanges, no interference with final-state exchanges

- change: integral along initial lepton direction over charge and potential

$$\delta\sigma_i \sim \int_{\text{lepton line}}^{\text{initial}} \rho(z)\,\mathrm{d}z \int \frac{\mathrm{d}^2\vec{q}_\perp}{(2\pi)^2} |v(\vec{q}_\perp)|^2 \left(\sigma_0\left(\vec{k}+\vec{q}_\perp, \vec{k}'\right) - \sigma_0\left(\vec{k}, \vec{k}'\right)\right)$$

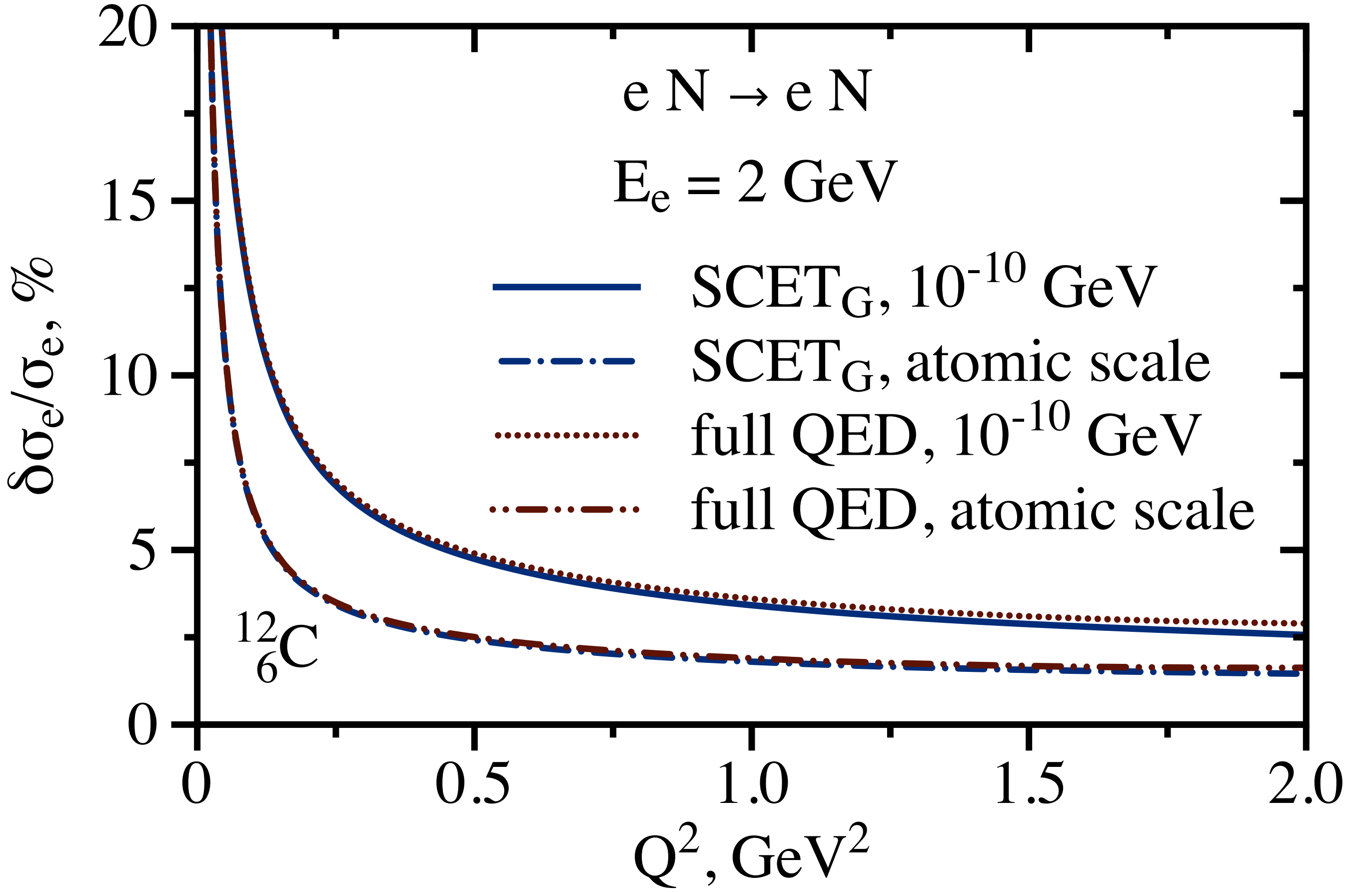
- change: integral along final lepton direction over charge and potential

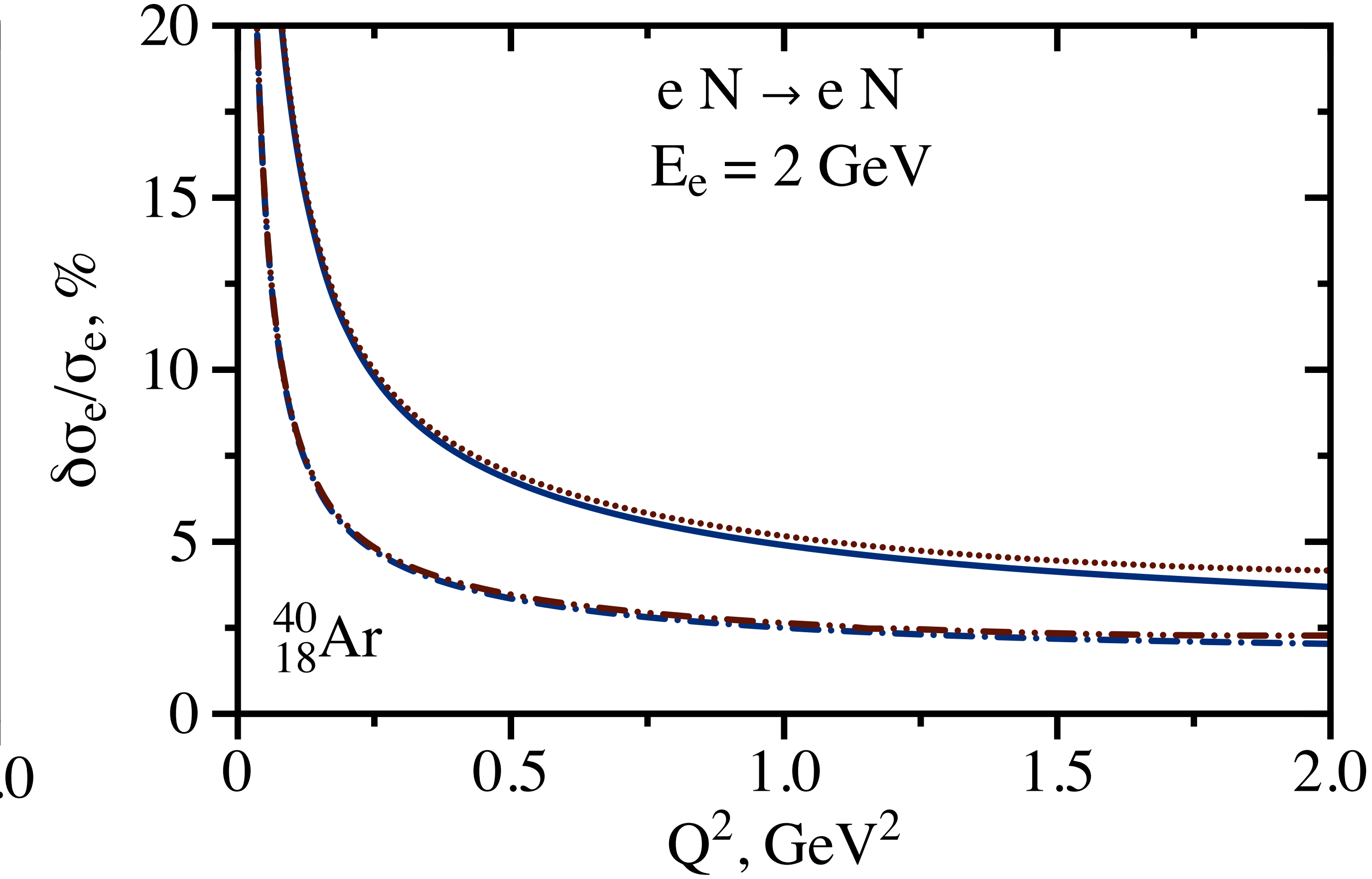
$$\delta\sigma_f \sim \int_{\text{lepton line}}^{\text{final}} \rho(z)\,\mathrm{d}z \int \frac{\mathrm{d}^2\vec{q}_\perp}{(2\pi)^2} |v(\vec{q}_\perp)|^2 \left(\sigma_0\left(\vec{k}, \vec{k}'-\vec{q}_\perp\right) - \sigma_0\left(\vec{k}, \vec{k}'\right)\right)$$

- leading-order cross sections are distorted
- EFT and full QED agree above the lepton mass scale

Electron scattering

- relative correction per nucleus after incoherent sum over nucleons

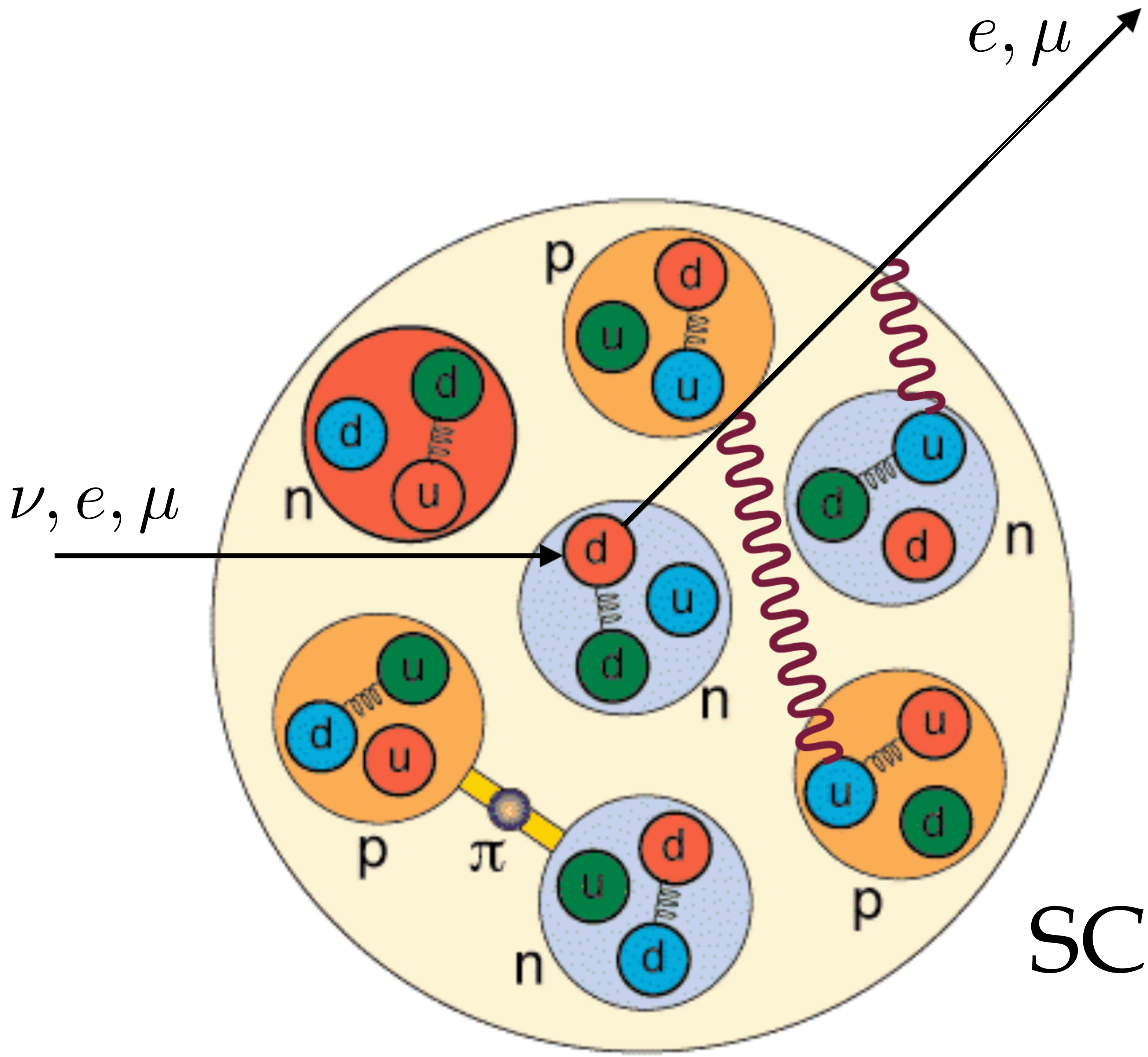




O. T. and Ivan Vitev, Phys. Lett. B 805, 135466 (2022)

- percent-level at low momentum transfers: $O(\alpha^2)$ correction
- critical new effect for electron scattering experiments

QED medium effects

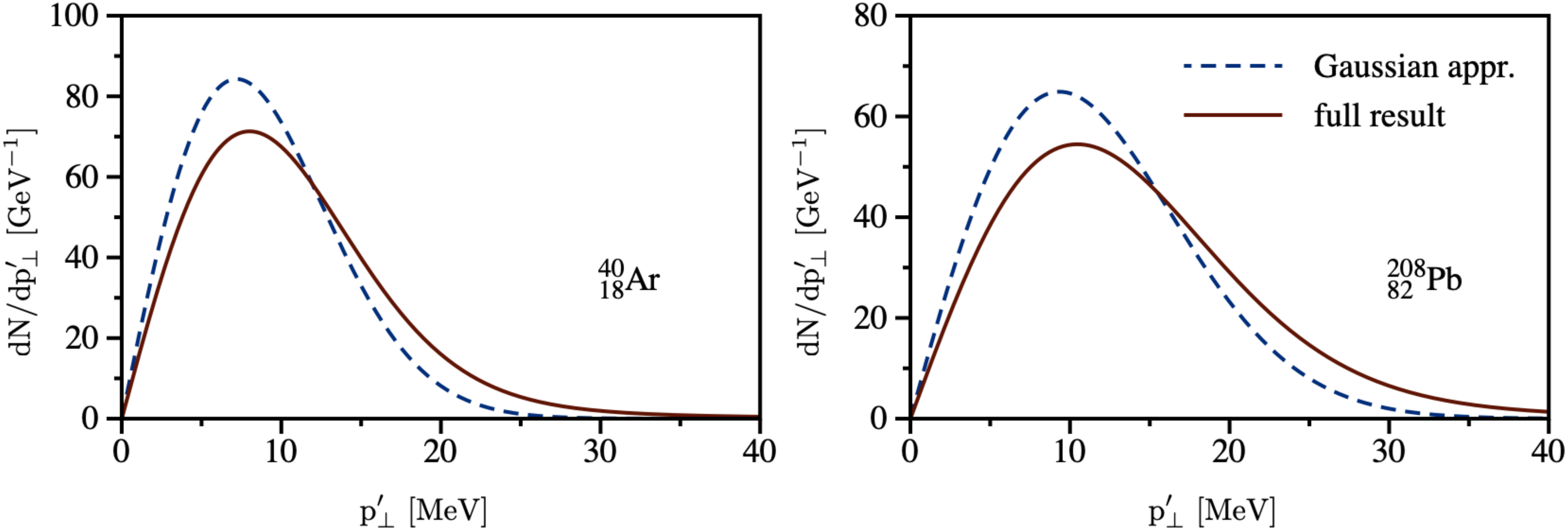


SCET$_G$ only

- >10000 interactions along the lepton trajectory resumed

Broadening of electron tracks

- multiple re-scattering generates transverse momentum



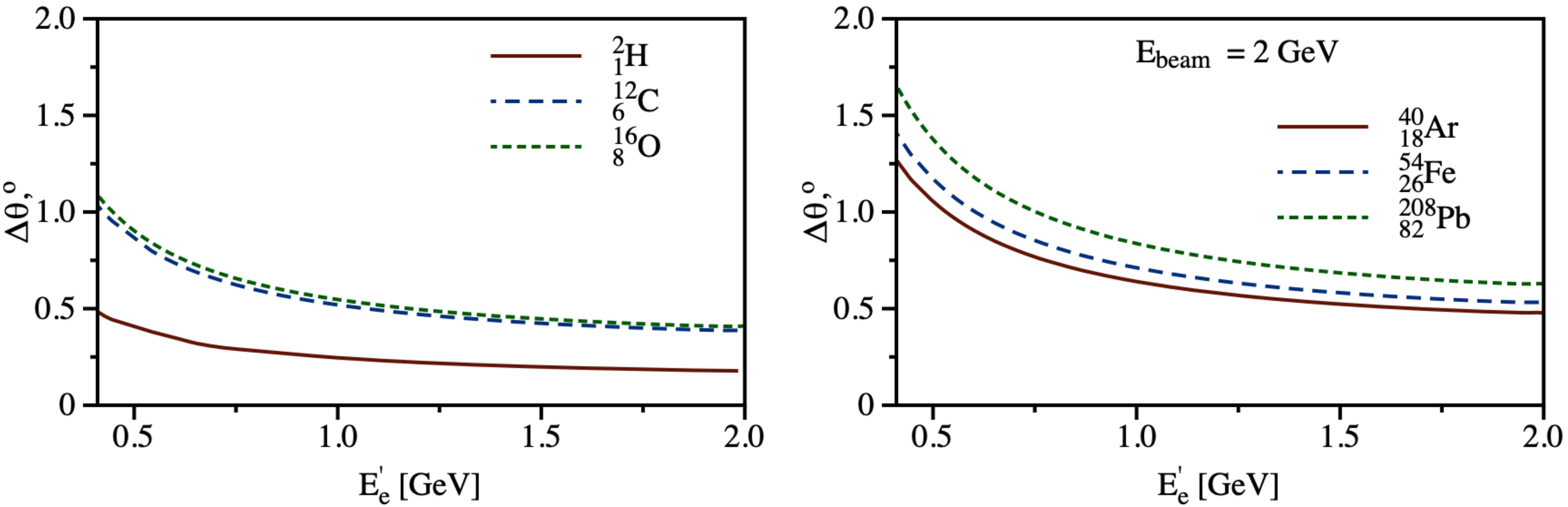
O. T. and Ivan Vitev, Phys. Rev. D 108 9, 9 (2023)

- exact resummation vs Gaussian approximation: nuclear size scale

- Glauber exchange induces 10-30 MeV transverse momentum

Broadening of electron tracks

- r. m. s. deflection angle after multiple rescattering



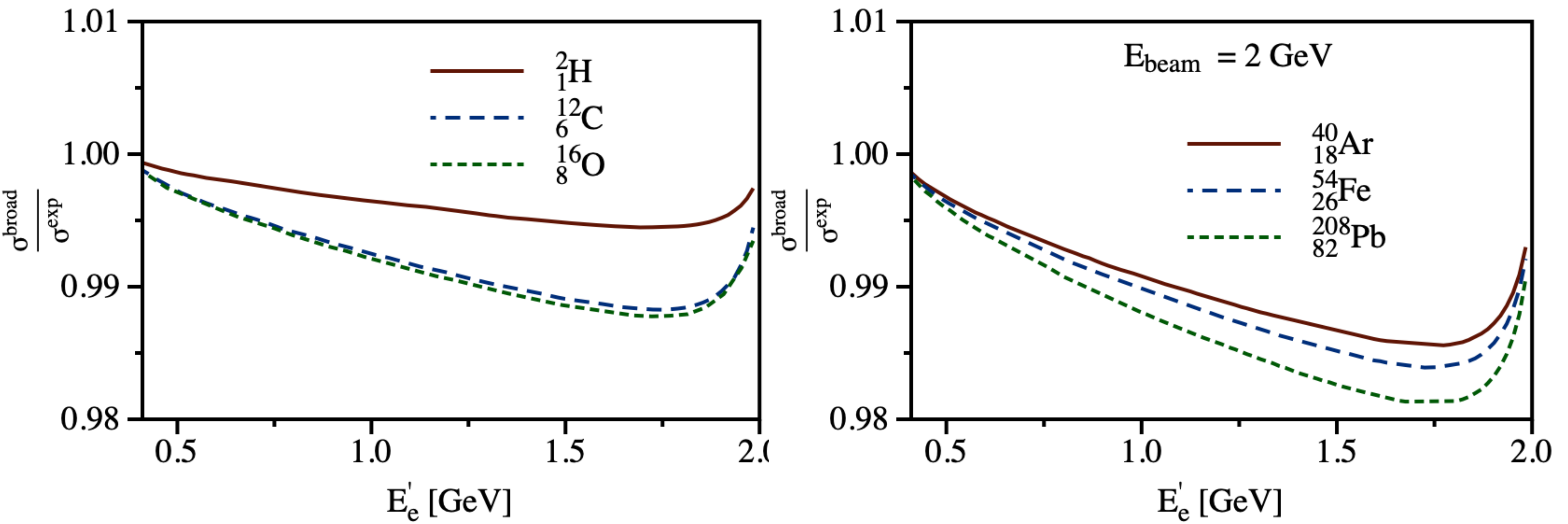
- nucleus approximated as sphere of constant density

- sizable deflection of electron tracks $\sqrt{< (\Delta\theta)^2 >} \sim 1/E$

Effect on unpolarized cross section

- initial and final re-scattering is taken into account

- momentum transfer from electron kinematics



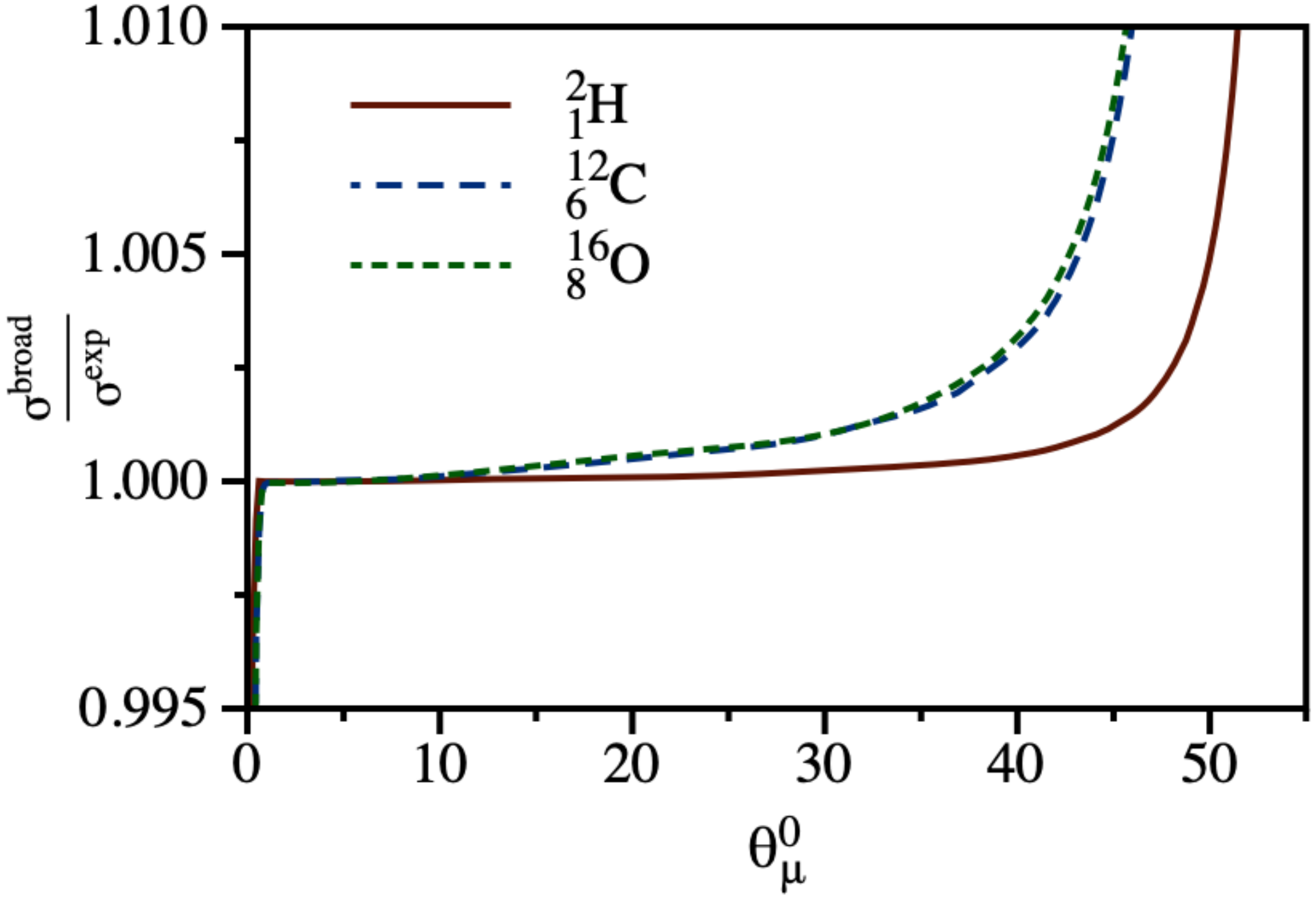
- nucleus approximated as sphere of constant density

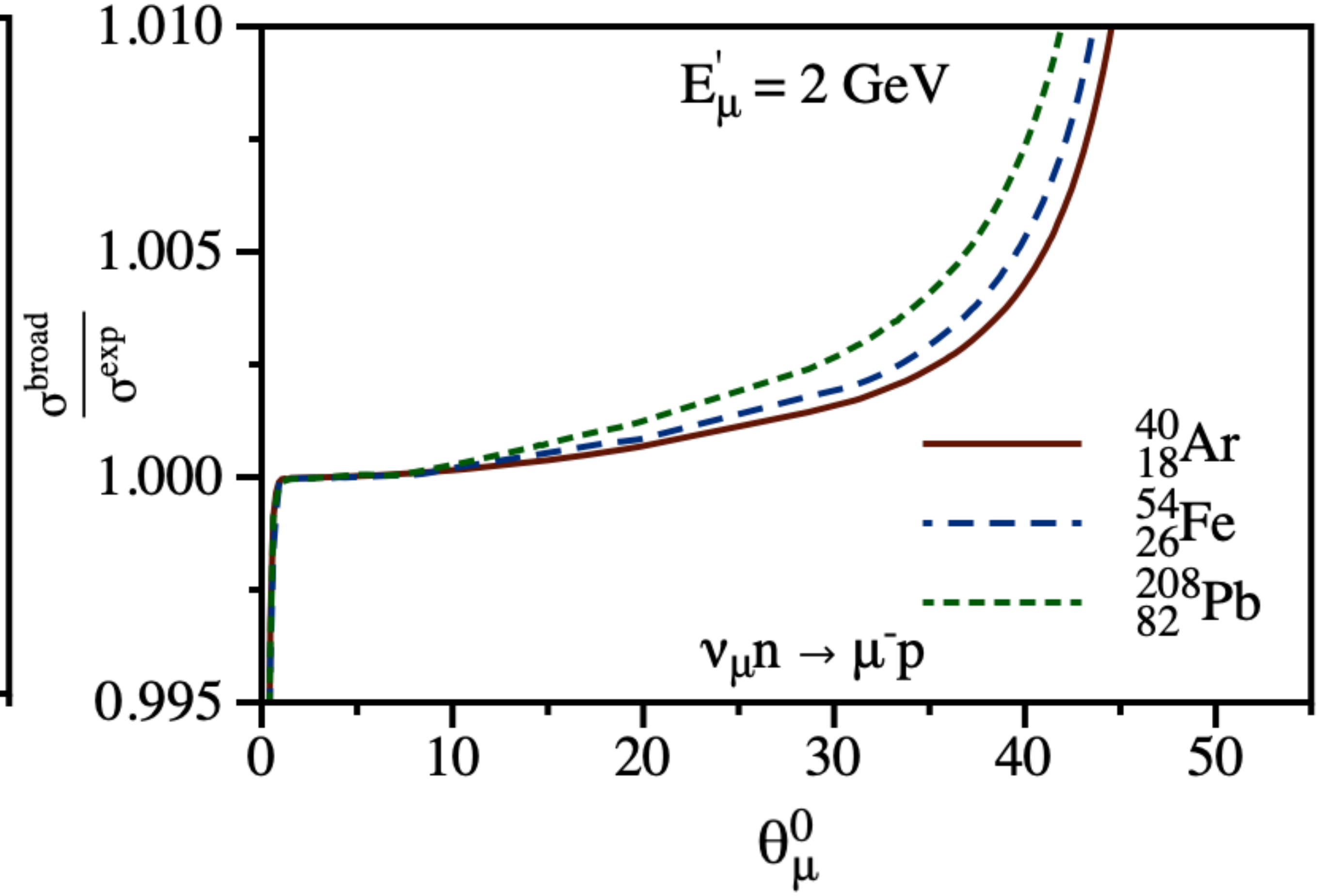
- percent-level electron-nucleus cross-section suppression

Effect on unpolarized cross section

- only final re-scattering present

$$E_\nu^r \approx \frac{E'_\mu - \frac{1}{2}\frac{E_B^2 - 2M_i E_B + m_\mu^2 + M_i^2 - M_f^2}{M_i - E_B}}{1 - \frac{E'_\mu}{M_i - E_B}\left(1 - \beta_\mu \cos\theta_\mu\right)}$$





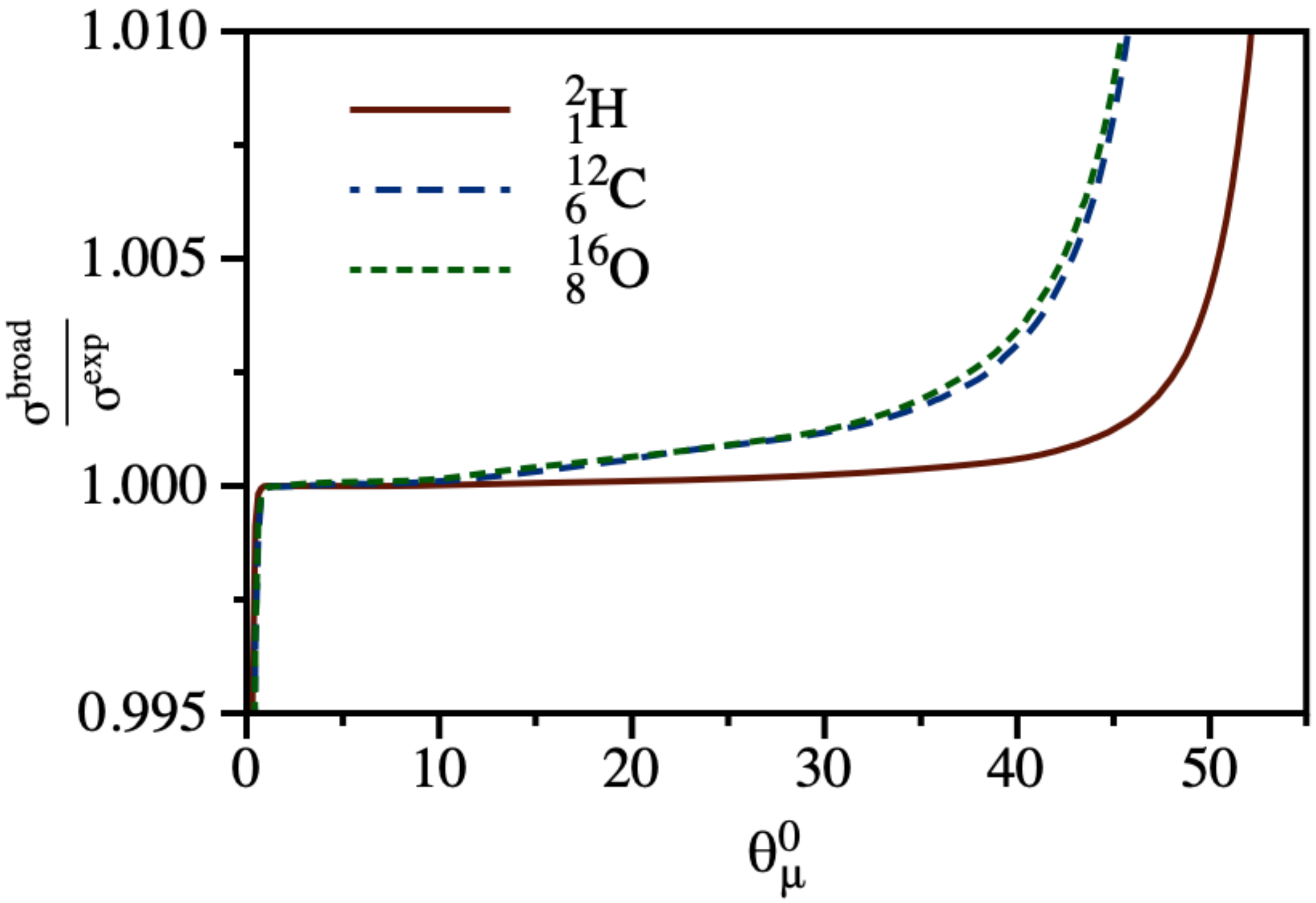
- nucleus approximated as sphere of constant density

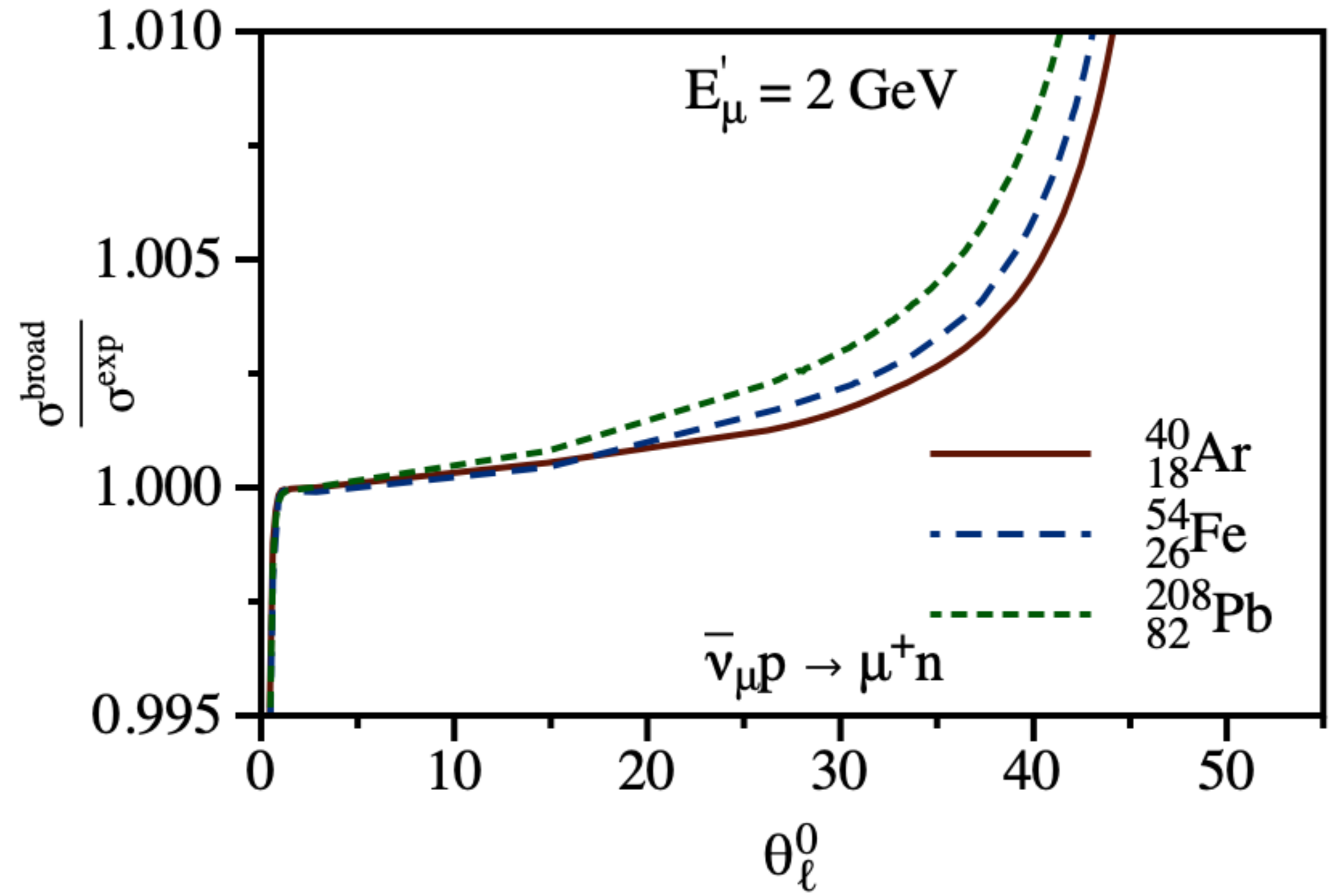
- neutrino-nucleus: percent-level at kinematic endpoints

Effect on unpolarized cross section

- only final re-scattering present

$$E_\nu^r \approx \frac{E'_\mu - \frac{1}{2}\frac{E_B^2 - 2M_i E_B + m_\mu^2 + M_i^2 - M_f^2}{M_i - E_B}}{1 - \frac{E'_\mu}{M_i - E_B}\left(1 - \beta_\mu \cos\theta_\mu\right)}$$

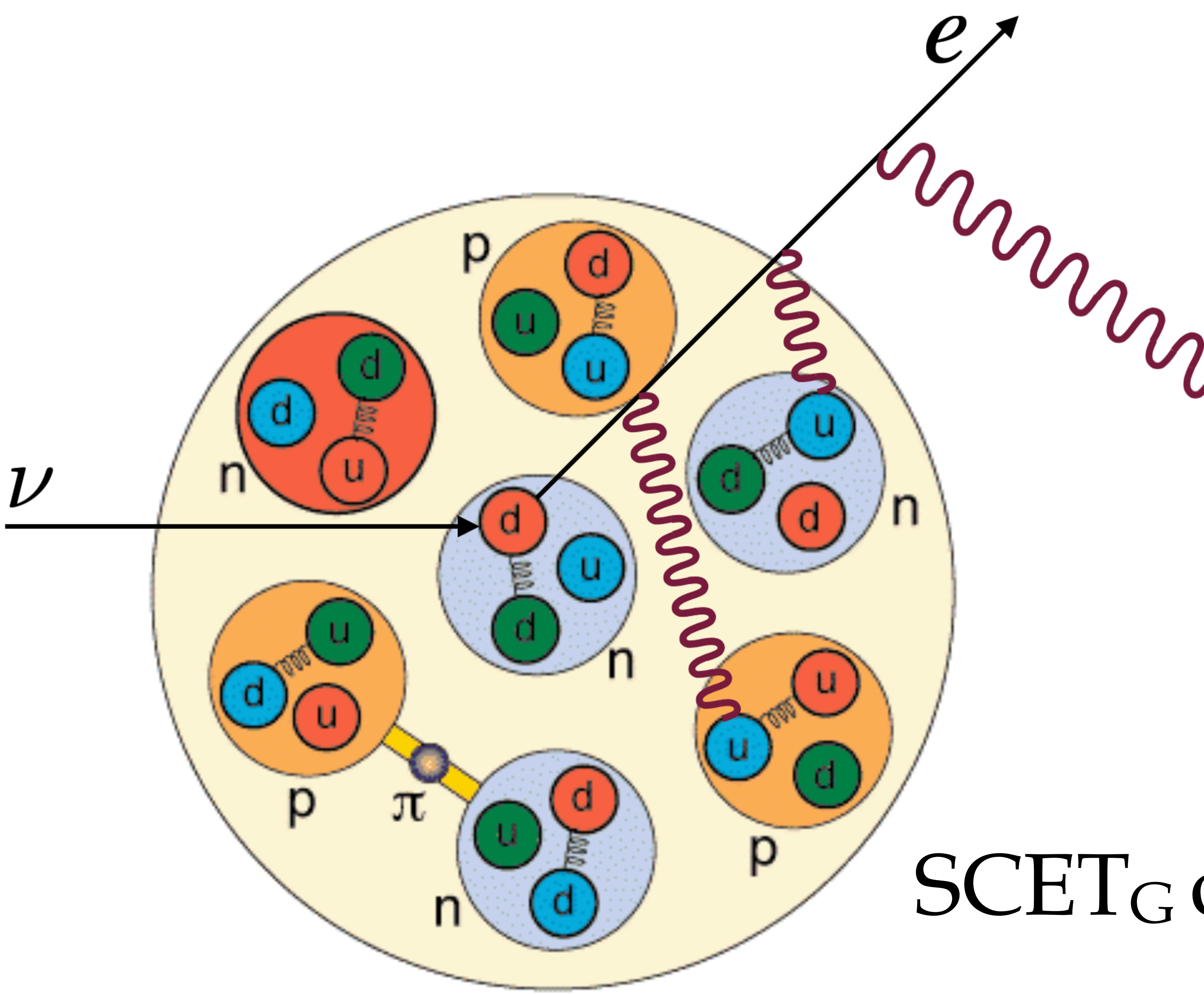




- nucleus approximated as sphere of constant density

- antineutrino-nucleus: percent-level at kinematic endpoints

QED medium-induced radiation



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- >10000 interactions along the lepton trajectory resumed

QED medium-induced radiation

broadening with radiation: p_T spectrum is multiplied with soft (collinear) function in vacuum for observables including soft (collinear) photons

soft (collinear) functions in vacuum:

O. T., Qing Chen, Richard J. Hill and Kevin S. McFarland, Nature Commun. 13 (2022), 1, 5286

O. T., Qing Chen, Richard J. Hill, Kevin S. McFarland and Clarence Wret editors suggestion in Phys. Rev. D (2022)

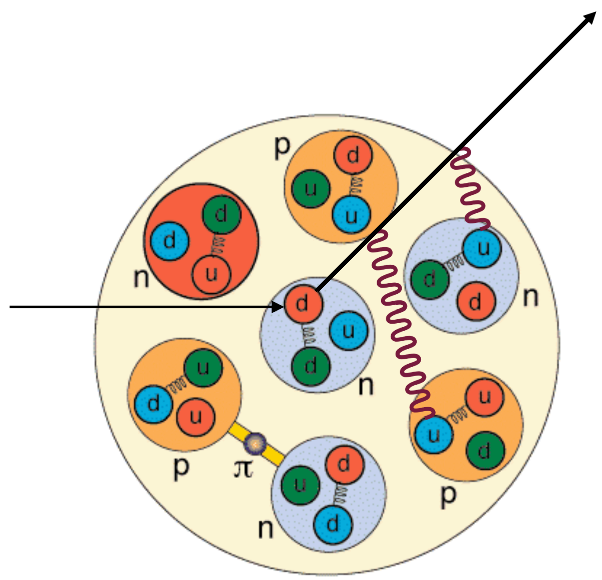
relative cross-section correction at each order in expansion on number of re-scatterings: the same for soft, collinear, and no-radiation cases

vanishing spectrum of soft or collinear medium-induced photons

O. T. and Ivan Vitev, Phys. Rev. D XXX X, X (2024)

- separation of scales: exact resummation of medium effects

Conclusions



formulation of

QED nuclear medium

effects

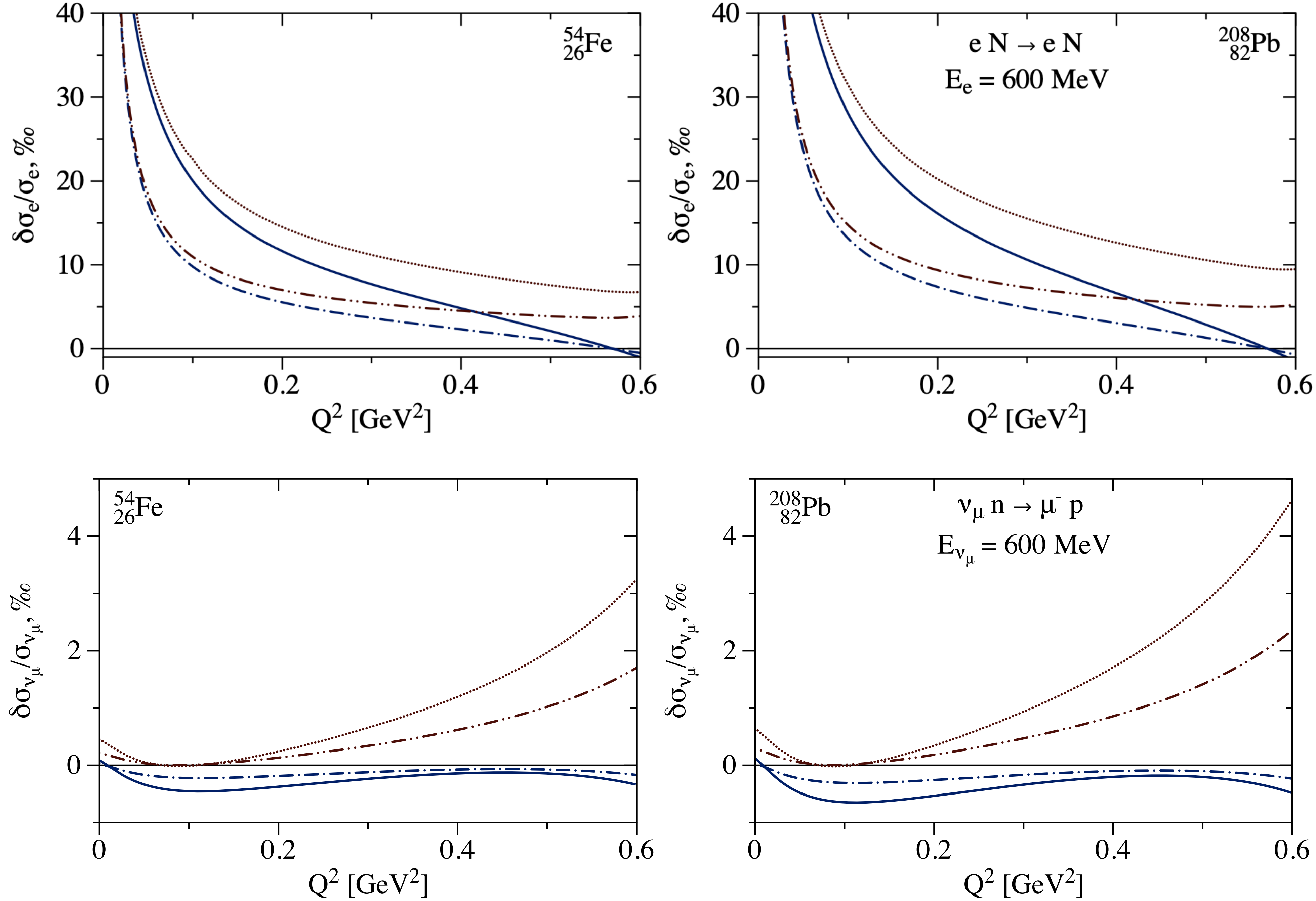
virtual corrections at 1st order in opacity: $\mathrm{SCET_G}$ and full QED

broadening and radiation: $\mathrm{SCET_G}$

verified: $\mathrm{SCET_G}$ works perfectly at GeV energies

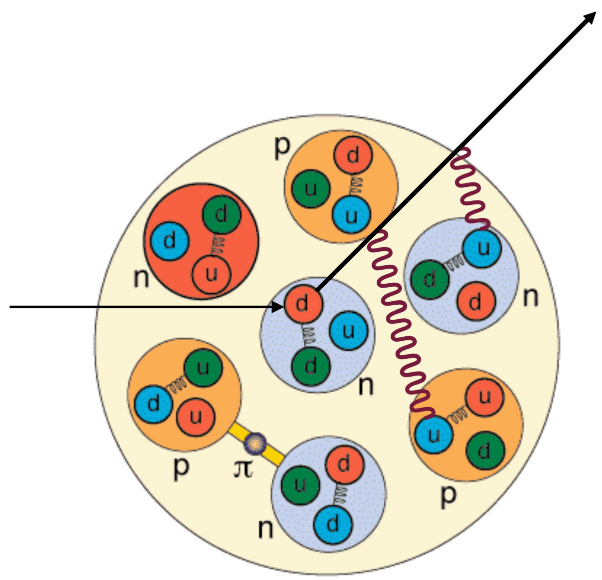
Cross sections at 600 MeV beam energy

1st order in opacity



- QED and SCET$_G$ significantly differ at 100 of MeV energy

Conclusions



formulation of

QED nuclear medium

effects

virtual corrections at 1st order in opacity: $\mathrm{SCET_G}$ and full QED

broadening and radiation: $\mathrm{SCET_G}$

verified: $\mathrm{SCET_G}$ works perfectly at GeV energies but not for 100th MeV !!!

found: a) sizable deflection of charged lepton tracks

b) multiple rescattering: ‰-level corrections at GeV energies

c) vanishing nuclear medium-induced photon energy spectra

d) radiation sizably (~10-20 %) modifies broadening

Thanks for your attention !!!