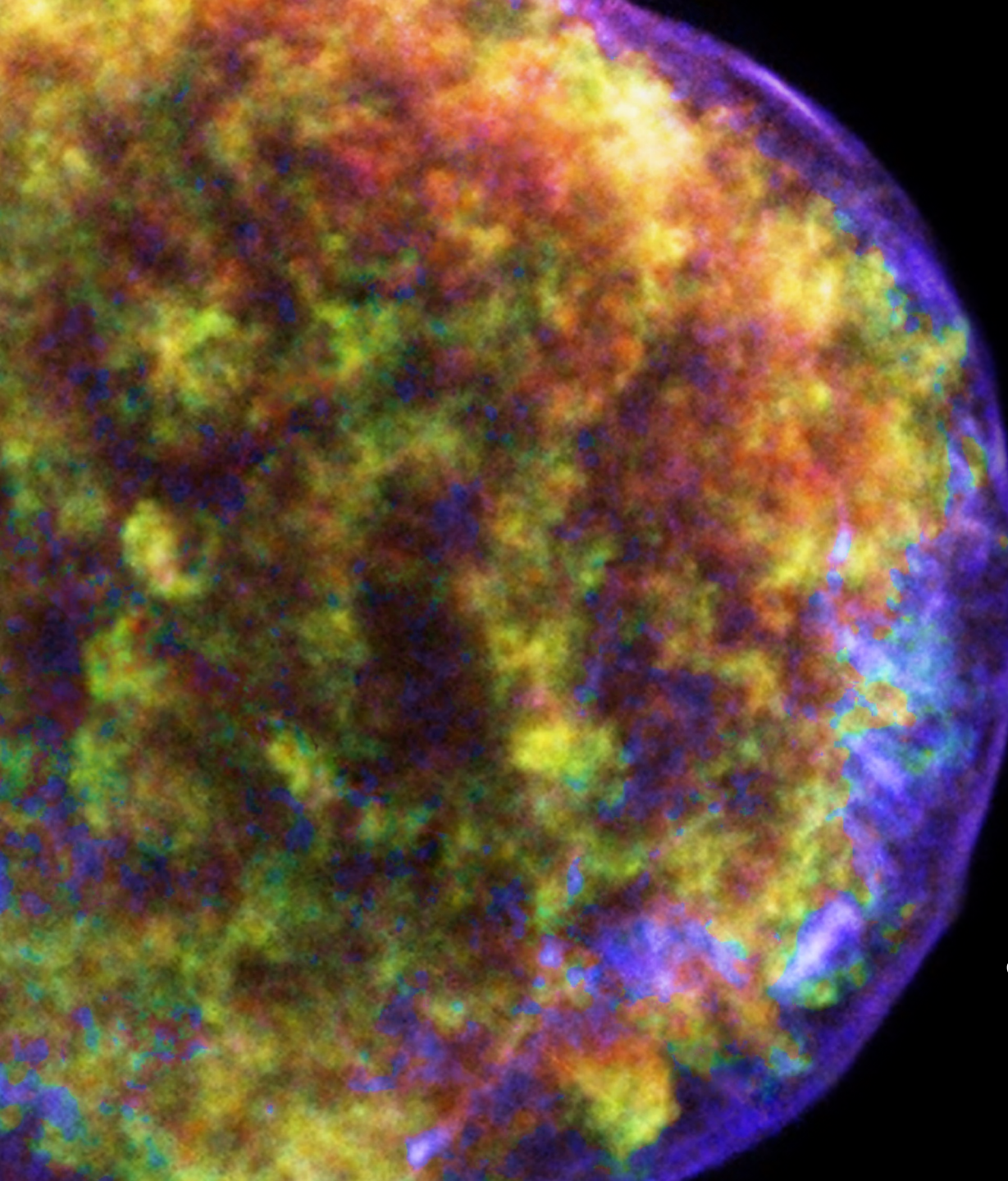


A vibrant, multi-colored image of the Tycho Supernova Remnant, showing a complex, textured surface with hues of yellow, orange, red, and purple against a black background.

The Maximum Energy of Shock- Accelerated Cosmic Rays

REBECCA DIESING

CDHY PEVATRON WORKSHOP

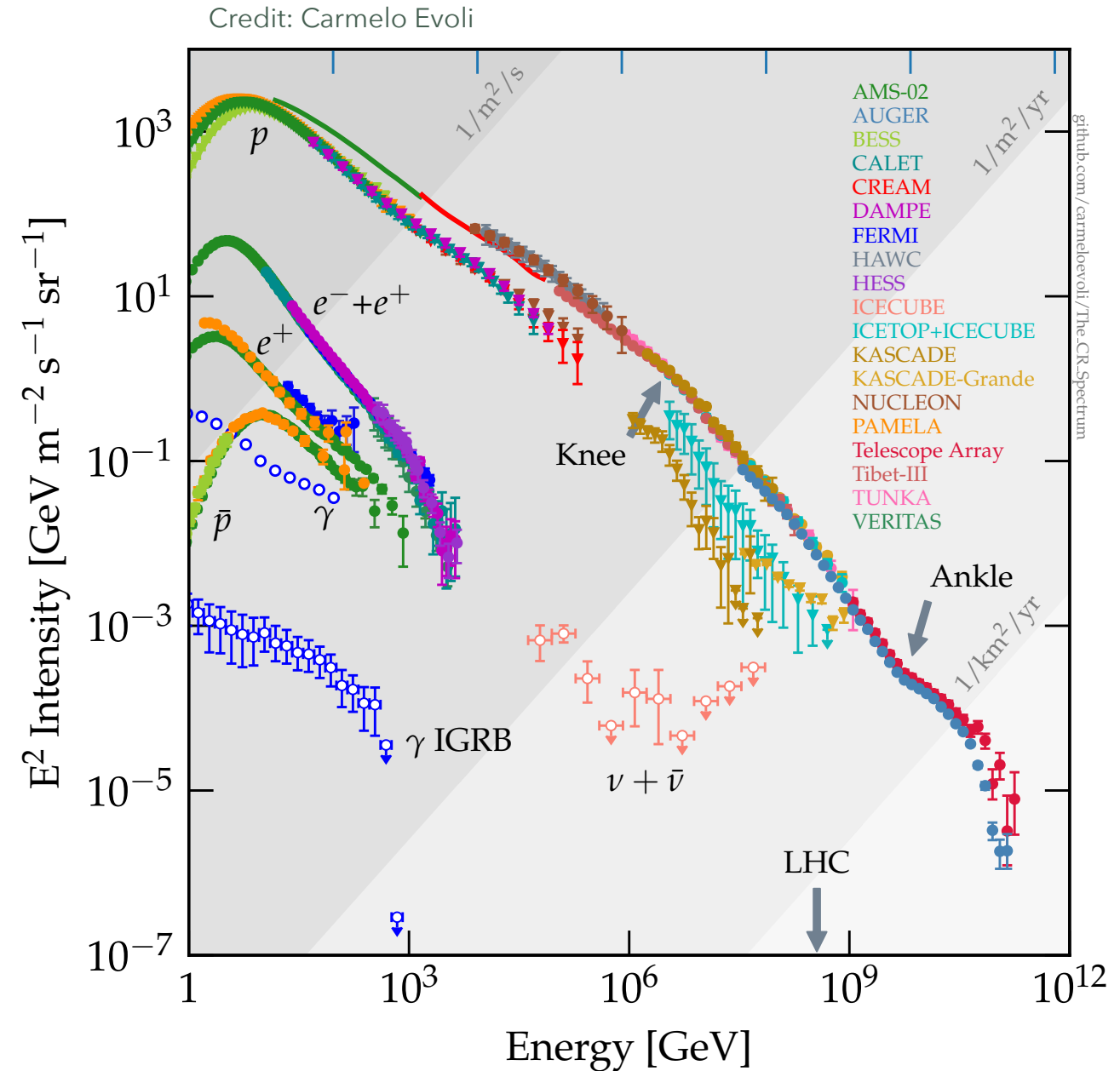
NEVIS LABS | OCT 8, 2025

 **COLUMBIA UNIVERSITY**
IN THE CITY OF NEW YORK

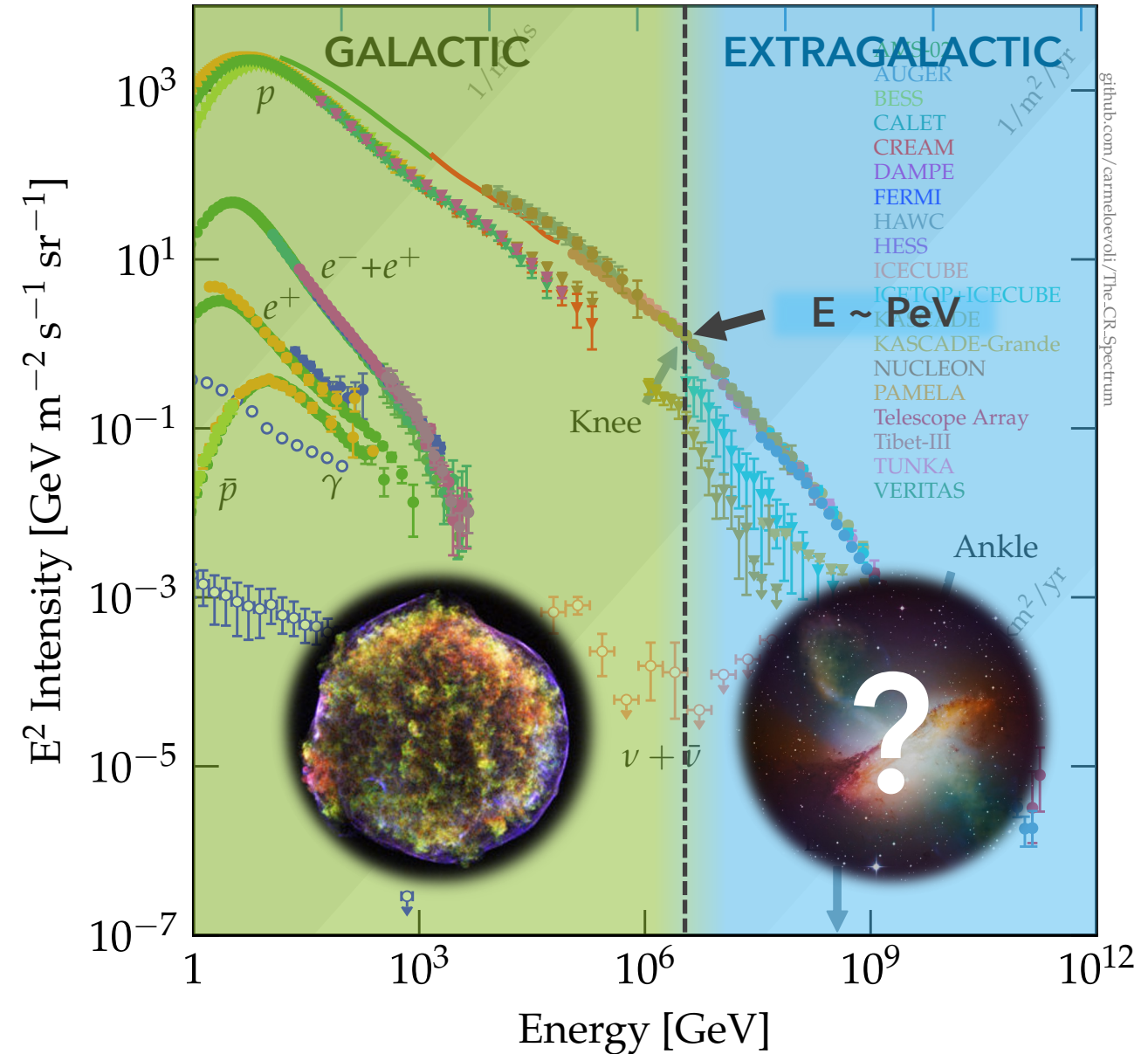
IAS | INSTITUTE FOR
ADVANCED STUDY

Image: Tycho Supernova Remnant; Credit: Chandra X-Ray Observatory

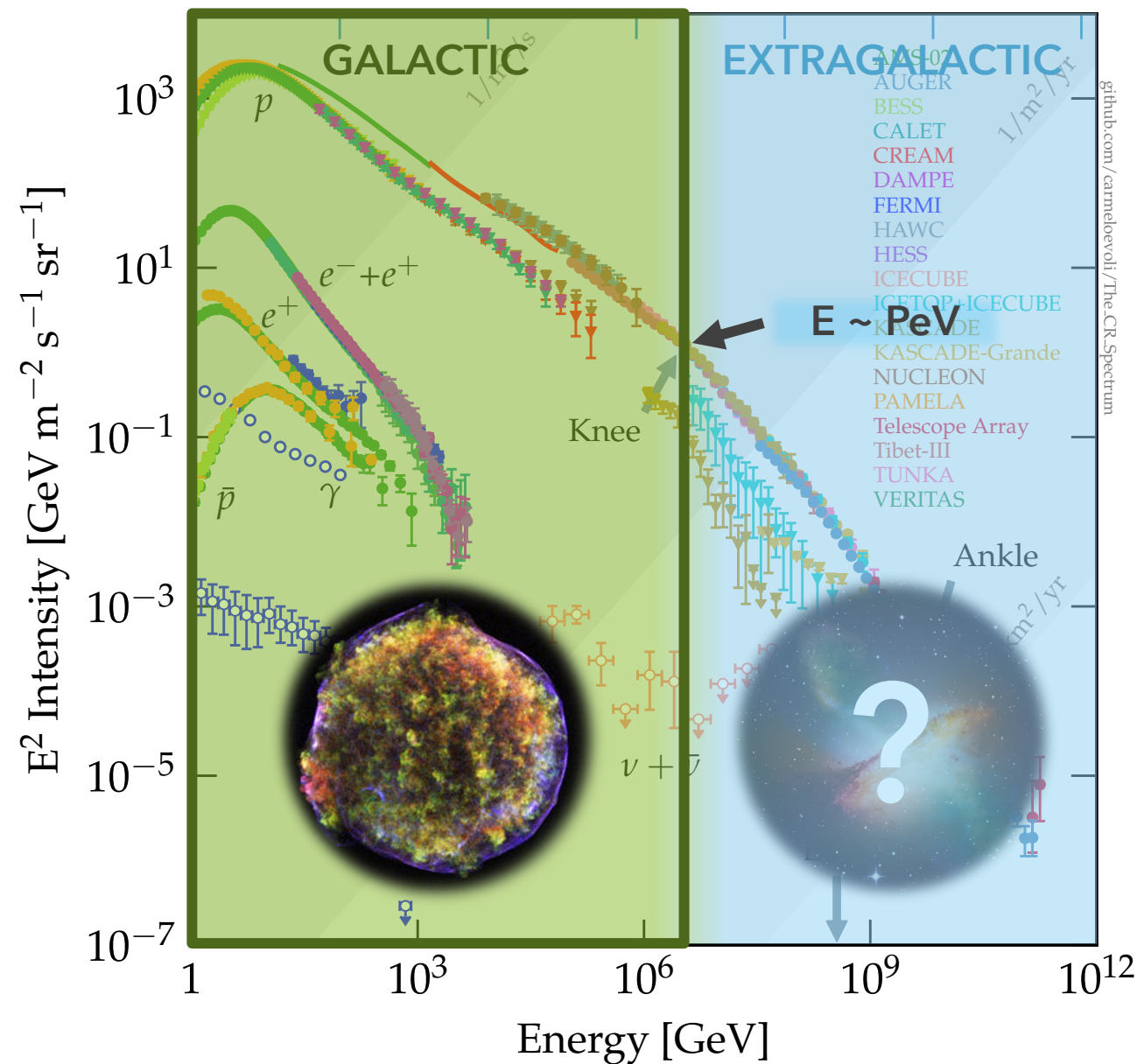
Where do cosmic rays come from?



Where do cosmic rays come from?

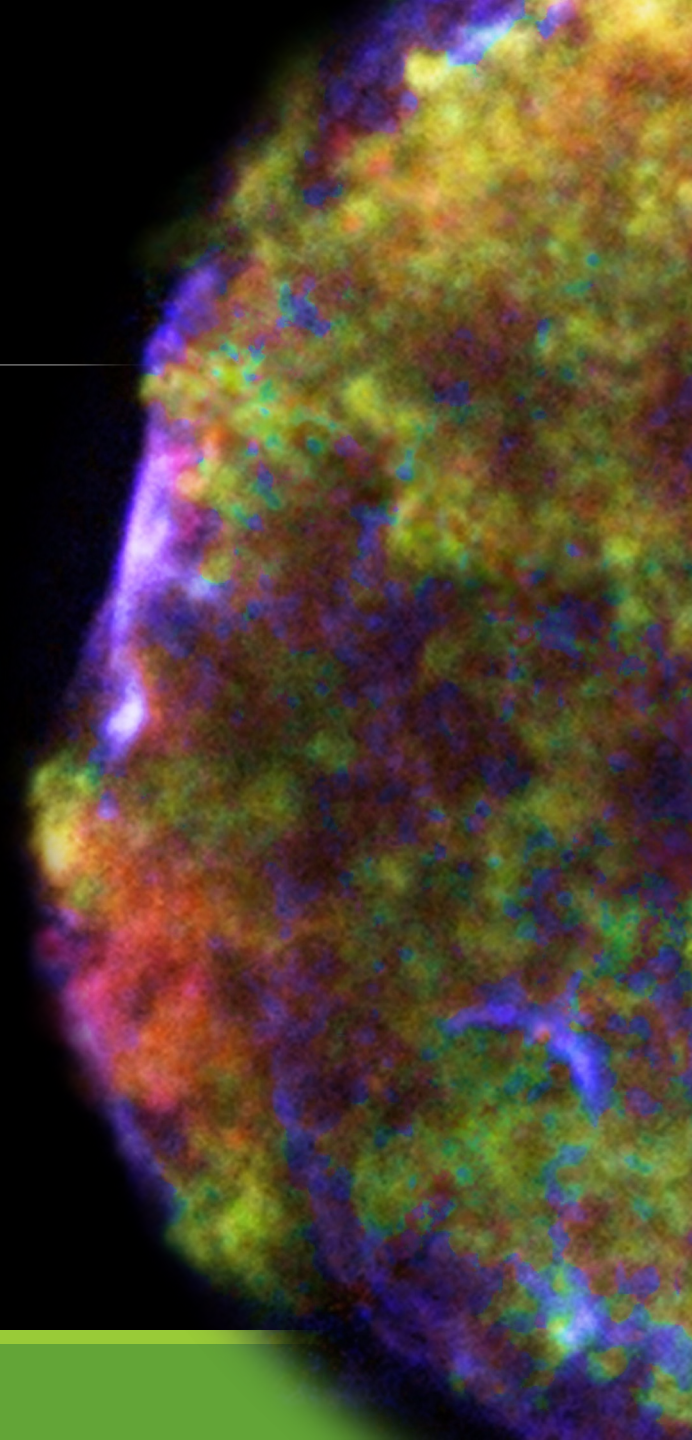


Are supernova
remnants
responsible for
all galactic
cosmic rays?



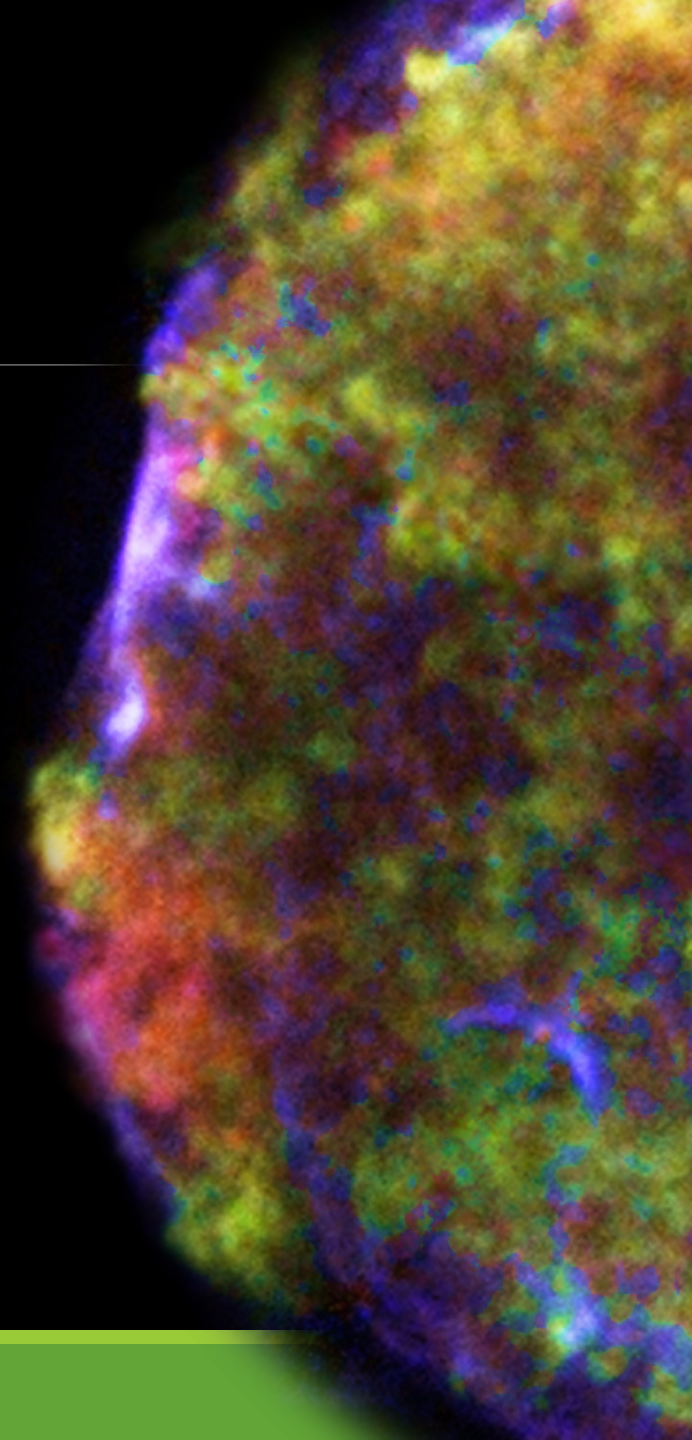
Summary

1. Can supernova remnants accelerate PeV protons?



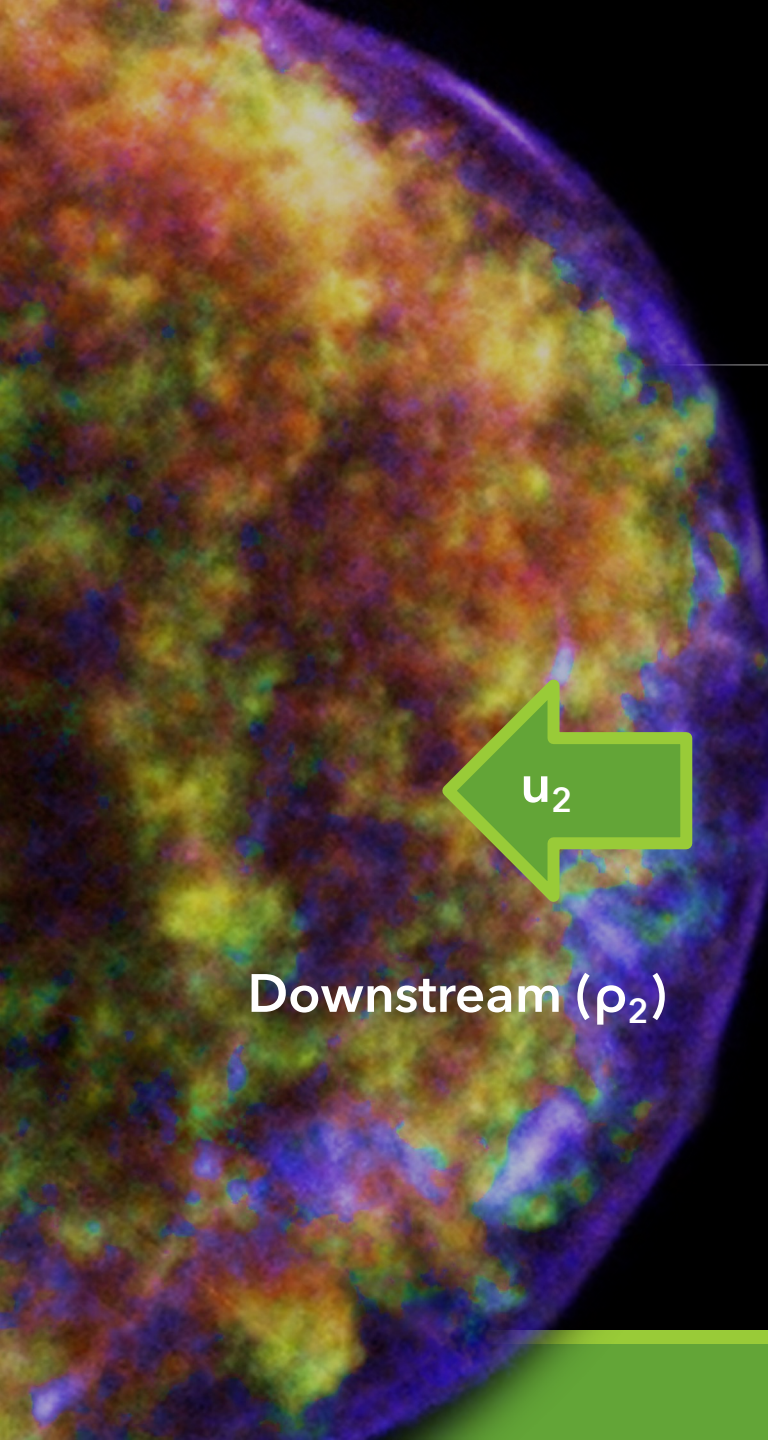
Summary

1. Can supernova remnants accelerate PeV protons?
2. Are supernova remnants the main source of PeV cosmic rays?



The standard paradigm of Galactic cosmic ray acceleration

Protons and electrons are accelerated via diffusive shock acceleration (DSA).*



u_2

Downstream (ρ_2)

u_1

Upstream (ρ_1)

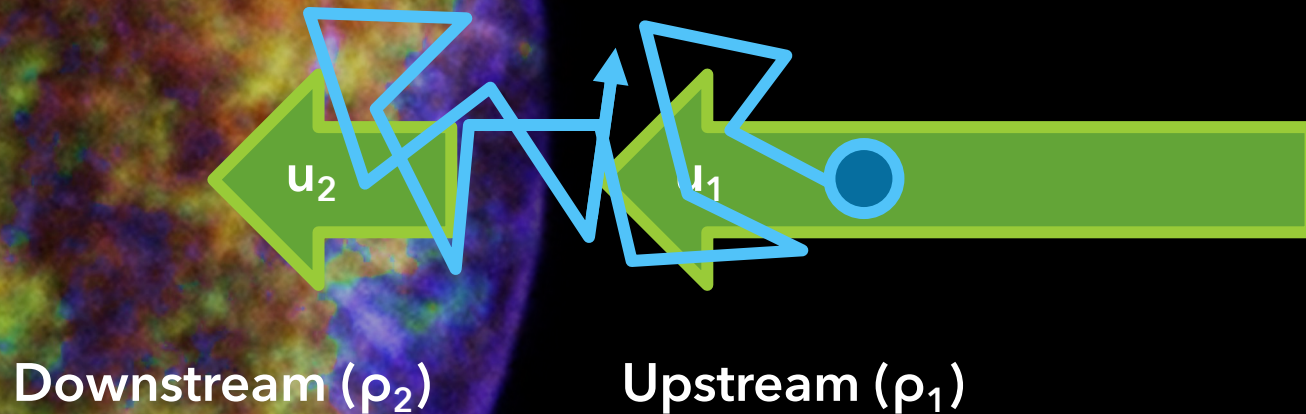
$$R \equiv \frac{\rho_2}{\rho_1} = \frac{u_1}{u_2}$$

For strong shocks, $R = 4$.

*Fermi54, Krymskii77, Axford+77, Bell78, Blandford+78

The standard paradigm of Galactic cosmic ray acceleration

Protons and electrons are accelerated via diffusive shock acceleration (DSA).*



$$R \equiv \frac{\rho_2}{\rho_1} = \frac{u_1}{u_2}$$

For strong shocks, $R = 4$.

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The standard paradigm of Galactic cosmic ray acceleration

DSA is a universal acceleration mechanism!

Supernova
remnants

Radio
supernovae

AGN lobes

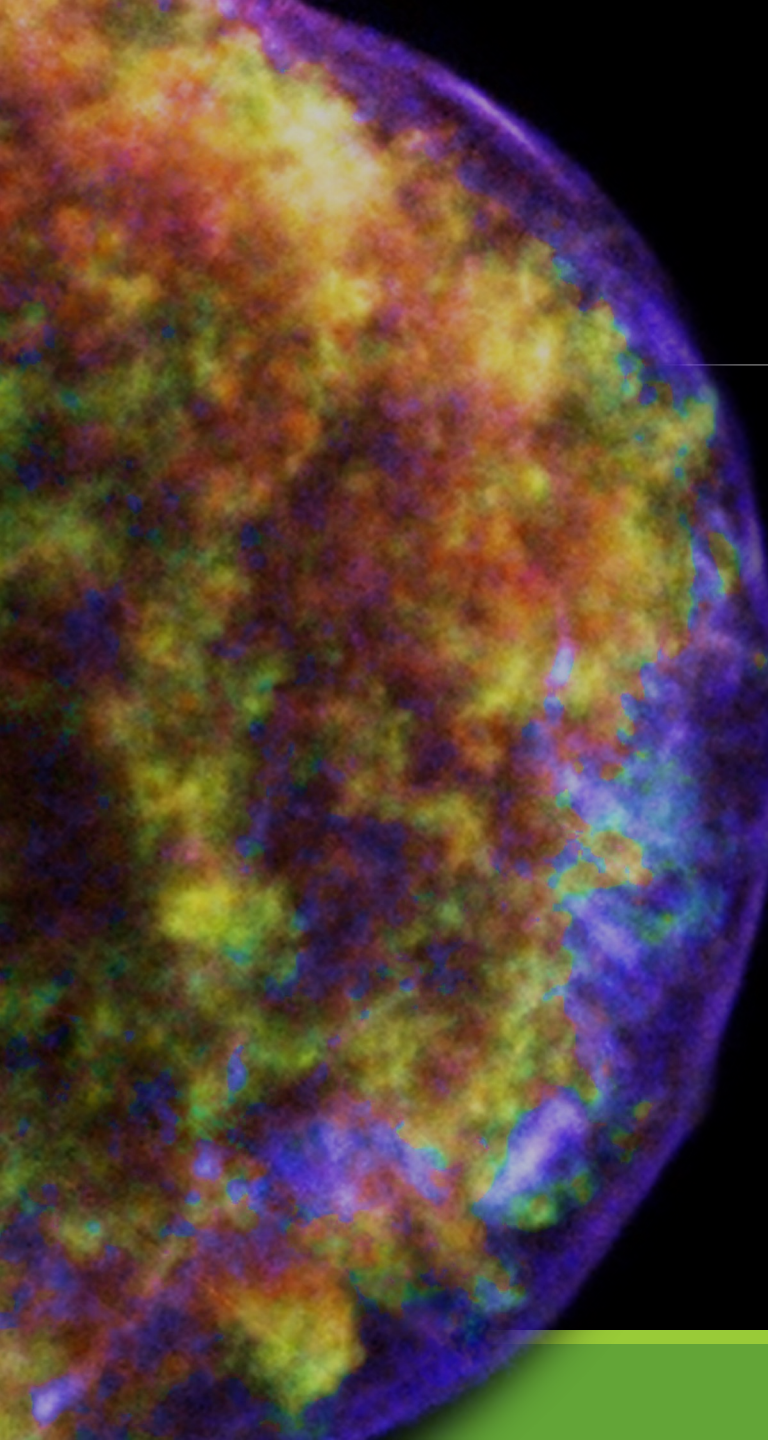
Galaxy cluster
shocks

Earth's bow
shock

Novae

Stellar bow
shocks

Interplanetary
shocks



The standard paradigm of Galactic cosmic ray acceleration

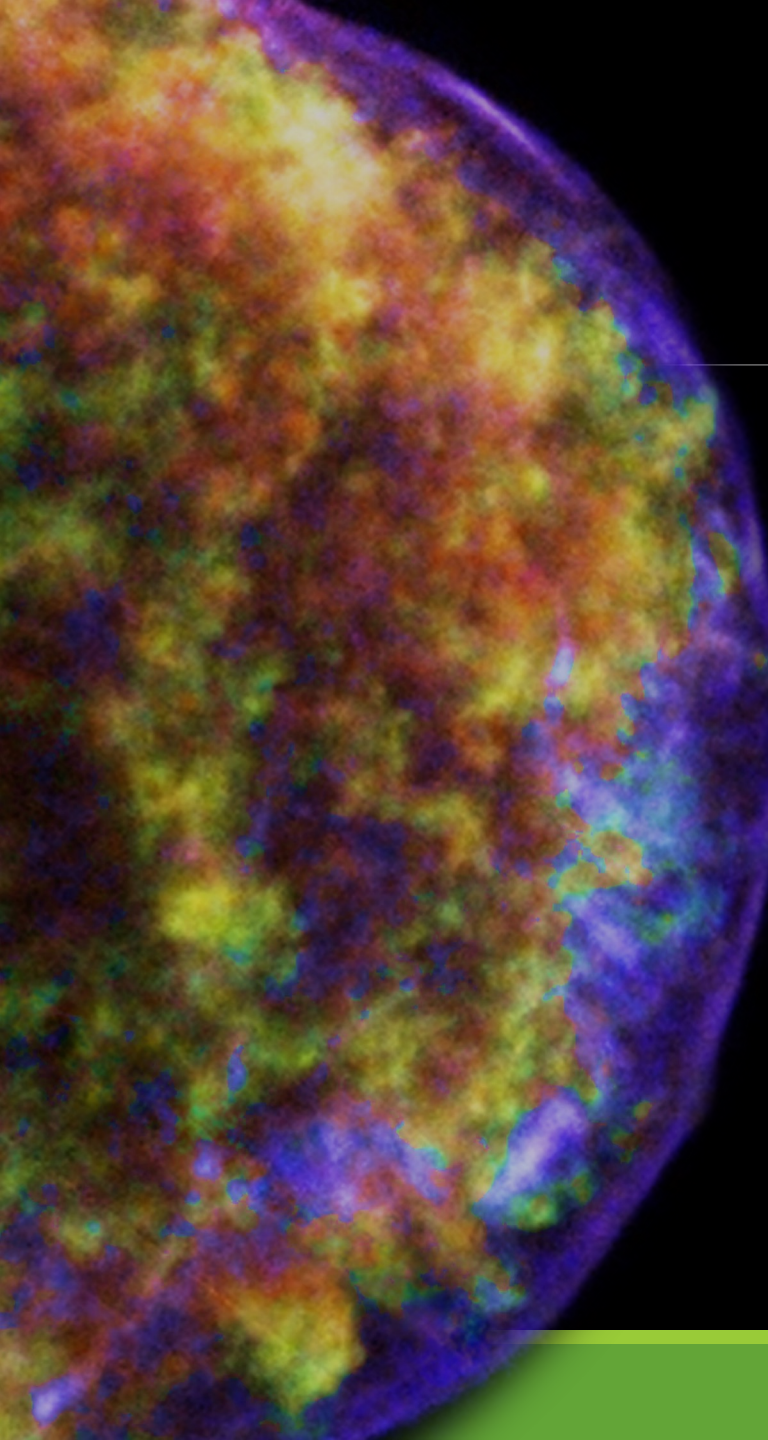
DSA predicts power law distributions of particles.*

$$\Phi \propto E^{-q}$$

$$q = \frac{R + 2}{R - 1}$$

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The standard paradigm of Galactic cosmic ray acceleration

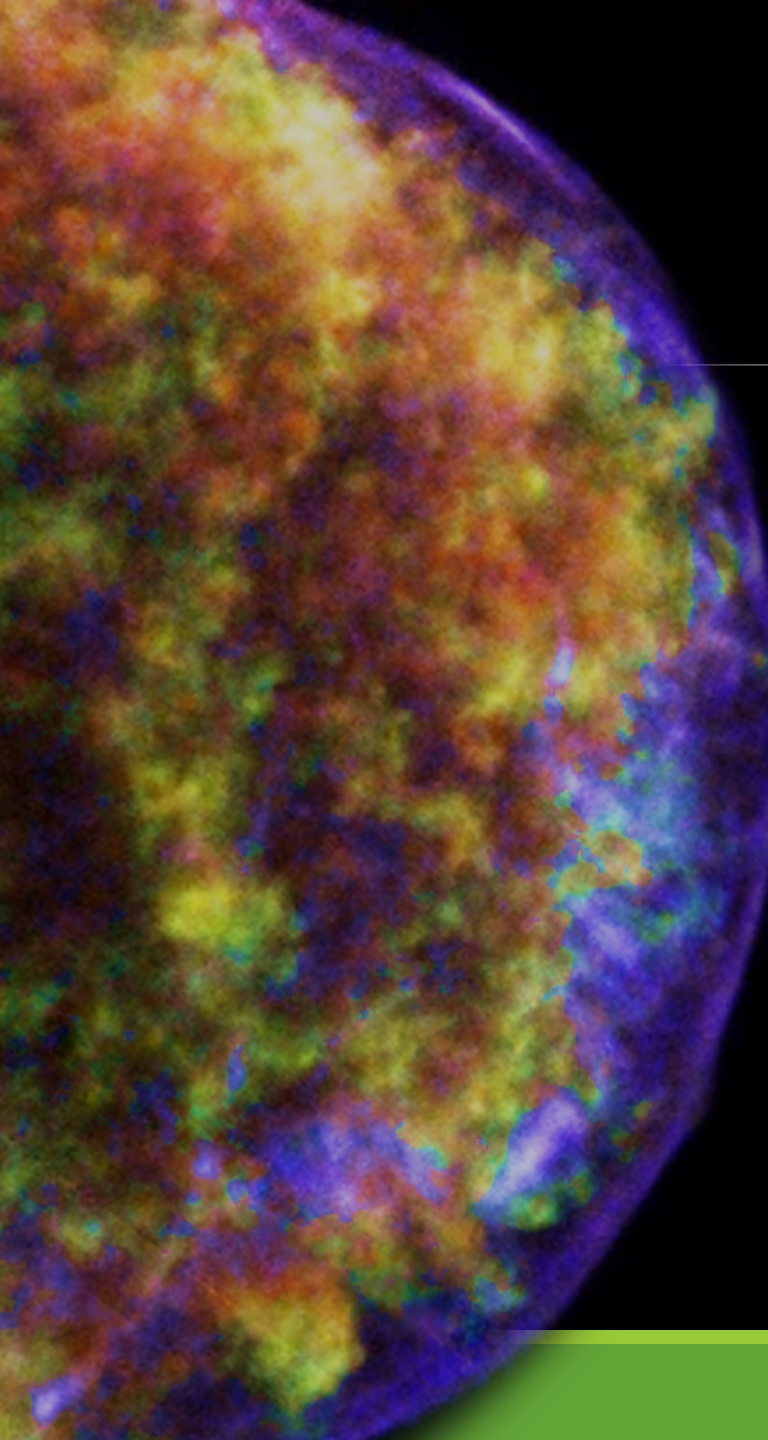
DSA predicts power law distributions of particles.*

$$\Phi \propto E^{-q}$$

$$q = \frac{R + 2}{R - 1}$$

For strong shocks, $q = 2$.
If acceleration is efficient, spectra will be flatter ($q < 2$).

*Fermi54, Krymskii77, Axford+77, Bell78, Blandford+78



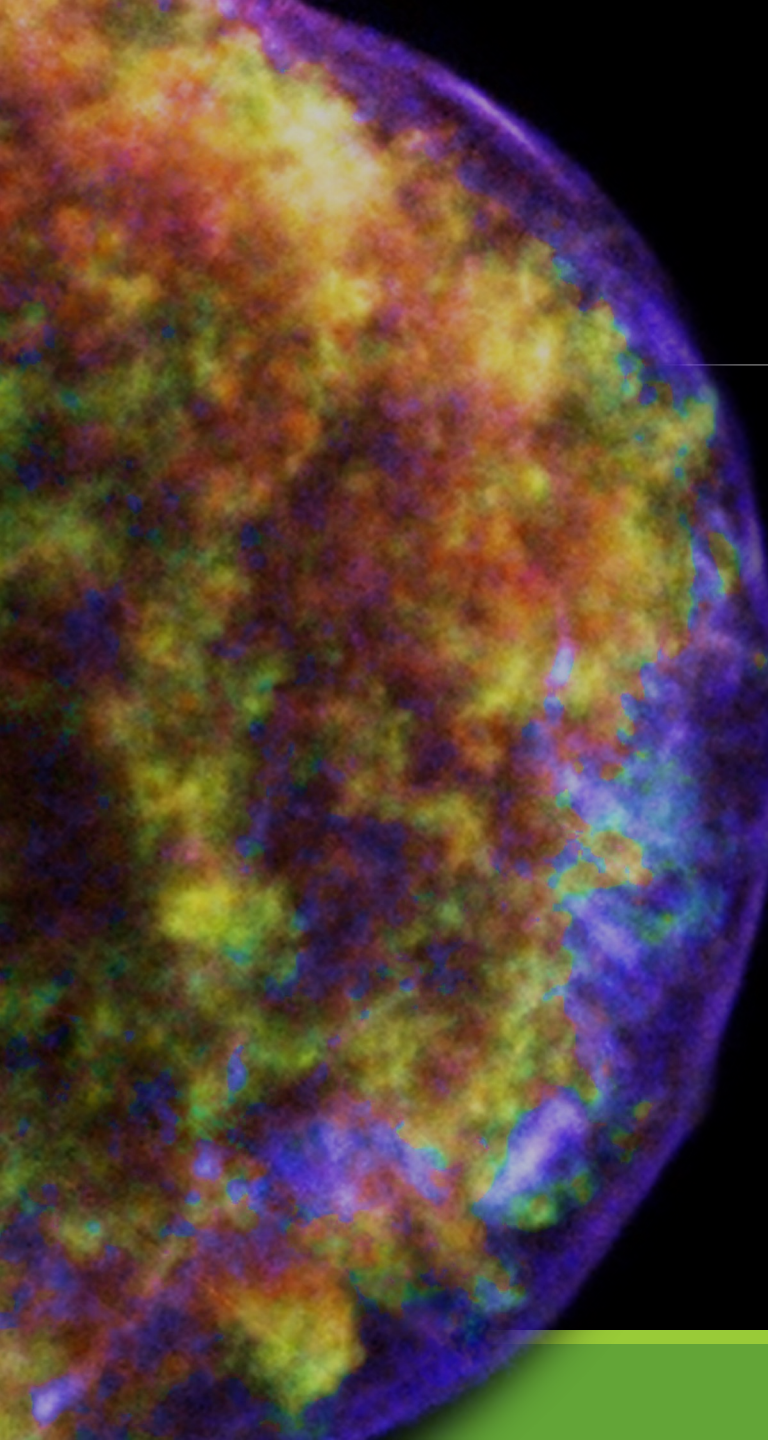
The maximum energy

LIMIT #1: AGE

The acceleration time must be shorter than the age of the system.

$$\tau_{\text{acc}} \sim \frac{D(E, B)}{v_{\text{sh}}^2} \leq \tau_{\text{age}}$$

Drury83



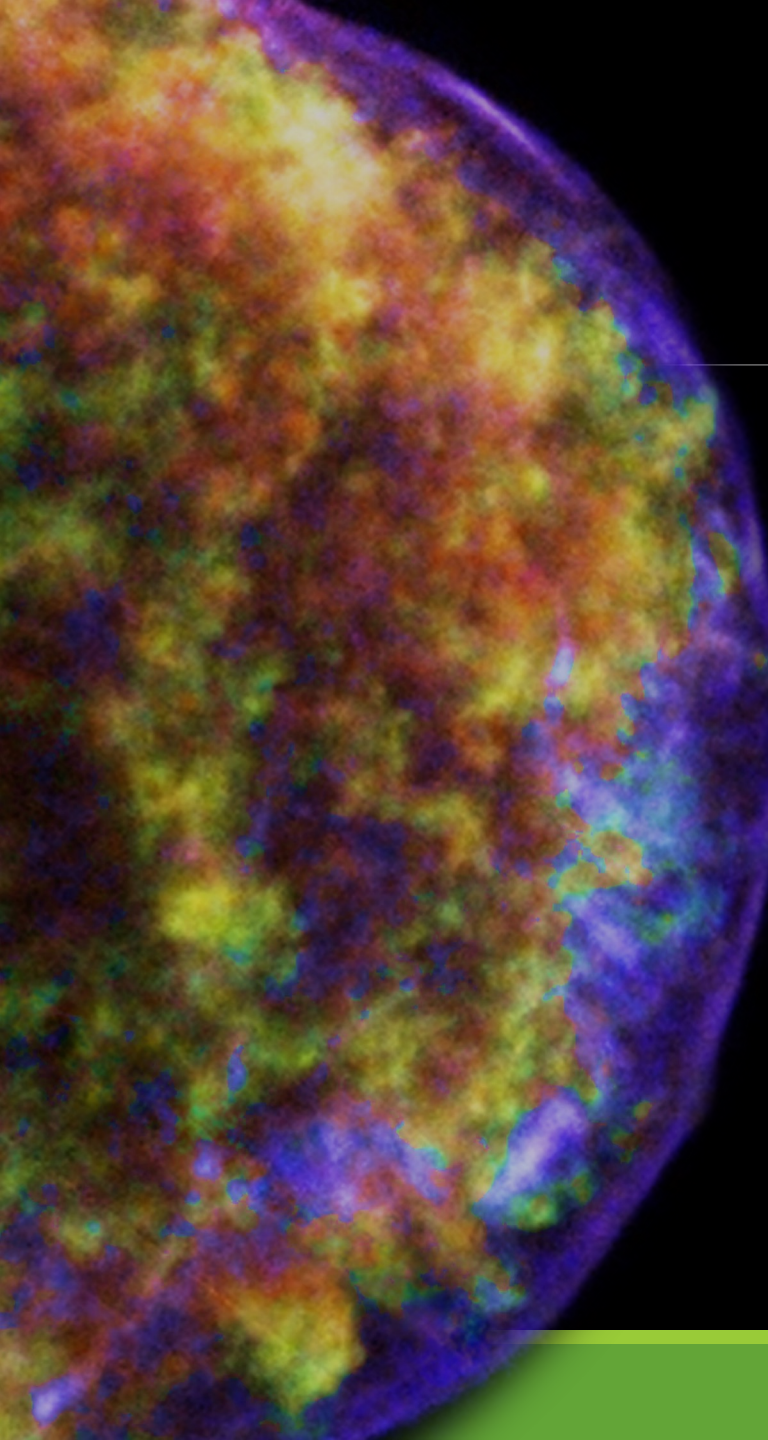
The maximum energy

LIMIT #2: CONFINEMENT

The diffusion length must be smaller than the size of the system.

$$\ell_{\text{diff}} \sim \frac{D(E, B)}{v_{\text{sh}}} \ll R_{\text{sh}}$$

Hillas84



The maximum energy

LIMIT #2: CONFINEMENT

The diffusion length must be smaller than the size of the system.

$$\frac{R_{\text{sh}}}{10} \approx \frac{D(E_{\text{max}}, B)}{v_{\text{sh}}}$$

Free-escape
boundary



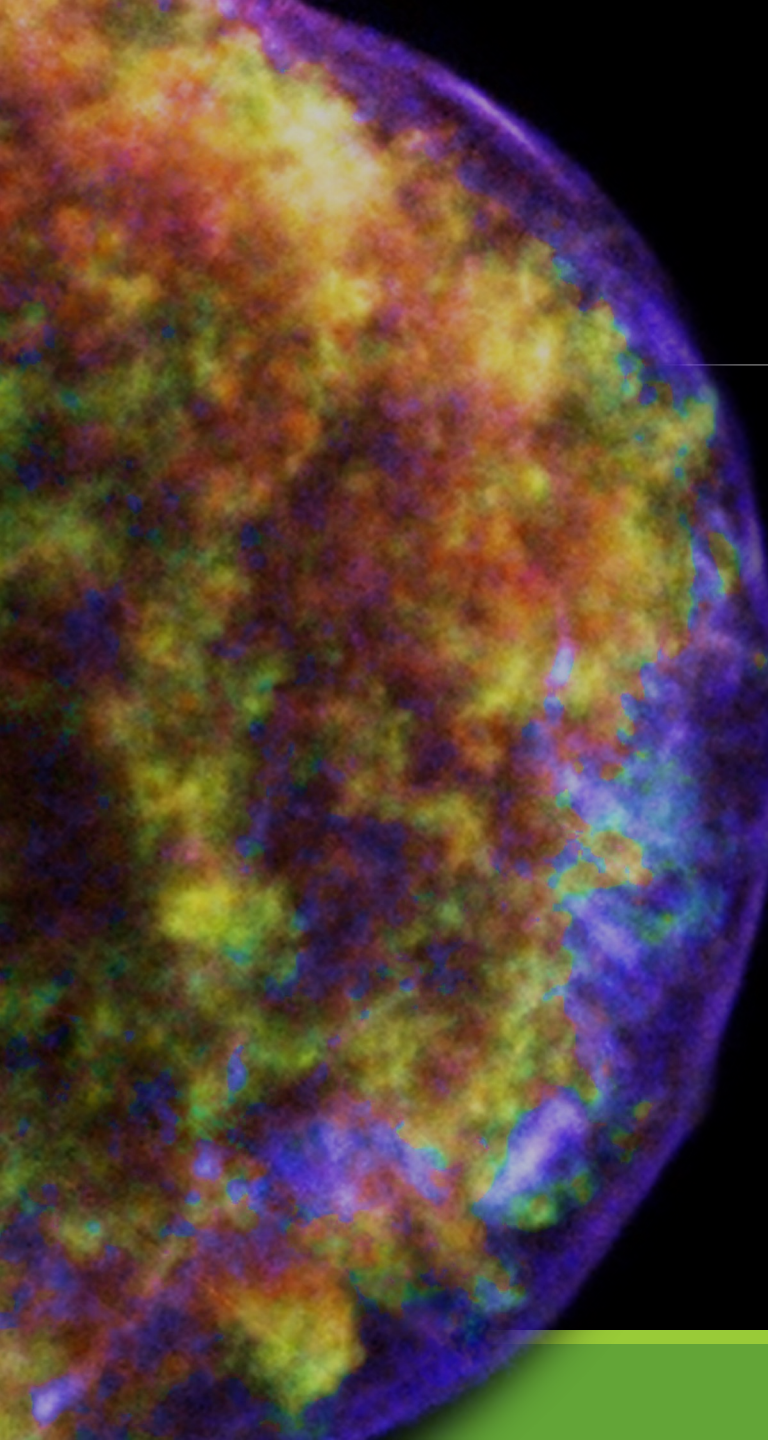
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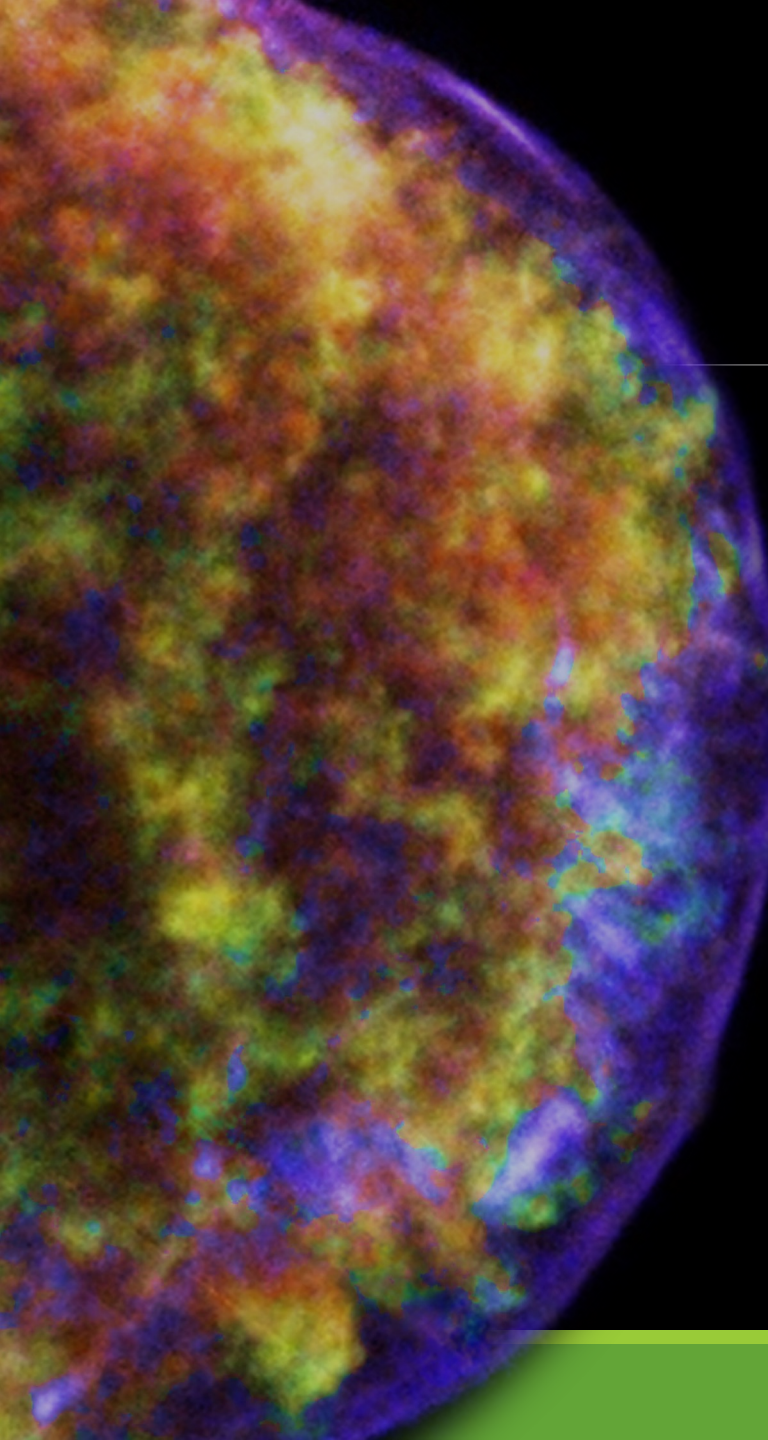
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Free-escape
boundary



The problem with the supernova remnant paradigm

LIMIT #2: CONFINEMENT

Assuming Bohm diffusion (optimistic),

$$E_{\text{max}} \approx 5 \text{ TeV} \left(\frac{v_{\text{sh}}}{5000 \text{ km s}^{-1}} \right) R_{\text{pc}} B_{\mu\text{G}}$$

This is inconsistent with TeV observations of SNRs!

A note about magnetic fields

Observations indicate that SNRs have enhanced magnetic fields ($\gtrsim 100\mu\text{G}$ compared to $\sim 3\mu\text{G}$ in the interstellar medium).*

*e.g., Bamba+05, Parizot+06, Morlino+10, Ressler+14

Thin synchrotron-emitting X-ray rims indicate electrons experience strong energy losses due to amplified magnetic field

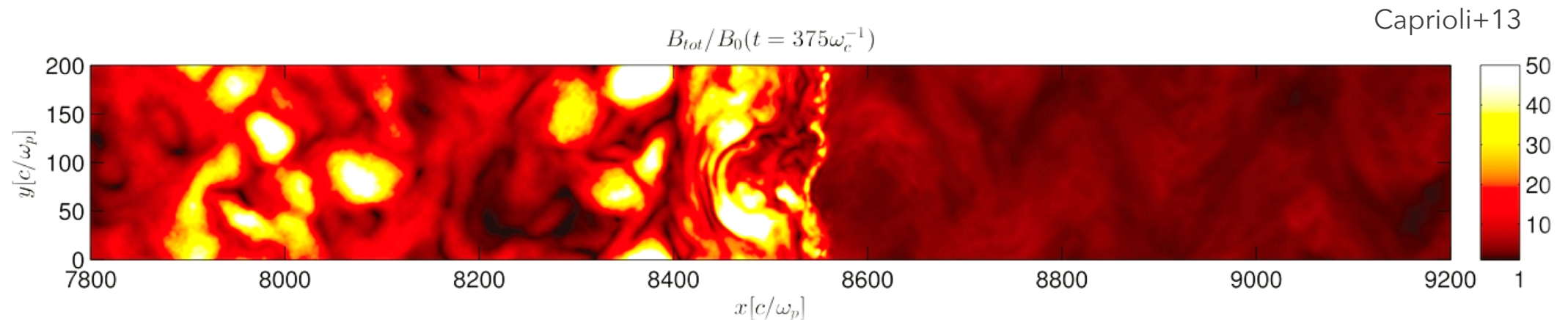
Tycho's SNR

Cassiopeia A

SN 1006

CR-Driven magnetic field amplification

CRs excite plasma instabilities which lead to turbulent, amplified magnetic fields.



Resonant: CRs excite waves with λ equal to their gyroradius; saturates when $\delta B/B \sim 1$.*

Non-resonant ("Bell"): CR currents upstream excite waves with initially small λ ; saturates when B-field pressure reaches equipartition with the net CR momentum flux.**

**e.g., Kulsrud+69, Skilling75, Bell78, Lagage+83; **Bell04, Zacharegkas+24

The problem with DSA

Observations point toward CR acceleration with spectra *steeper* than E^{-2} (i.e., $q > 2$).

The problem with DSA

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e.g., Caprioli11, Giordano+12, Saha+14, Aharonian+19

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1. **γ -ray emission from Galactic SNRs suggest $2.2 \lesssim q \lesssim 2.6$.**
e.g., Caprioli11, Giordano+12, Saha+14, Aharonian+19
2. **Radio emission from young extragalactic SNe (radio SNe) suggest $q \approx 3$.**
e.g., Chevalier+06, Chevalier+17, Soderberg+10, Soderberg+12, Kamble+16, Terreran+19

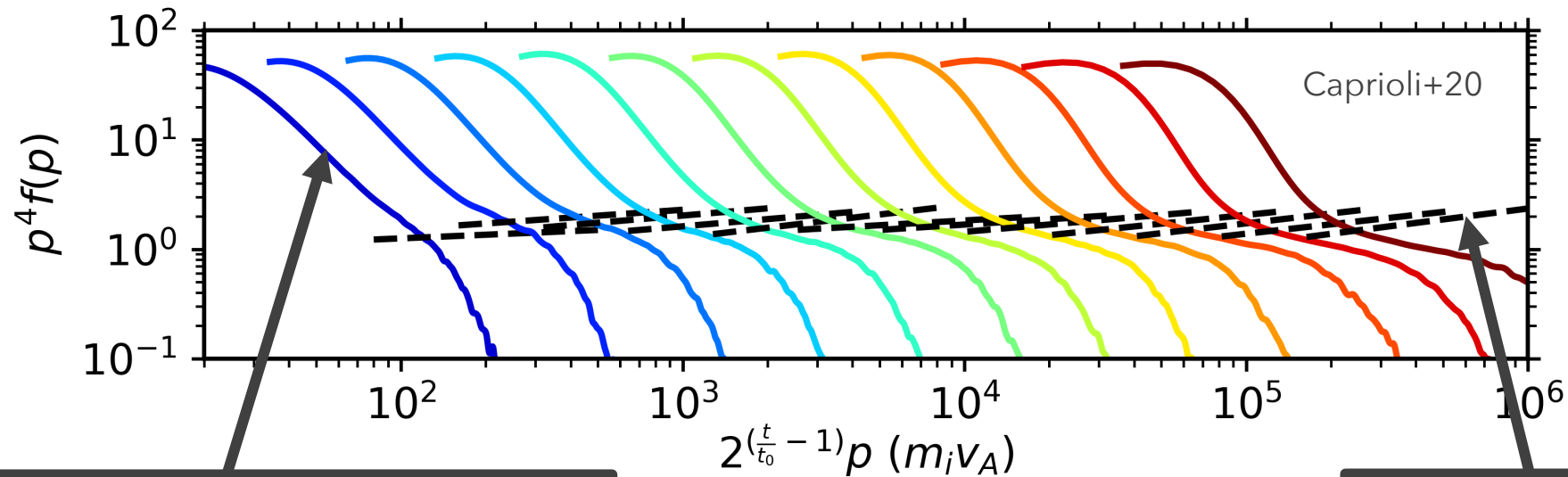
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e.g., Chevalier+06, Chevalier+17, Soderberg+10, Soderberg+12, Kamble+16, Terreran+19
3. **Observations of Galactic CRs require $2.3 \lesssim q \lesssim 2.4$.**
e.g., Evoli+19

Steep spectra in simulations

Kinetic simulations performed in Haggerty+20 and Caprioli+20 naturally reproduce steep spectra.

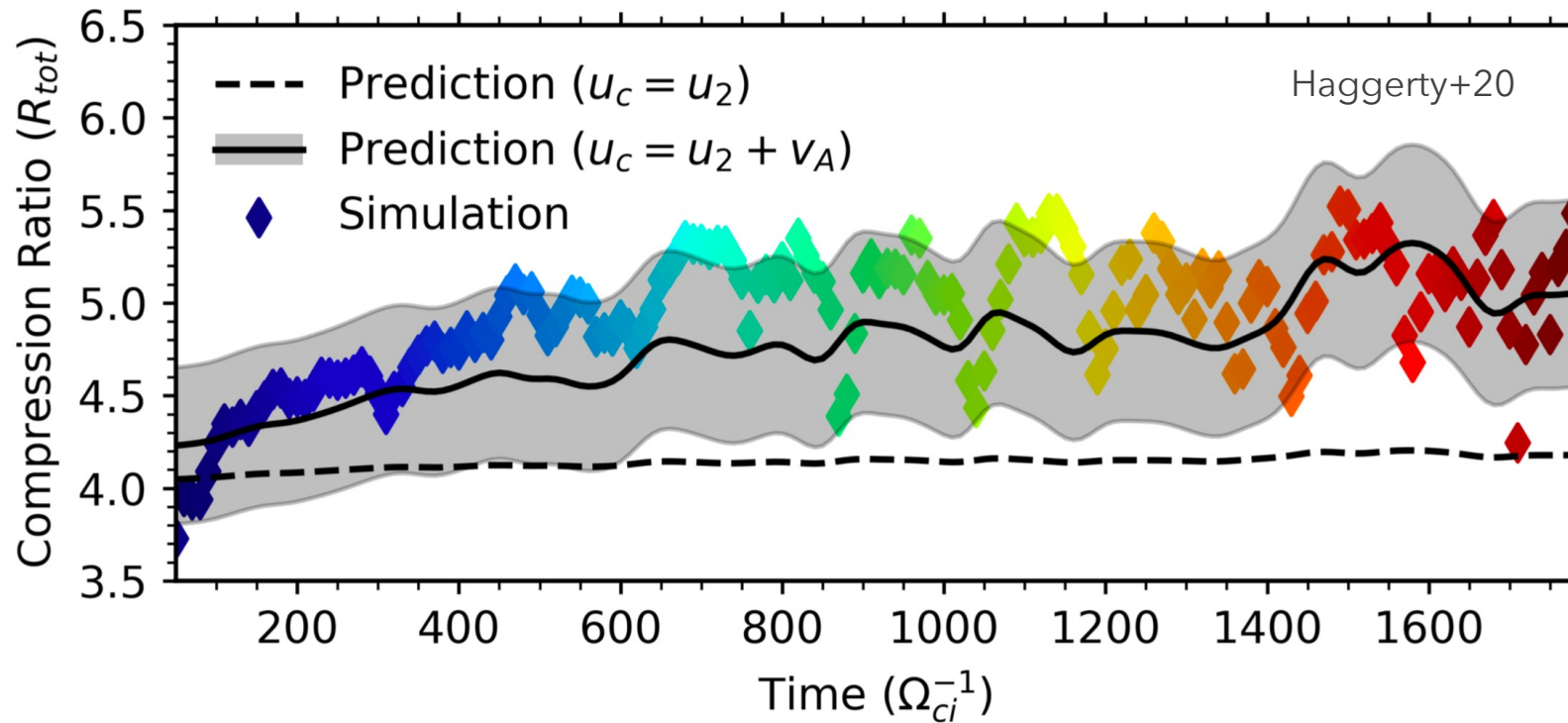


Simulated particle spectra (color scale denotes time)

Nonlinear DSA prediction

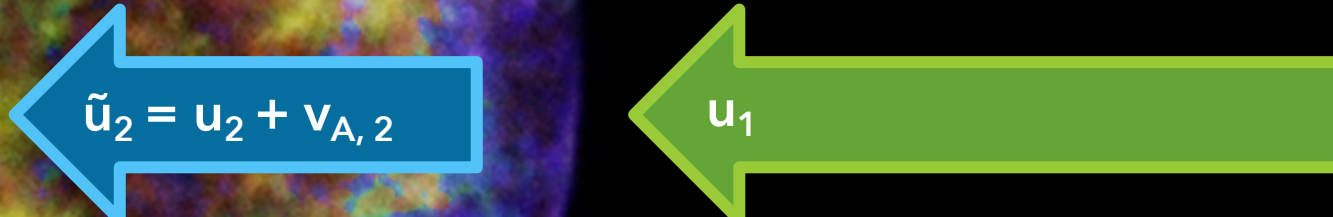
Enhanced compression ratios

Intriguingly, Haggerty+20 and Caprioli+20 also find fluid compression ratios significantly *larger* than 4.



The “postcursor”

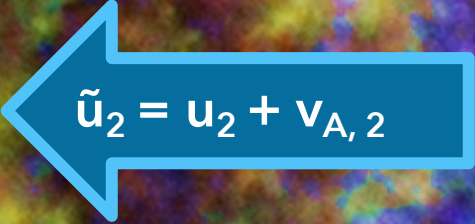
Magnetic fluctuations generated by cosmic rays (CRs) in the upstream retain their inertia over a non-negligible distance when advected into the downstream.


$$\tilde{u}_2 = u_2 + v_{A,2}$$

Fluctuations drift
at the local Alfvén
speed relative to
the plasma.

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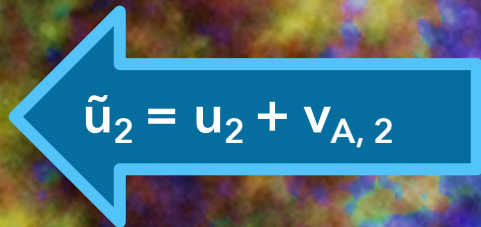
Fluctuations drift at the local Alfvén speed relative to the plasma.


$$u_1$$

CRs isotropize these magnetic fluctuations → a *postcursor* of drifting magnetic fluctuations and CRs enhances escape from the acceleration region, raising the fluid compression ratio while steepening the CR spectrum.

The “postcursor”

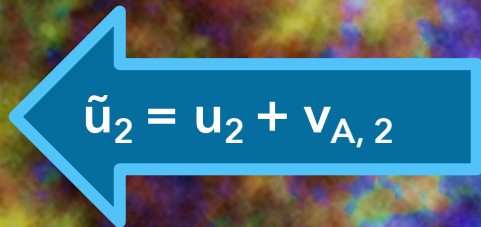
Equivalently, since particles are scattered by magnetic fluctuations, the postcursor modifies the compression ratio that particles “see.”



$$\tilde{R} = \frac{u_1}{u_2 + v_{A,2}}$$

The “postcursor”

Equivalently, since particles are scattered by magnetic fluctuations, the postcursor modifies the compression ratio that particles “see.”



$$\tilde{R} = \frac{u_1}{u_2 + v_{A,2}}$$

Efficient CR acceleration and thus B-field amplification yield $0.5 < v_{A,2}/u_2 < 1$ and spectra steeper than E^{-2} .

Modeling shock acceleration

Calculate the CR proton spectrum by solving the Parker transport equation.

Assume a fraction η of particles crossing the shock are injected into DSA.

$$\tilde{u}(x) \frac{\partial f(x, p)}{\partial x} = \frac{\partial}{\partial x} \left[D(x, p) \frac{\partial f(x, p)}{\partial x} \right] + \frac{p}{3} \frac{d\tilde{u}(x)}{dx} \frac{\partial f(x, p)}{\partial p} + Q(x, p)$$

Diagram illustrating the Parker transport equation components:

- Advection:** $\tilde{u}(x) \frac{\partial f(x, p)}{\partial x}$
- Diffusion:** $\frac{\partial}{\partial x} \left[D(x, p) \frac{\partial f(x, p)}{\partial x} \right]$
- Adiabatic compression:** $\frac{p}{3} \frac{d\tilde{u}(x)}{dx} \frac{\partial f(x, p)}{\partial p}$
- Injection:** $Q(x, p)$

The equation is solved for the CR proton spectrum $f(x, p)$.

Modeling shock acceleration

Calculate the CR proton spectrum by solving the Parker transport equation.

To include a postcursor, we consider $\tilde{u}(x)$, the velocity of magnetic scattering centers.

The diagram illustrates the Parker transport equation with four color-coded components and arrows indicating their physical processes:

- Advection:** Represented by the first term $\tilde{u}(x) \frac{\partial f(x, p)}{\partial x}$ in a green box, with a green arrow pointing up to it.
- Diffusion:** Represented by the second term $\frac{\partial}{\partial x} \left[D(x, p) \frac{\partial f(x, p)}{\partial x} \right]$ in a green box, with a green arrow pointing up to it.
- Adiabatic compression:** Represented by the third term $+\frac{p}{3} \frac{d\tilde{u}(x)}{dx} \frac{\partial f(x, p)}{\partial p}$ in a teal box, with a teal arrow pointing up to it.
- Injection:** Represented by the fourth term $+Q(x, p)$ in a blue box, with a blue arrow pointing up to it.

Arrows from the text box above point to the first and third terms of the equation.

$$\tilde{u}(x) \frac{\partial f(x, p)}{\partial x} = \frac{\partial}{\partial x} \left[D(x, p) \frac{\partial f(x, p)}{\partial x} \right] + \frac{p}{3} \frac{d\tilde{u}(x)}{dx} \frac{\partial f(x, p)}{\partial p} + Q(x, p)$$

Modeling shock acceleration

Calculate the CR proton spectrum by solving the Parker transport equation.

Assume Bohm diffusion (optimistic, but consistent with sims of parallel shocks*)

$$\tilde{u}(x) \frac{\partial f(x, p)}{\partial x} = \frac{\partial}{\partial x} \left[D(x, p) \frac{\partial f(x, p)}{\partial x} \right] + \frac{p}{3} \frac{d\tilde{u}(x)}{dx} \frac{\partial f(x, p)}{\partial p} + Q(x, p)$$

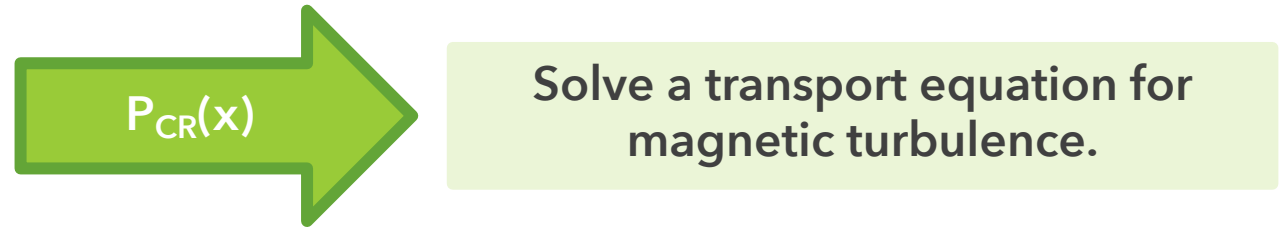
Diagram illustrating the Parker transport equation components:

- Advection** (green arrow pointing to the left-hand side term)
- Diffusion** (green arrow pointing to the diffusion term)
- Adiabatic compression** (teal arrow pointing to the adiabatic compression term)
- Injection** (blue arrow pointing to the source term $Q(x, p)$)

*e.g., Caprioli+14

Modeling shock acceleration

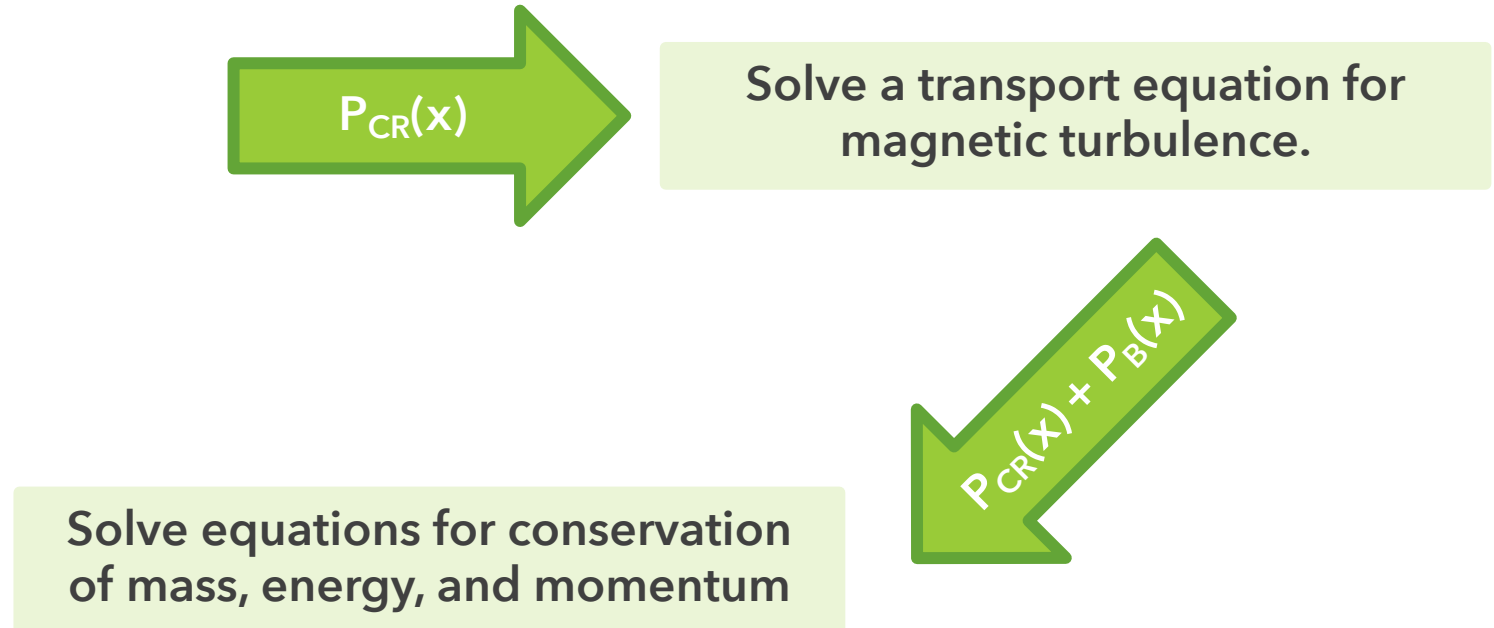
Use a semi-analytic model of non-linear DSA which self-consistently accounts for particle acceleration and magnetic field amplification.



See also Amato+06, Caprioli+10; Caprioli12.

Modeling shock acceleration

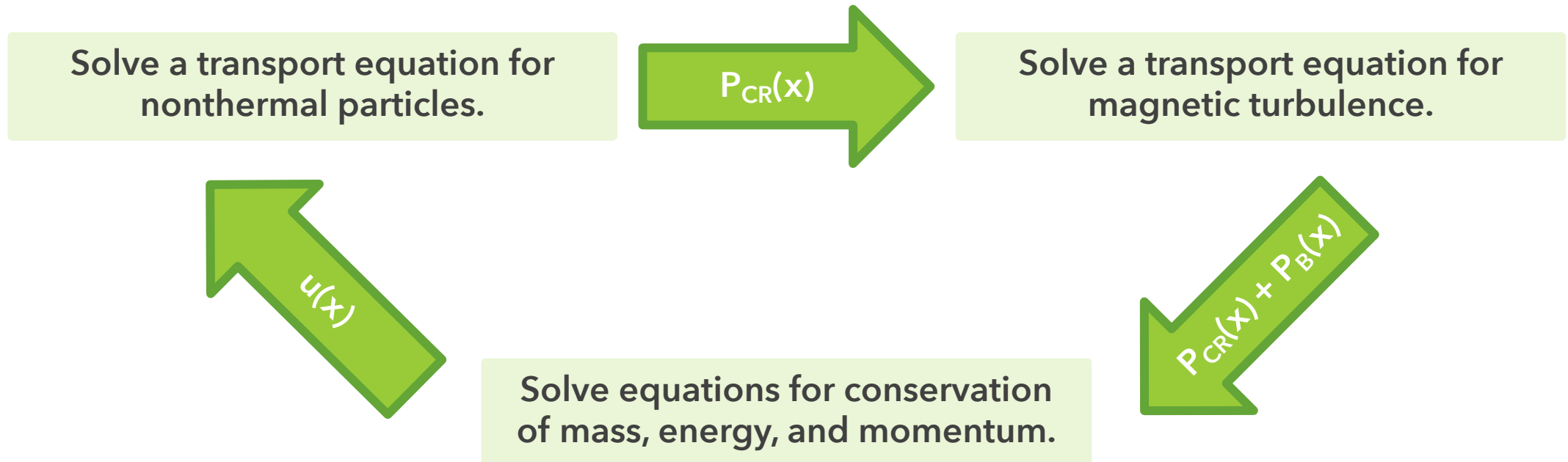
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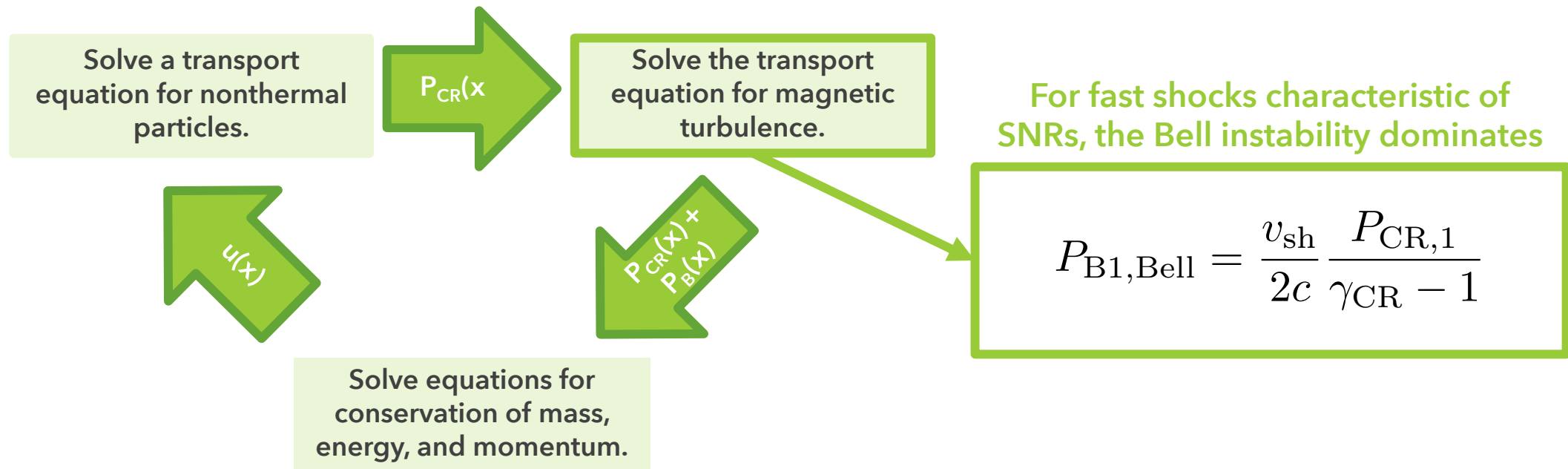
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Magnetic field amplification

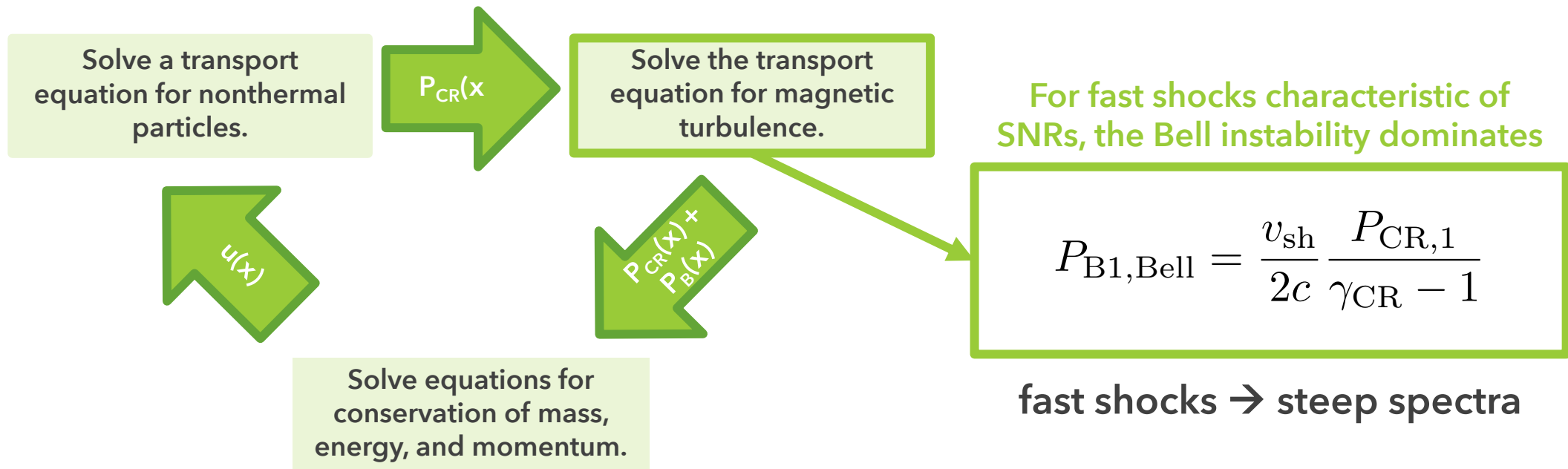
We model magnetic field amplification by assuming contributions from both the resonant streaming instability* and the non-resonant (Bell) instability.**



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Magnetic field amplification

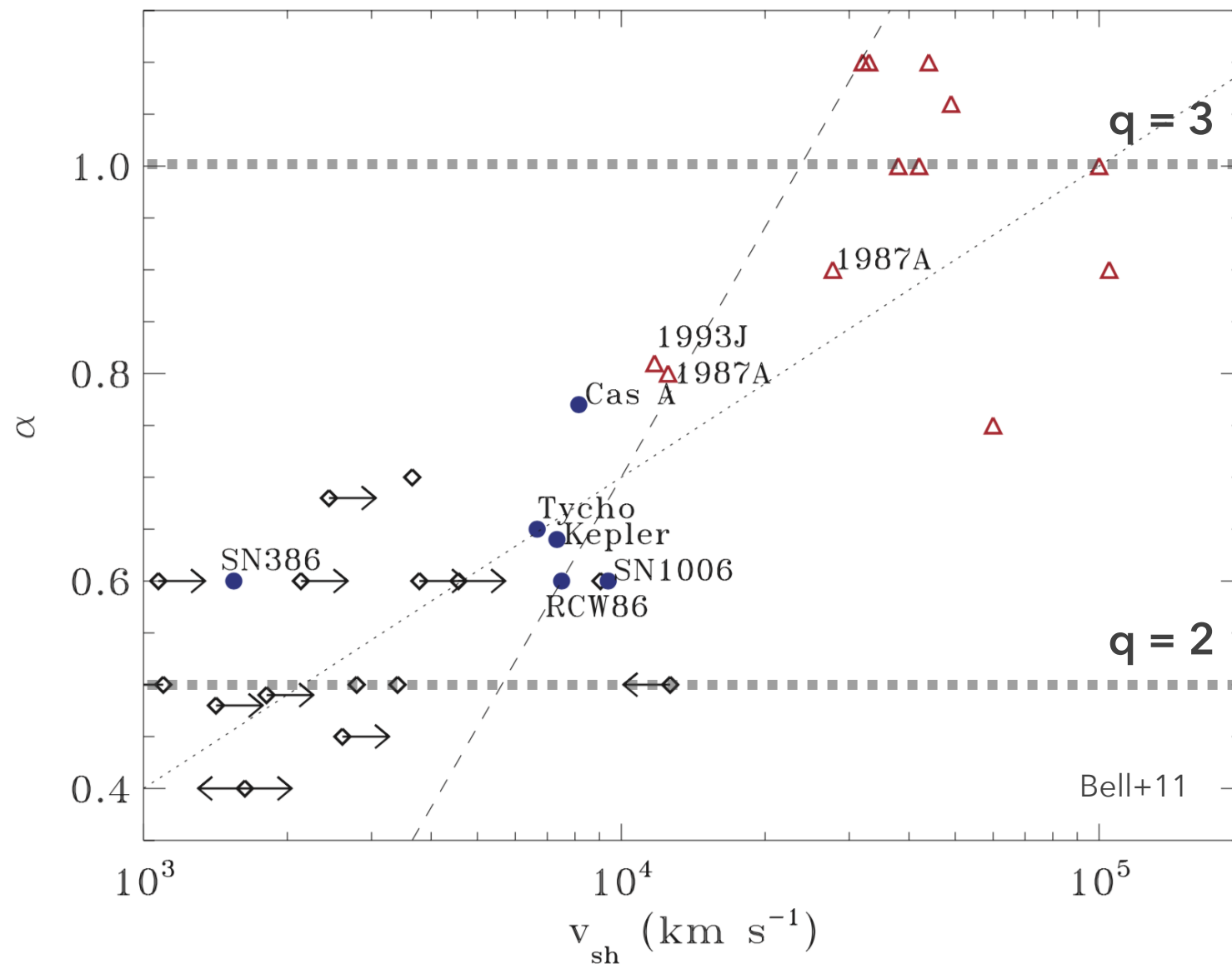
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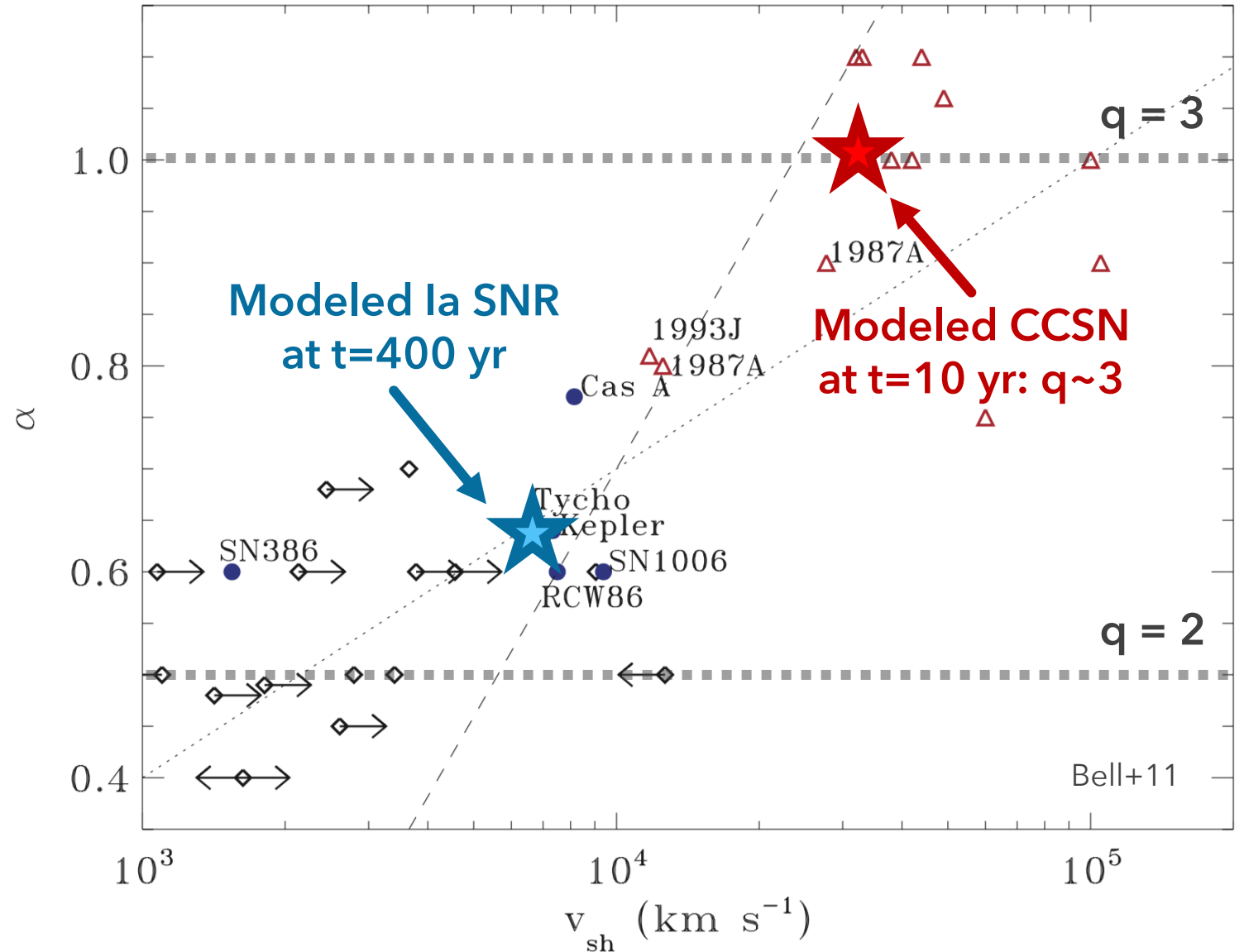
Comparison
to observations:

Fast shocks $\rightarrow$
steep spectra



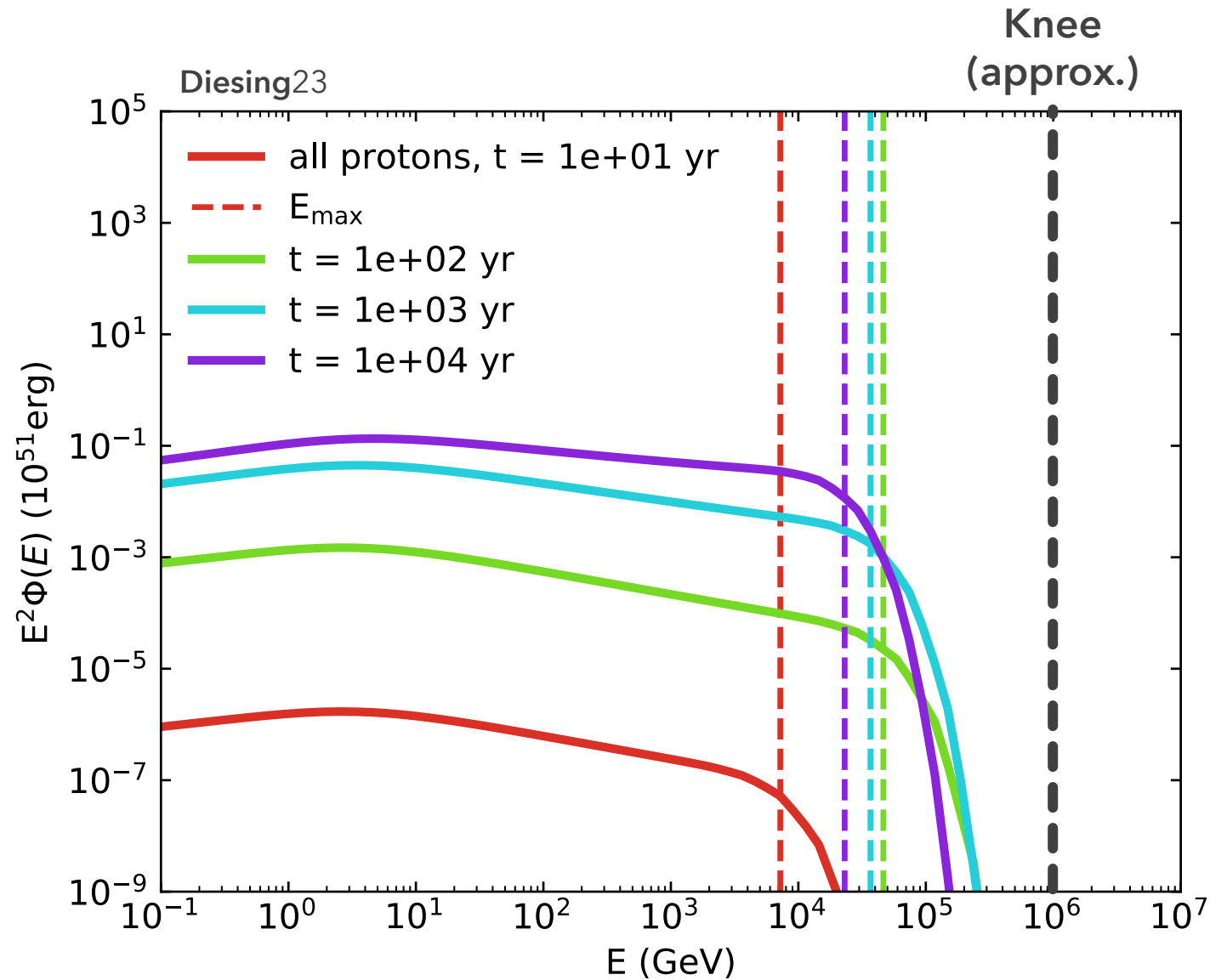
Comparison
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Can supernova remnants reach the knee?

Typically, no.

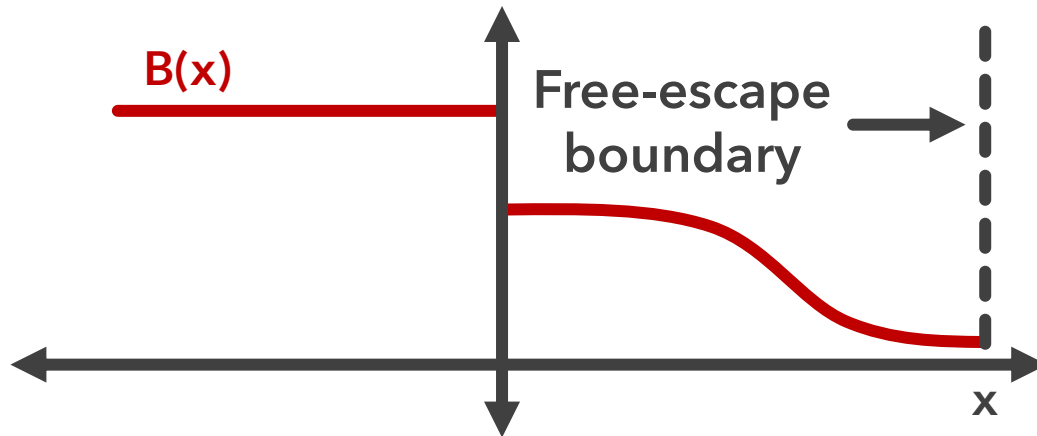


Bracketing uncertainties

To account for uncertainties in the nature of magnetic field amplification (specifically, what sets the saturation point of the non-resonant instability), we consider two cases:

CASE 1: THE "PESSIMISTIC" SCENARIO

Amplification is driven by diffusing particles and therefore takes place near the shock $\rightarrow P_B(x) \sim P_{CR}(x)$ upstream.

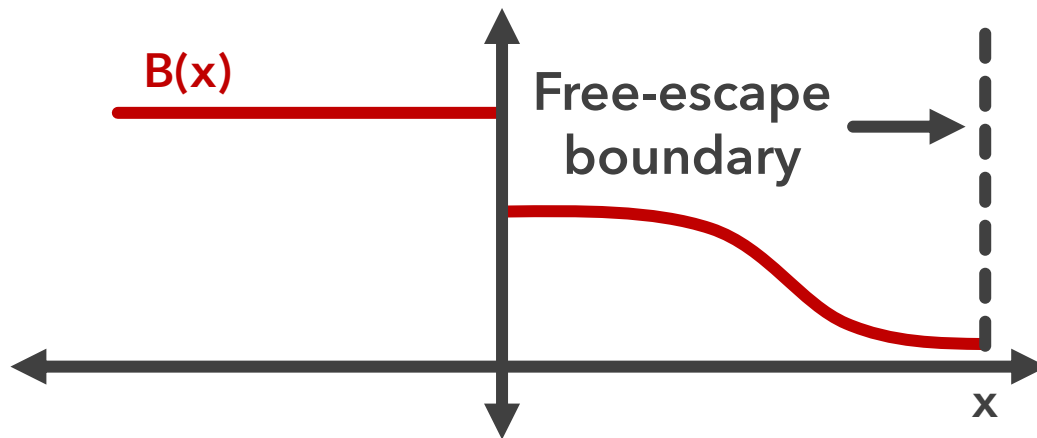


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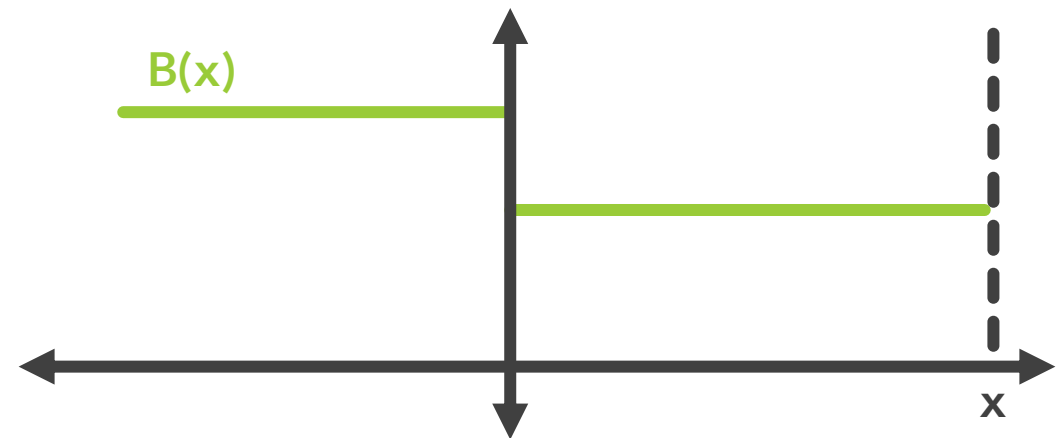
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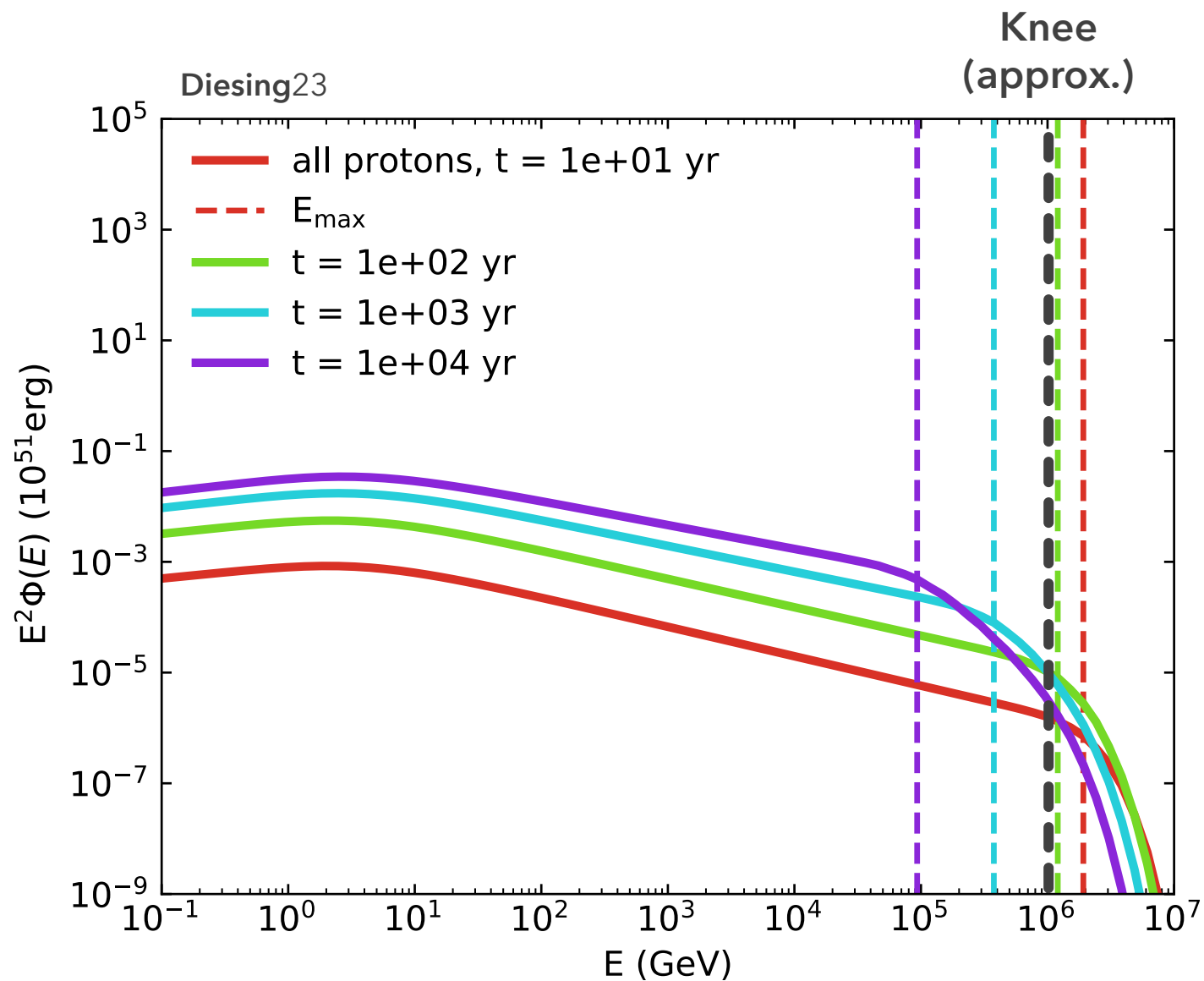
CASE 2: THE "OPTIMISTIC" SCENARIO

Amplification is driven by escaping particles and therefore takes place far upstream $\rightarrow P_B(x) \sim \text{constant}$ upstream.



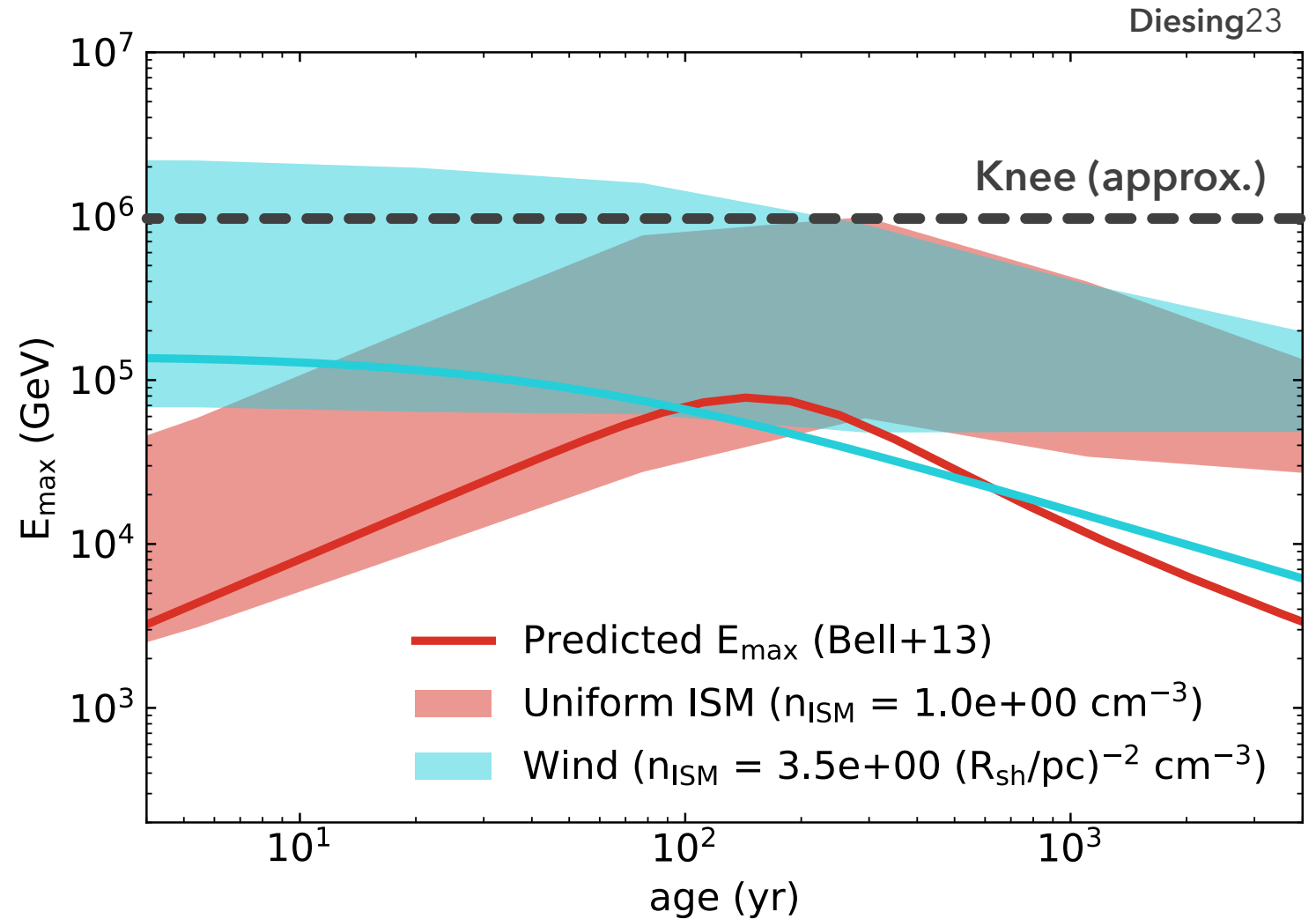
Can supernova remnants accelerate PeV protons?

Optimistically, yes.



Can supernova remnants accelerate PeV protons?

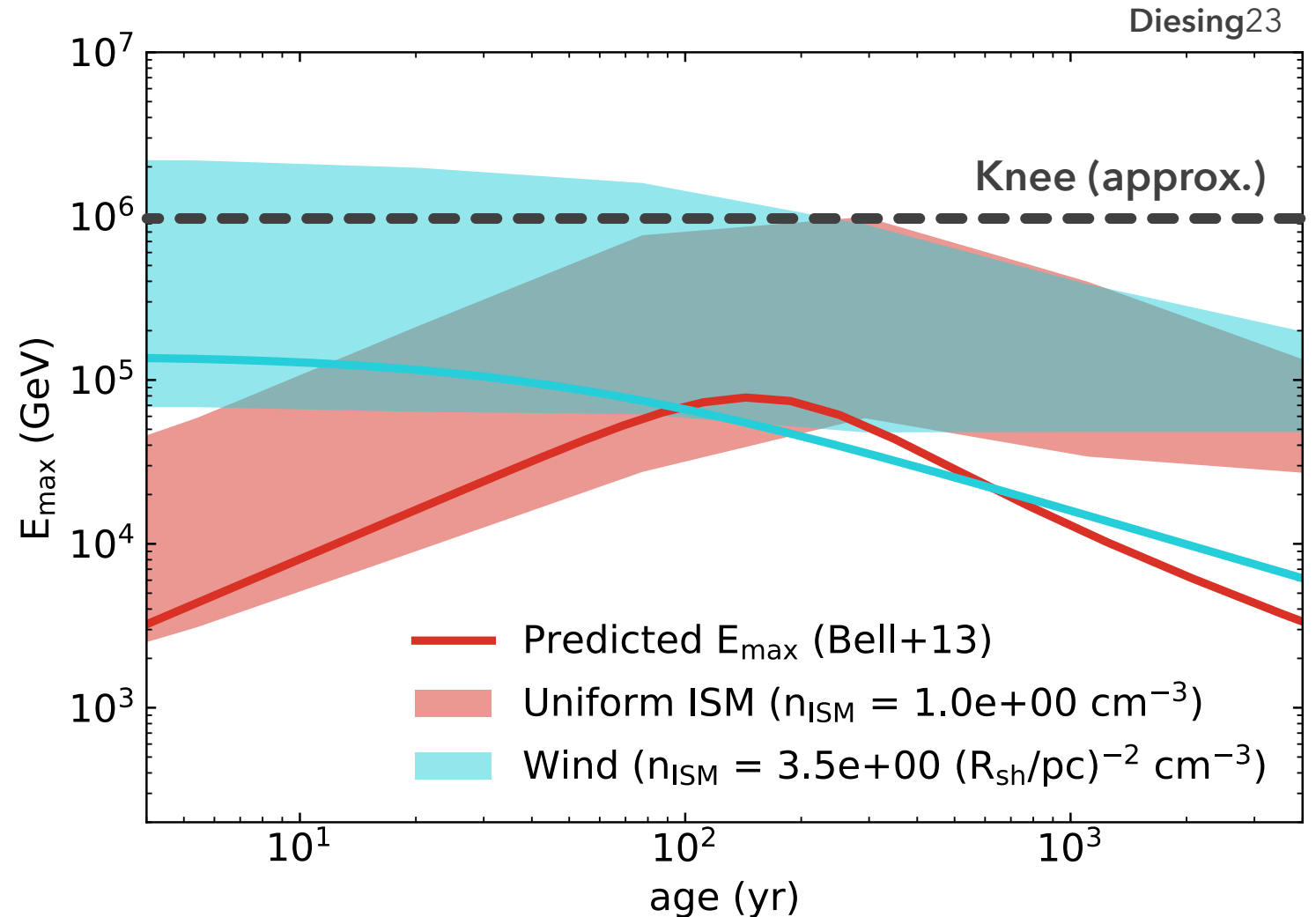
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Are supernova remnants the main source of PeV cosmic rays?

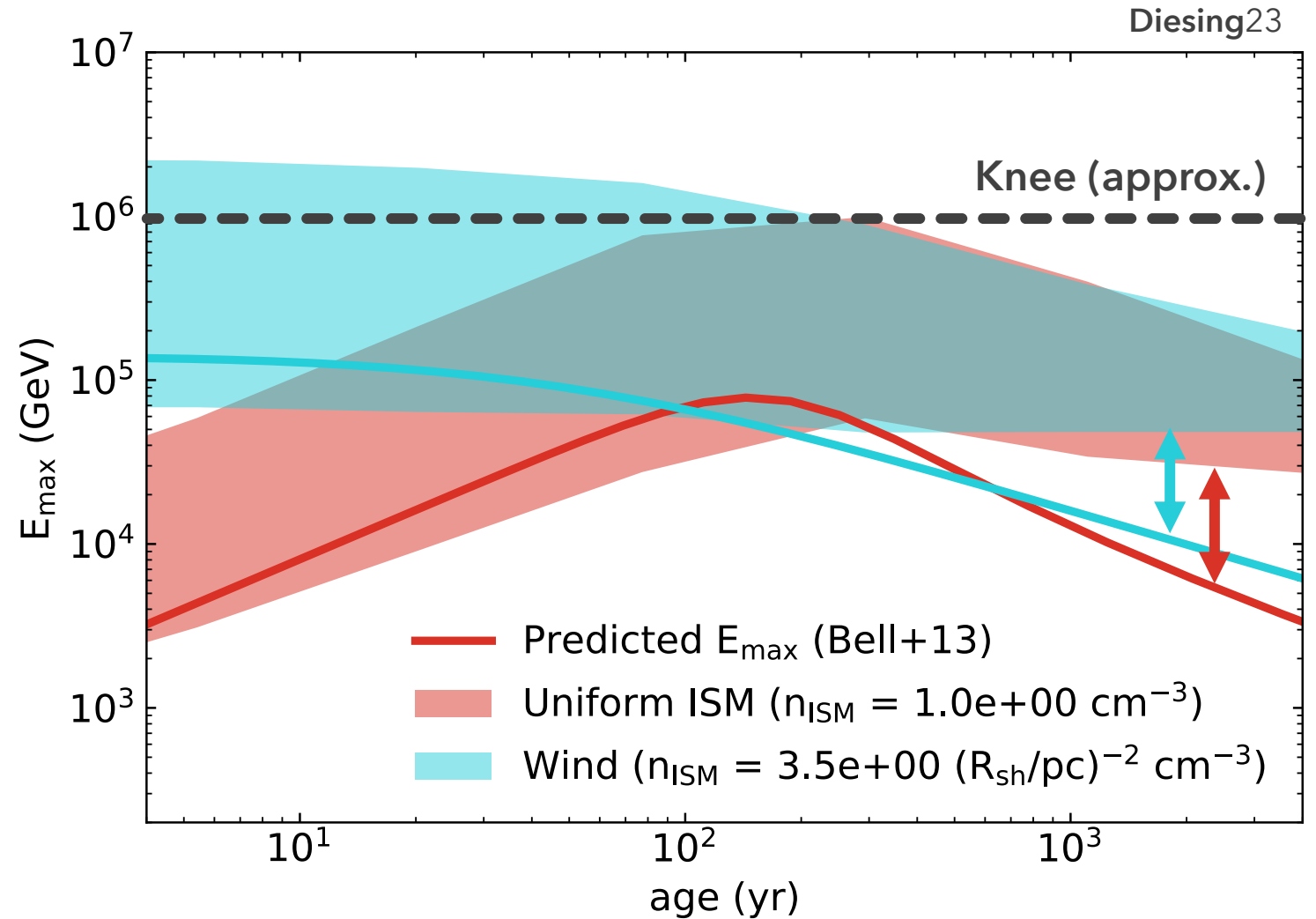
No.

To accelerate PeV particles, supernova remnants must be exceptionally young, fast, and expanding into dense media.



A consideration for observers

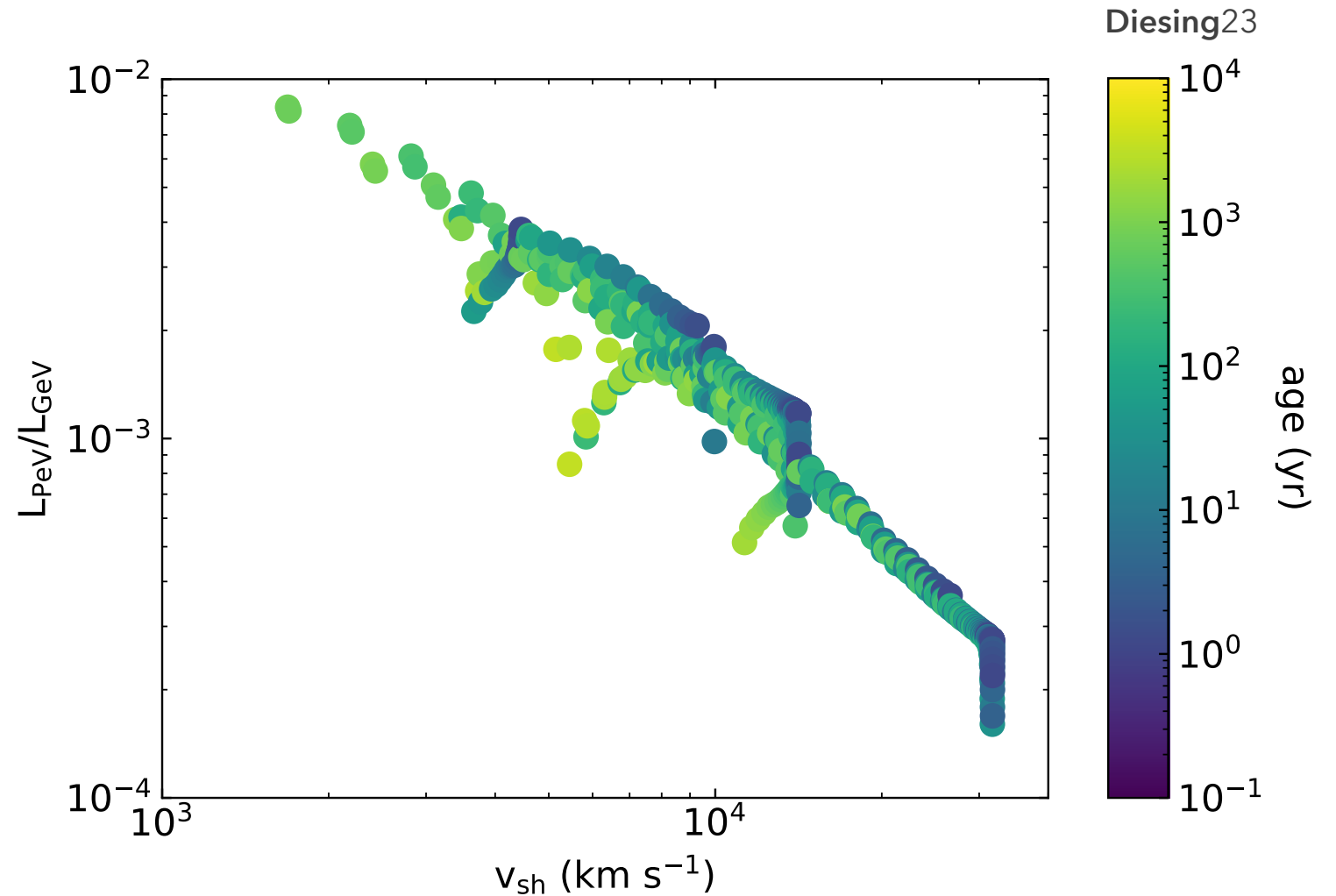
Particles accelerated at early times contribute to a higher maximum energy at later times, and may produce observable gamma-ray signatures.



Are supernova remnants the main source of PeV cosmic rays?

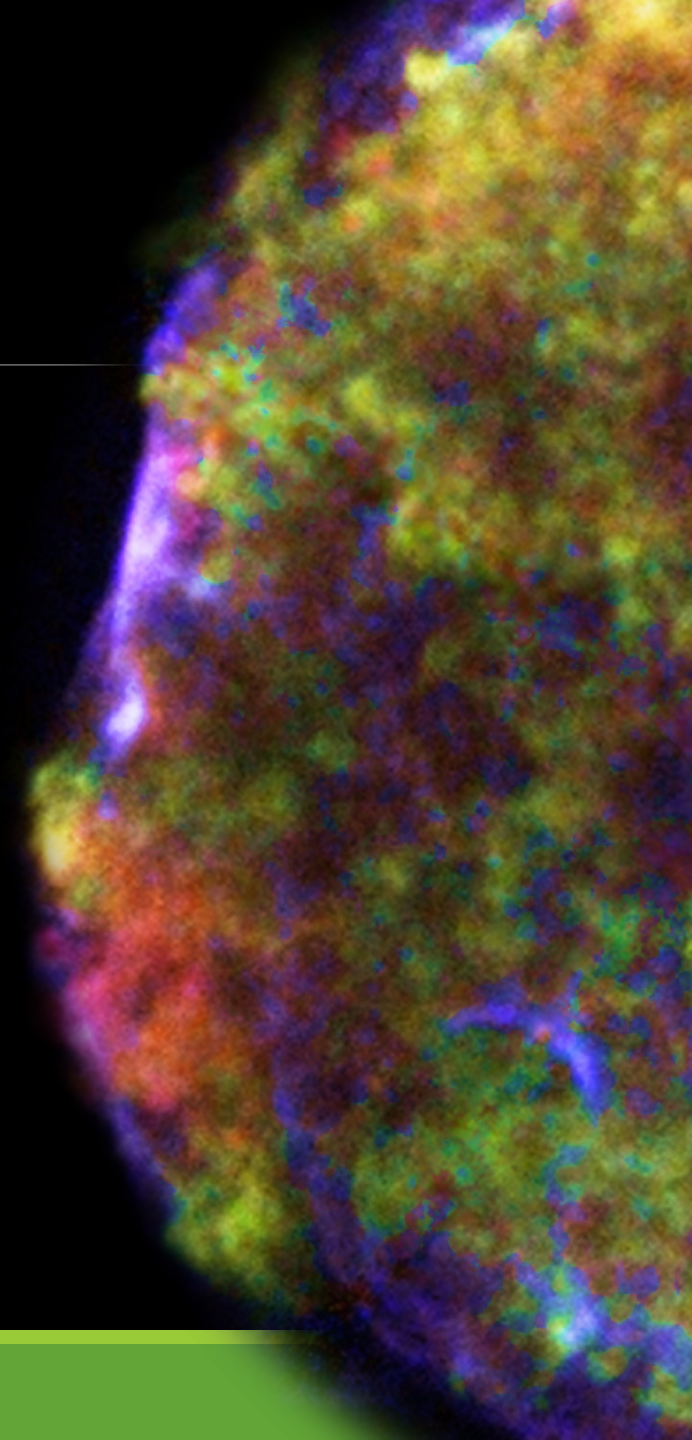
No.

In general, faster shocks are more likely to be PeVatrons. However, they also have steeper spectra.



Summary

1. Can supernova remnants accelerate PeV protons? **Yes.*†**

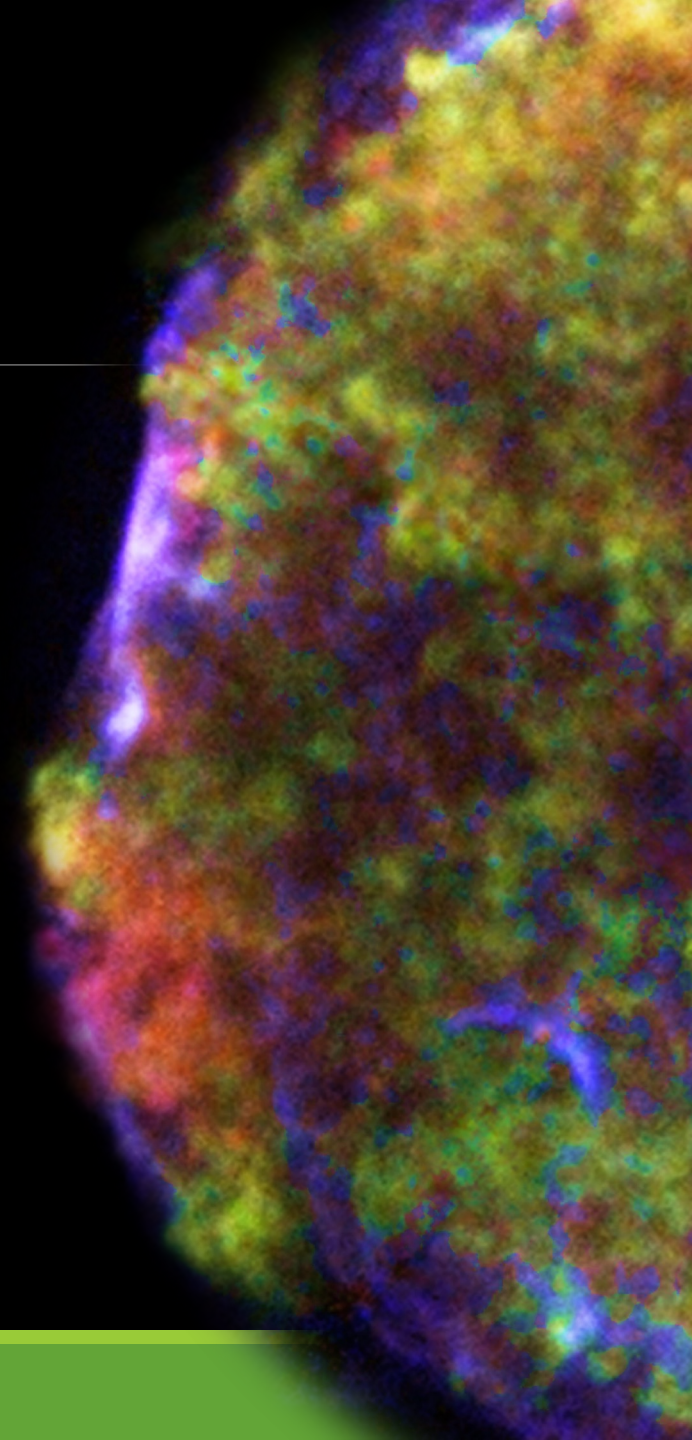


Summary

1. Can supernova remnants accelerate PeV protons? **Yes.*†**

*Only under optimistic assumptions about B-field amplification

†Only young, fast CCSNe ($v_{sh} \gtrsim 10,000 \text{ km s}^{-1}$, $n_{ISM} \gtrsim 1 \text{ cm}^{-3}$)



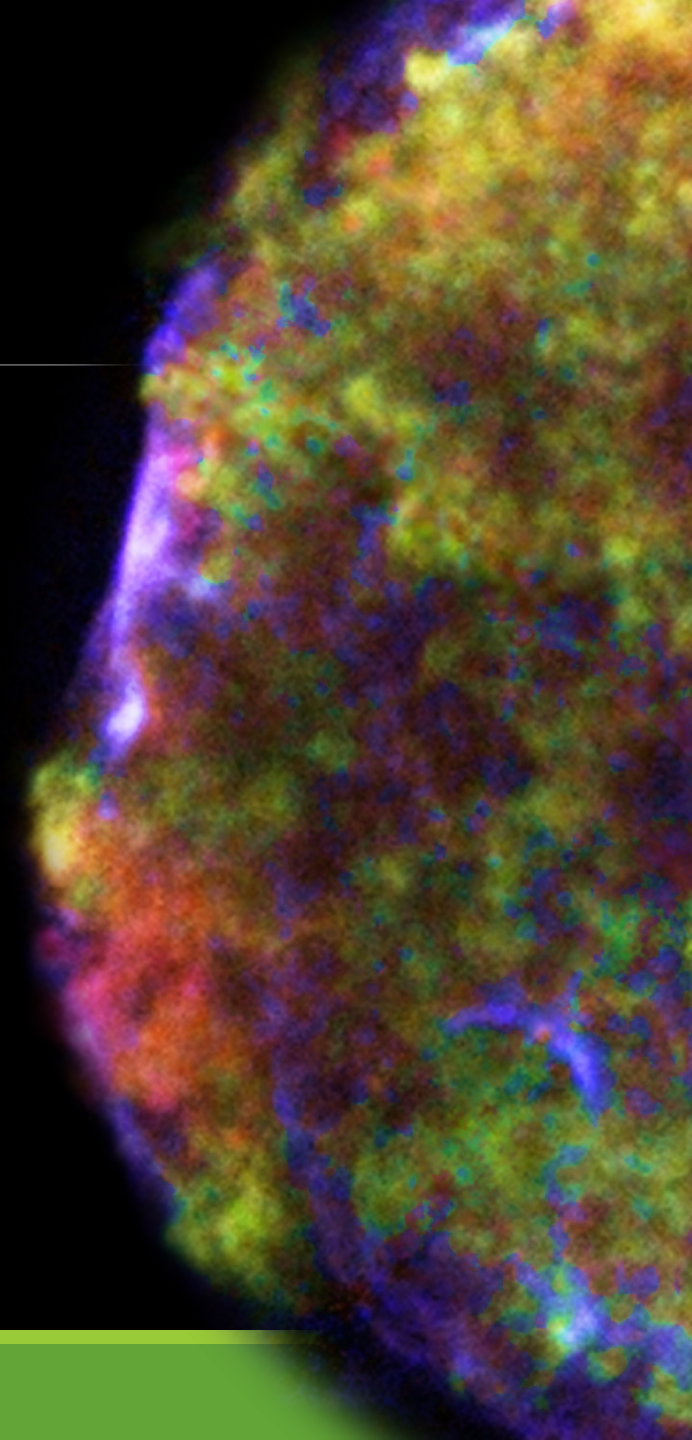
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2. Are supernova remnants the main source of PeV cosmic rays? **No.**

Even under optimistic assumptions, PeVatron SNRs are too short-lived to reproduce the PeV flux

Fast shocks tend to produce steep particle spectra.

